Piezoelectric Materials

Characteristics and Modeling

By

Amro Shafik and Ridha Ben Mrad

Table of Contents

1. Introduction

In the last century, a new category of materials that is called smart materials emerged. Smart materials are reactive materials that change their properties when they are exposed to an external stimulus, such as, stress, strain, electric field, pressure or temperature. Smart materials are a family of materials that include piezoelectric materials, shape memory alloys, and magnetostrictive materials. **Table 1** shows the relationship between different stimuli and the corresponding responses for various materials [1]. The physical phenomena associated with the smart materials are shown in the non-diagonal cells of the table.

Table 1: Stimulus-response relations and various effects in smart materials

		Output (response)				
		Strain	**Electric charge**	**Magnetic flux**	**Temperature**	**Light**
Input (stimulus)	**Stress**	Elasticity	Piezo-electricity	Magneto-striction		Photo-elasticity
	Electric field	Piezo-electricity	Permittivity			Electro-optic effect
	Magnetic field	Magneto-striction	Magneto-electric effect	Permeability		Magneto-optic
	Heat	Thermal expansion	Pyro-electricity		Specific heat	
	Light	Photostriction	Photo-voltaic effect			Refractive index

2. Piezoelectric Materials

2.1 Piezoelectricity

Piezoelectric materials generate a charge when they are subject to pressure. The word "piezoelectricity" is a Greek word which means electricity resulting from pressure. As shown in **Table 1**, piezoelectricity is listed twice. This is because the piezoelectric effect and its inverse both exist in piezoelectric materials. The direct piezoelectric effect is when the input is a mechanical stress and the output is an electric charge. The converse piezoelectric effect takes place when the input is the electric field and the output is a strain. Therefore, piezoelectric materials can be used as transducers to transform mechanical energy to electrical energy and vice versa. Mainly, the direct effect is utilized in sensors and the converse effect is utilized in actuators. **Figure 1** shows the direct and converse effects in action.

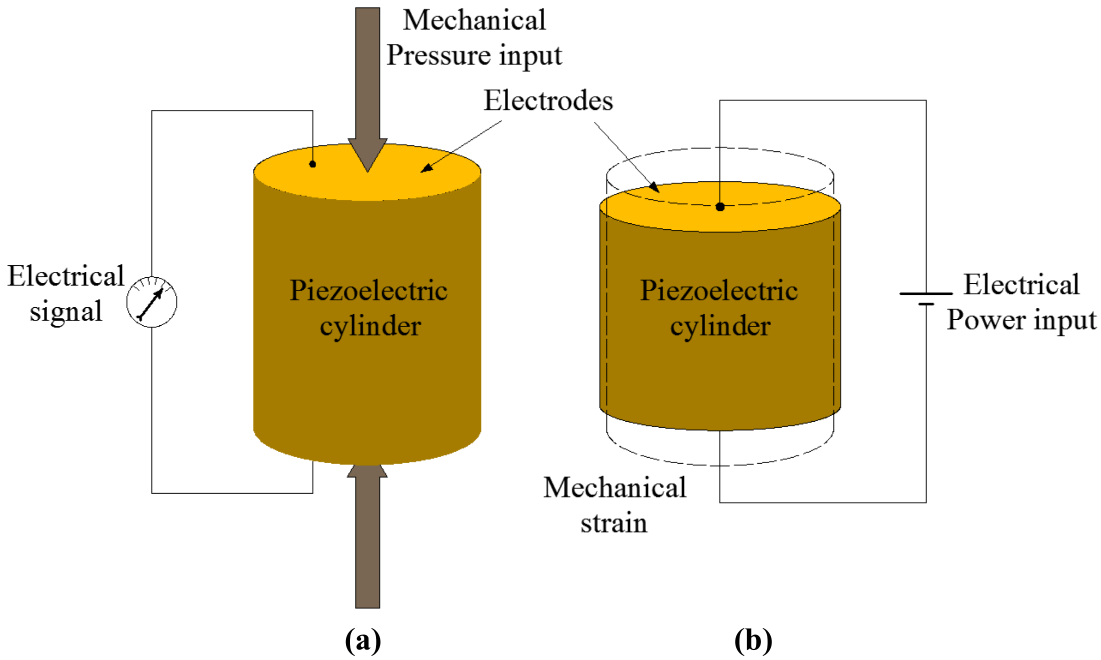


Figure 1. Piezoelectric effect (a) direct, (b) converse.

