1) $$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

Initial cond $u = \begin{cases} 0 & 0 \le x < 0.4 \\ 1 & 0.4 \le x \le 0.6 \\ 0 & 0.6 \le x \le 1 \end{cases}$

FTCS, 64 grid point $\Delta t = \Delta x$.

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \nu \frac{\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)}{(\Delta x)^2}$$

$$u_i^{n+1} = u_i^n \quad \frac{\nu \Delta t}{(\Delta x)^2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

2) $$\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}.$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -c \frac{u_i^n - u_{i-1}^n}{\Delta x}.$$

$$u_i^{n+1} = u_i^n - c\left(\frac{\Delta t}{\Delta x}\right)\left(\frac{u_i^n - u_{i-1}^n}{\Delta x}\right)$$

3) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -u_i^n\left(\frac{u_i^n - u_{i-1}^n}{\Delta x}\right) + \nu\left(\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}\right)$$

Lax W $$\frac{u_j^{n+1} - u_j^n}{\Delta t} + u_j^n \frac{U_{j+1}^n - U_{j-1}^n}{2\Delta x} - \frac{(u_j^n)^2}{2\Delta x^2}\left(U_{j+1}^n - 2U_j^n + U_{j-1}^n\right)$$

$$= \frac{\nu}{2(\Delta x)^2}\left(U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}\right)$$

$$= \nu\, (u_{j+1}^n)^2.$$

~~Ritchmayer~~

$$\cancel{u_{i+1/2}^{n+1/2} = \frac{(u_{i+1}^{n} + u_{i-1}^{n})}{2} - \frac{\Delta t}{2\Delta x}\left(f(u_{i+1}^{n}) - f(u_i^{n})\right)}$$

$$\cancel{u_{i+1}^{n+1/2} = \frac{u_{i+1}^{n} + u_{i-1}^{n}}{2} - \frac{\Delta t}{2\Delta x}\left[-u_i^{n}(u_{i+1}^{n} - u_i^{n})\right] - \frac{\Delta t\,\nu}{2(\Delta x)^2}\left[u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}\right]}$$

$$\cancel{u_{i+1}^{n+1/2} = u_i^{n} + u_{i+1}}$$

Rachtmayer

$$u_i^{n+1/2} = \frac{1}{2}(u_{i+1}^{n} + u_{i-1}^{n}) - \frac{u_i^{n}\Delta t}{2\times 2\Delta x}(u_{i+1}^{n} - u_{i-1}^{n}) + \frac{\nu\Delta t}{2\times 2(\Delta x)^2}\left[u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}\right]$$

$$u_i^{n+1} = u_i^{n} - \frac{u_i \Delta t}{2\Delta x}\left(u_{i+1}^{n+1/2} - u_{i-1}^{n+1/2}\right) + \frac{\nu\Delta t}{2(\Delta x)^2}\left[u_{i+1}^{n+1/2} - 2u_i^{n+1/2} + u_{i-1}^{n+1/2}\right]$$

Mackcormac

$$u_i^{*} = u_i^{n} - \frac{u_i\Delta t}{\Delta x}(u_{i+1}^{n} - u_i^{n}) + \frac{\nu\Delta t}{(\Delta x)^2}\left(u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n}\right)$$

$$\cancel{u_i^{n+1} = u_i^{n+1/2}}$$

$$u^{n+1/2} = \frac{(u_i^{n} + u_i^{*})}{2}$$

$$u_i^{n+1} = u_i^{n+1/2} - \frac{u_i\Delta t}{2\Delta x}(u_i^{*} - u_{i-1}^{*}) + \frac{\nu\Delta t}{2(\Delta x)^2}\left[u_{i+1}^{*} - 2u_i^{*} + u_{i-1}^{*}\right]$$

Beam Warming Scheme

$$u_i^{n+1} = u_i^n - \frac{u\Delta t}{\Delta x}\left(3u_i^n - 4u_{i-1}^n + u_{i-2}^n\right)$$
$$+ \frac{u^2\Delta t^2}{2\Delta x^2}\left(u_i^n - 2u_{i-1}^n + u_{i-2}^n\right)$$
$$+ \nu\frac{\Delta t}{\Delta x^2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$