Modeling of Mechatronic Systems – An Example: Capacitive and Piezoelectric Accelerometers

Linear Accelerometers

Accelerometers are electromechanical sensors used to measure acceleration. The most commonly used types of accelerometers are MEMS capacitor accelerometers and piezoelectric accelerometers.

Piezoelectric Accelerometers

A piezoelectric material is a transducer that converts mechanical energy into electrical energy and vice-versa. Quartz and PZT are examples of piezoelectric materials. By changing their shape, they generate an electric current. A piezoelectric accelerometer is chosen due to its large bandwidth, allowing measurements at high frequencies. No input voltage is required to operate.

Applications include various laboratory and industry environments to measure dynamic motions such as shock and vibration.

Image courtesy of Noliac, 2012. MH and ME series models.

Capacitive Accelerometers

The capacitive type accelerometer is chosen due to its high sensitivity and accuracy over a wide range of temperatures. MEMS capacitive accelerometers can be made extremely small, and can be made to measure accelerations in all 3 dimensions and rotations (gyroscopes). These accelerometers require input voltage to generate a capacitance.

Applications include and not limited to the following:
- Motion controlled gaming remotes
- Tilt functionality for mobile devices, such as phones and tablets
- Image stability for cameras
- Free fall detection in hard drives to quickly shut off and prevent damage to the disk
- Car airbag detection systems
- Sport and fitness devices such as pedometers

Image courtesy of STMicroelectronics, 2011.

Pieozoelectric materials and sensors

The piezoelectric effect refers to a phenomenon in which forces applied to a segment of material lead to the appearance of electrical charge on the surfaces of the segment. The source of this phenomenon is the distribution of electric charges in the unit cell of the piezoelectric material. The reverse piezoelectric effect refers to a deformation of the material due to the application of an electric field across the material.

A complete model of the piezoelectric materials depends upon the motor-generator action of the crystal and will be discussed in more detail in a subsequent lecture. In 1987 a standards committee of the IEEE published a renowned mathematical description of the piezoelectric phenomenon. In this documentation IEEE adopted the linear theory of piezoelectricity, coined the constitutive equations, where the linear elasticity is coupled to charge by means of piezoelectric constants:

$$\boxed{\begin{aligned} S &= s^E T + dE \\ D &= dT + \varepsilon^T E \end{aligned}}$$

strain = compliance at constant E*applied stress + piezoelectric charge constant*E
dielectric displacement = piezoelectric charge constant*applied stress + material permittivity*E

where $\boldsymbol{T}$ represents the stress applied, $\boldsymbol{S}$ the strain and $\boldsymbol{s^E}$ denotes the compliance of the medium at constant field strength $\boldsymbol{E}$. The dielectric displacement is denoted by $\boldsymbol{D}$, where $\boldsymbol{\varepsilon^T}$ is the permittivity of the medium at constant $\boldsymbol{T}$ and $\boldsymbol{d}$ is the piezoelectric charge constant that relates the mechanical and electrical domains. Note that all variables in the constitutive equations are tensors.

The sensors

Piezoelectric sensors

The application of a load to the piezoelectric material leads to an accumulation of charge according to the following expression:

$$Q \text{ (charge)} = \mathbf{d}\, F$$

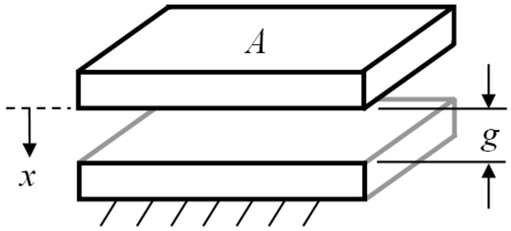
The force is a vector quantity, and the **d** (piezoelectric coefficient) is a 3x3 matrix. Forces along any axis (e.g. the x axis) produce charge accumulation along the x, y, and z axes, with the charge along any of the axis given by the corresponding d_{nm} coefficient. Typical values of the piezoelectric charge coefficients are 1-100 pico-coulombs/N which implies that depending on the capacitance of the piezo used, an amplification circuit needs to be used to obtain a voltage range of values that is easy to measure.

Piezoelectrics are not generally very good dielectrics and are are somewhat leaky. Therefore, aa charge placed on a pair of electrodes gradually leaks away. Piezoelectrics are not very useful for static measurement conditions and are typical used for dynamic measurements.

Piezoelectric elements have several important advantages over other sensing mechanisms. The device generates its own voltage so the sensor element does not need to have power applied to it in order to function. For applications where power consumption is a significant constraint, piezoelectric devices can be very valuable. In addition to this, the piezoelectric effect has some interesting scaling laws which suggest it is useful in small devices. As indicated above, the primary disadvantage of piezoelectric sensing is that it is inherently sensitive only to time varying signals. Many application require sensitivity to static quantities, and piezoelectric sensing simply does not work for such applications.

Capacitive Sensors – Parallel Plate

this case is non-linear… piezo is linear model



$$C(g)=\frac{\varepsilon\varepsilon_o A}{g} \qquad C(g-x)=\frac{\varepsilon\varepsilon_o A}{g-x}$$

$$C(g-x)=C(g)+x\frac{\partial C(g)}{\partial x}+\frac{x^2}{2}\frac{\partial^2 C(g)}{\partial x^2}+\ldots$$
$$\approx\frac{\varepsilon\varepsilon_o A}{g}\left(1+\frac{x}{g}+\frac{x^2}{g^2}\right)$$

All terms after and including the cubic term, $\frac{x^3}{g^3}$, are small and negligible.

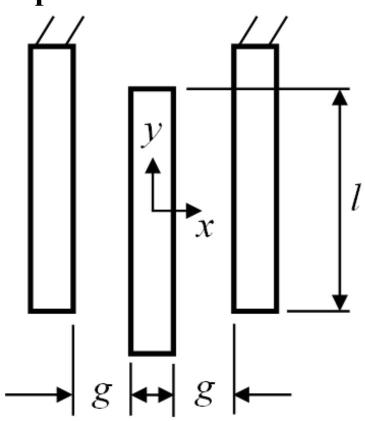
When $x \ll g$, the change in capacitance is linear with respect to displacement; however, when x is larger, the change becomes non-linear.

Sensitivity

$$\frac{dC}{dx}=\frac{\varepsilon\varepsilon_o A}{(g-x)^2}=\frac{\varepsilon\varepsilon_o A}{g^2}\left(\frac{1}{(1-x/g)^2}\right)$$

As the initial gap, g, becomes very small, the sensitivity becomes very large. This suggests capacitive sensors with very small g (however, this is limited by dielectric breakdown).

Capacitive Sensors – Differential



$$\Delta C=C_2-C_1=\frac{\varepsilon\varepsilon_o A}{g-x}-\frac{\varepsilon\varepsilon_o A}{g+x}$$
$$=\frac{\varepsilon\varepsilon_o A}{g}\left(1+\frac{x}{g}+\frac{x^2}{g^2}+\ldots\right)-\frac{\varepsilon\varepsilon_o A}{g}\left(1-\frac{x}{g}+\frac{x^2}{g^2}-\ldots\right)$$
$$\approx\frac{\varepsilon\varepsilon_o A}{g}\left(\frac{2x}{g}\right)$$

Note: $A=l\times t$, where t is the thickness

The intended sensing direction is in the x-direction. The change in capacitance is linear with respect to displacement since the squared term is subtracted away. The nonlinearity appears in higher order terms which are small enough to be neglected.

Sensitivity

Assuming the moveable finger has the ability to move in the *y*-direction, we can revise the equation to the following.

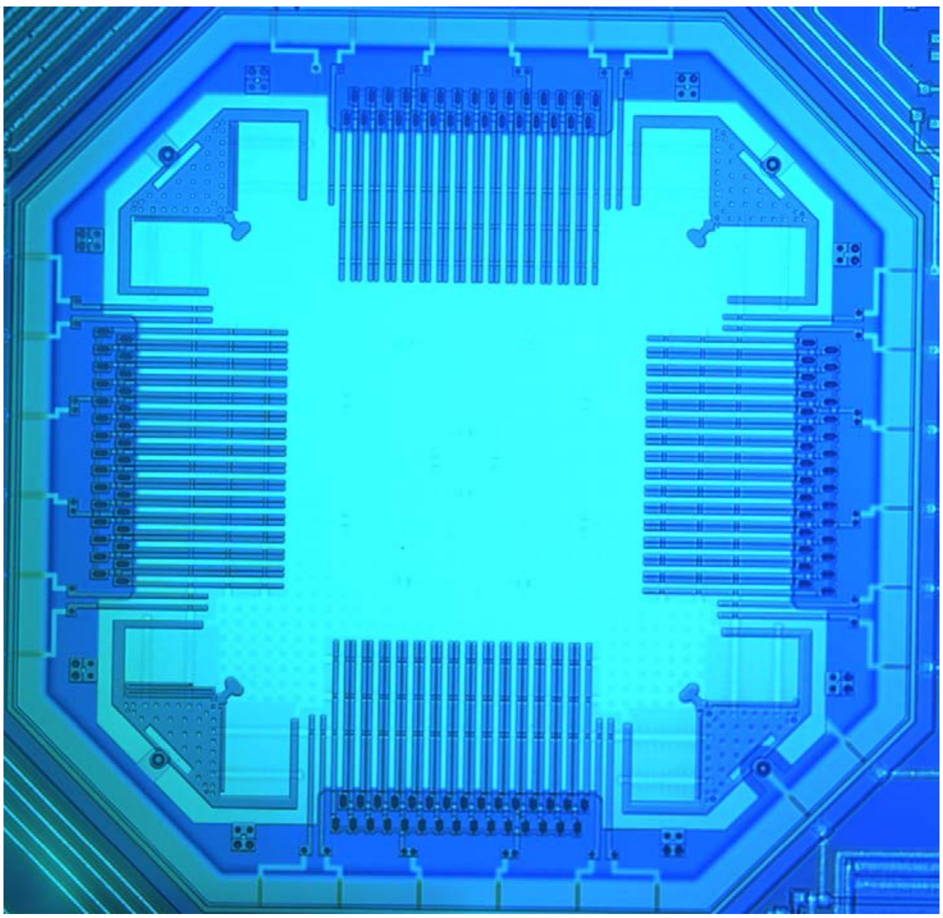
$$\Delta C = \frac{2\varepsilon\varepsilon_o t(l+y)}{g^2}x$$

where, $A = t(l+y)$. If $l >> y$, then $(l+y) \approx l$, and thus the difference in capacitance is not affected by any *y* displacement.

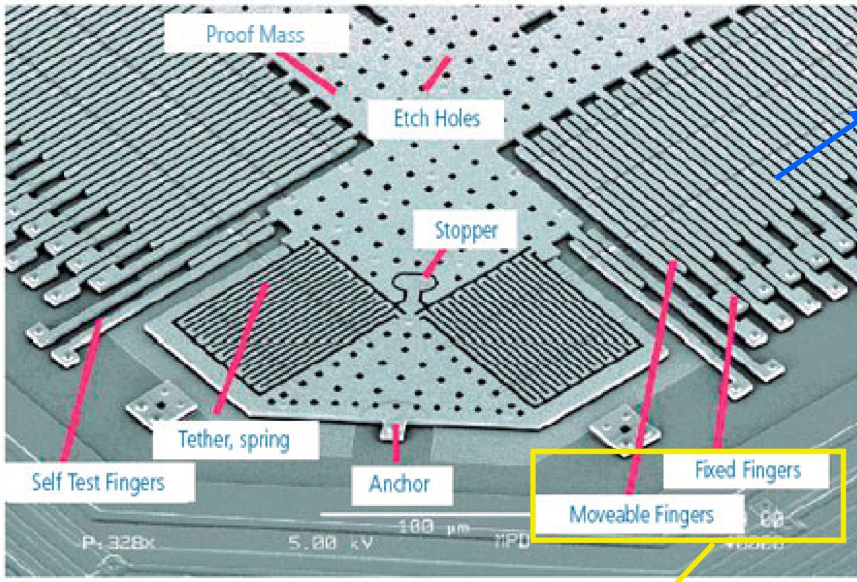
$$\frac{\partial \Delta C}{\partial y} = \frac{2\varepsilon\varepsilon_o tx}{g^2}$$

$$\frac{\partial \Delta C}{\partial x} = \frac{2\varepsilon\varepsilon_o t(l+y)}{g^2}$$

If *y* can move by the same amount as *x*, and $l >> y$, then $(l+y) >> x$ and $\frac{\partial \Delta C}{\partial x} >> \frac{\partial \Delta C}{\partial y}$.



Analog Devices *ADXL202* taken by microblog.net, 2007 (superseded by *ADXL203*).



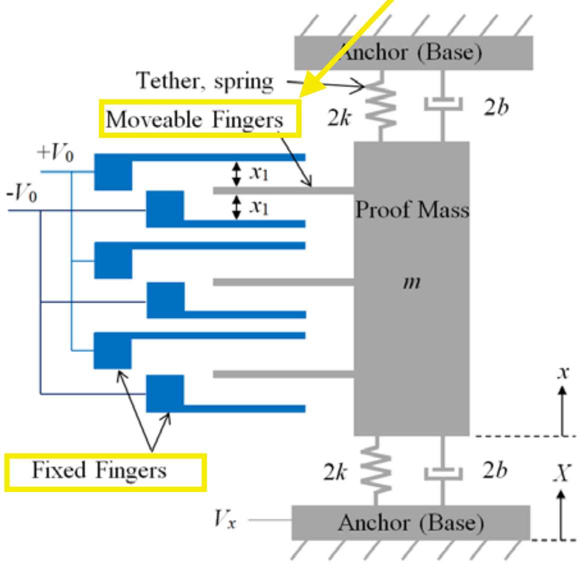
g (gaps between plates/fingers) very small -> highly efficient performance

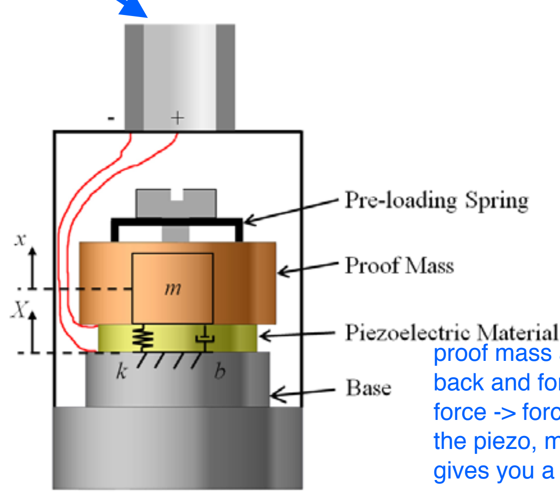
Image courtesy of Analog Devices. *ADXL202* (superseded by *ADXL203*).

The self test fingers are added which when excited, force the proof mass to move. So it can electronically test the accelerometer.

Equivalent Models (Lumped Parameter Models)

Below is a simplified model of a single linear accelerometer for a (left) capacitive accelerometer and a (right) piezoelectric accelerometer.





proof mass accelerating back and forth (generating force -> force is acting on the piezo, multiplied by d gives you a charge

X is the position of the base (frame)
x is the position of the proof mass

Equation of motion:

velocity of proof mass relative to base

$$m\frac{d^2x}{dt^2} = b\boxed{\frac{d(X-x)}{dt}} + k(X-x)$$

$$Z = X - x$$

$$x = X - Z$$

$$\therefore m\frac{d^2X}{dt^2} = m\frac{d^2Z}{dt^2} + b\frac{dZ}{dt} + kZ$$

Development of Transfer Function

We know the solution for the displacement is,

$$X = X_0 e^{i\omega t} \qquad Z = Z_0 e^{i\omega t}$$

Thus,

$$-m\omega^2 X_0 e^{i\omega t} = -m\omega^2 Z_0 e^{i\omega t} + i\omega b Z_0 e^{i\omega t} + kZ_0 e^{i\omega t}$$

Where,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{b}{2\sqrt{km}}$$

$$Z_0 = \frac{m\omega^2 X_0}{m\omega^2 - i\omega b - k} = \frac{X_0}{\sqrt{\left(1-\left(\frac{\omega_n}{\omega}\right)^2\right)^2 + \left(2\zeta\frac{\omega_n}{\omega}\right)^2}}$$

In the Laplace domain,

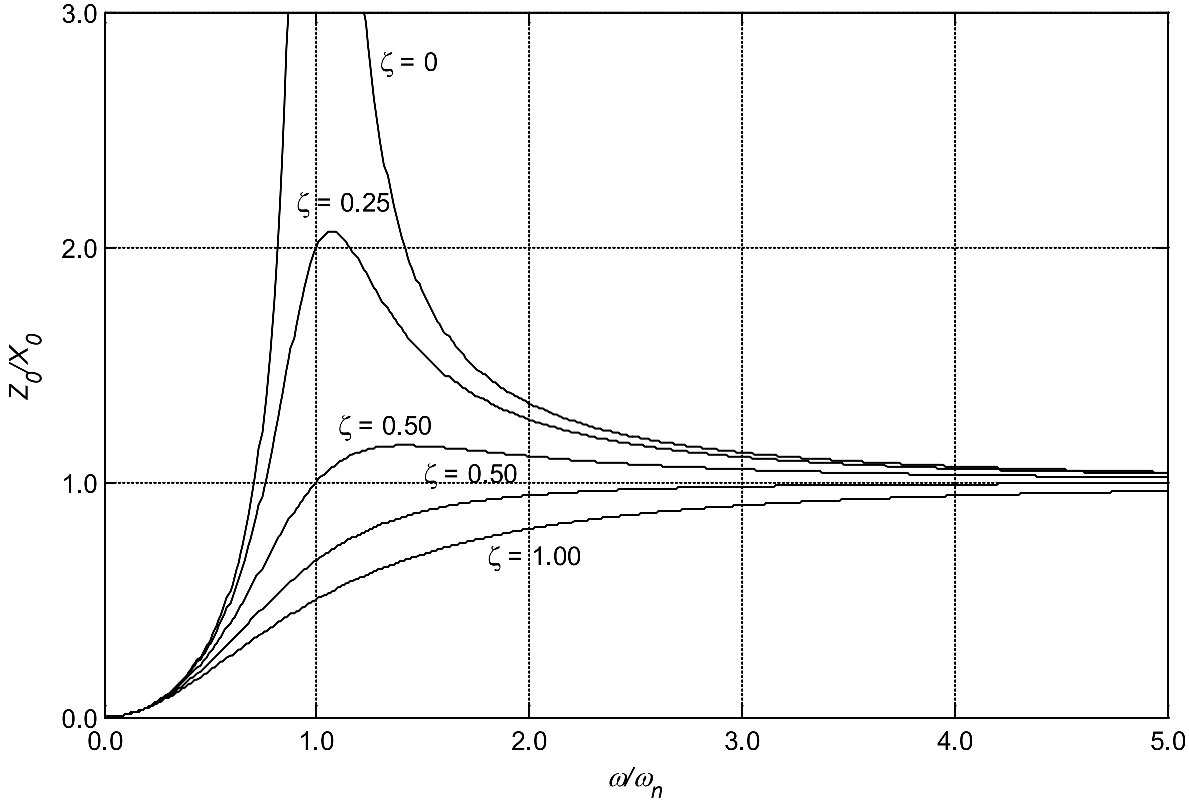
$$ms^2 X(s) = ms^2 Z(s) + bsZ(s) + kZ(s) = \left(ms^2 + bs + k\right)\cdot Z(s)$$

Define,

$$T(s) = \frac{Z(s)}{X(s)} = \frac{ms^2}{ms^2 + bs + k} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

laplace transform

Bode plot, zeta is the damping ratio



If $b = 0$ (no damping), there is no solution when $\omega = \omega_n$.

When $\omega < \omega_n$, the expression becomes

$$Z_0 \approx \frac{\omega^2 X_0}{\omega_n^{\ 2}} = \frac{A}{\omega_n^{\ 2}} \qquad \text{and} \qquad T(s) = s^2$$

at low frequencies (less than natural freq) -> there is acceleration

$$\therefore Z(s) = s^2 X(s) \rightarrow Z(t) = \frac{d^2 X(t)}{dt^2}$$

In this case, we are measuring acceleration and the sensor behaves as an accelerometer.

When $\omega > \omega_n$, the expression becomes

$$Z_0 \approx X_0 \qquad \text{and} \qquad T(s) = 1$$

at high freq: Z and X are equal when the frequency is higher than the natural frequency

In this case, the mass cannot respond to high frequency changes in x and the sensor behaves as a vibrometer.

select mass/structure according to the properties that you need and that way you can get the freq you need