2.2 Piezoelectric materials

Piezoelectric materials were discovered in 1880 by the Curie brothers. They discovered that electricity can be produced by applying pressure on certain types of single-crystal materials such as quartz and tourmaline. In World War I, an ultrasonic detector was made from quartz and used with a hydrophone to detect submarines, and that was the first piezoelectric transducer [2]. During World War II, Barium Titanate was discovered as the first synthetic piezoelectric material that has ferroelectricity. The Lead Zirconate Titanate (PZT) family of piezoelectric ceramics which is widely used today was discovered in the 1950s. A summary of the main events in the history of piezoelectricity is shown in **Table 2**.

Piezoelectric materials can be classified according to three main categories [3]: Inorganic, organic, and piezoelectric composites materials. The inorganic type includes two big families which are the piezoelectric ceramics and the piezoelectric single crystals, also called, monocrystalline piezoelectric materials. Single crystal piezoelectric materials were discovered first and were the main category of piezoelectric materials for few decades. However, piezoelectric ceramics once developed became more popular because of their high sensitivity and high piezoelectric constants, as well as the ease of fabrication as they contain a large number of fine crystals. The second category is the organic piezoelectric materials, also called piezoelectric polymers. Piezoelectric polymers can be considered the third generation of piezoelectric materials and they possess excellent flexibility (i.e. low brittleness and ease of deformation) [4]. The third category is the piezoelectric composites, which can be considered the fourth generation of piezoelectric materials. The composites are a combination of piezoelectric polymers and piezoelectric ceramics together and possess unique advantages. A detailed comparison is shown in **Table 3**.

Table 2: History of piezoelectric materials

Year	Event	Scientist
1880	Piezoelectricity was initially discovered in Quartz (single crystal). Examples of single crystal piezoelectric materials include: Quartz, topaz, tourmaline, cane sugar, and Rochelle salt	Jacques and Pierre Curie [5]
1917	The first piezoelectric device is developed in World War 1, which was an ultrasonic submarine detector made of quartz crystals.	Paul Langevin
1946	Scientists discovered that by applying an electric field to a $BaTiO_3$ ceramic, it shows the piezoelectric effect. This led to the discovery of piezoceramics.	
1954	Discovery of the Lead Zirconate Titanate (PZT) family.	Jaffe et al.

Table 3: Piezoelectric materials classification

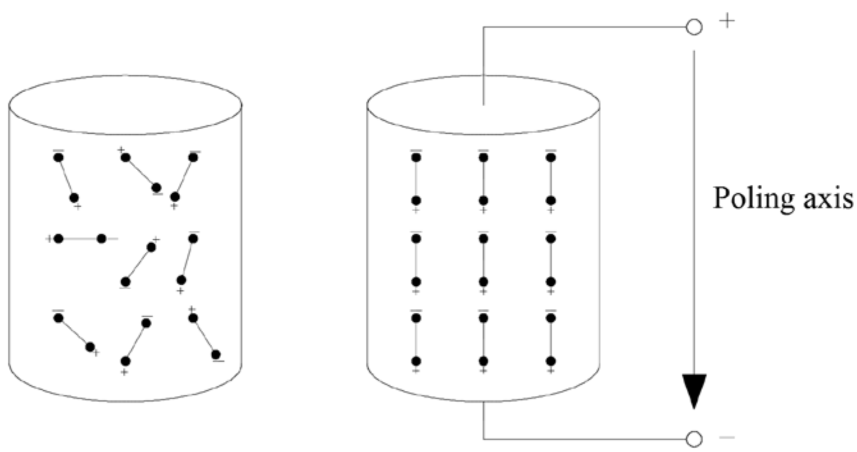
	Inorganic piezoelectric materials		**Organic piezoelectric materials**	**Piezoelectric composites**
	Piezoelectric ceramics	Piezoelectric single crystals (Monocrystalline)	Piezoelectric Polymers	Piezoelectric ceramics and polymers are incorporated together
Advantages	• Strong piezoelectricity • High dielectric constant • Can be easily fabricated into various shapes cheap	• High mechanical quality factor • Excellent stability expensive, efficient - good for energy harvesting	• Excellent flexibility (i.e. can be deformed) • Low density • Small impedance • Reasonable piezoelectric coefficient	• Large piezoelectricity • Strong strength • Low density • Excellent machinability
Disadvantages	• