Applied CFD (AE-661A)

Term: Spring 2023

Instructor: Prof. Rajesh Ranjan

Assignment-II/Home Exam

Topics: Numerical Schemes and Solution

Due Date: 12 Noon, Apr. 11, 2023 **Total points:** 50 (Wt: 10%)

Instructions

- Write your name and roll number as the first line of your each code.
- There should be one code for each problem below and they should be called as pbm1_#roll.ext, pbm2_#roll.ext, pbm3_#roll.ext ('ext' is the extension of file). All subproblems should be employed as functions.
- You can employ any coding language (MATLAB/Python/Julia) of your choice.
- You can submit codes and report online. No need to submit a hard copy.
- You will get partial marking for good honest attempts. However, if two codes are found to be copied from one another, both will get **zero**.

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1. Consider a 1-D diffusion equation.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

   (a) Using $0 \leq x \leq 1$ and initial square wave ($u = 1$ for $0.4 \leq x \leq 0.6$ and 0 elsewhere), write a code to solve the problem using a finite difference FTCS approach using 64 grid points. Take $\nu = 0.501$ and $nt = nx$ with $t_{max} = 0.06$. Plot the solution at $t = 0.00, 0.01$ and 0.05. **(5)**

   (b) Now, plot the solution at $t = 0.06$, and 0.07 (for later use $nt = nx$ with $t_{max} = 0.07$). Can you comment on the stability of the scheme? How can the scheme be made more stable to obtain correct solutions? **(5)**

2. Consider a 1-D linear convection equation.

$$\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}$$

   (a) For an initial sinusoidal wave of wavenumber $k = 4\pi$ (2 periods in 1 m), and convection speed of 1 m/s, Solve this problem using an upwind approach, Lax-Wendroff Approach, and the Lax-Fredrichs approach and compare with the theoretical solution at $t = 1.5$ s. You can choose the time step of your choice for numerical stability but justify your choice. **(15)**

   (b) Comment on relative features of numerical approaches employed in terms of numerical accuracy and stability. **(5)**

3. Write a code for solving the 1-D Non-linear Convection-Diffusion (Burger's) equation for the initial input of a Heaviside function using the following approaches (you are allowed to choose your spatial and temporal discretization steps, and the final simulation time of your choice)
   (a) Richtmyer **(5)**
   (b) MacCormack **(5)**
   (c) Beam-Warming (implicit) **(10)**

All the best !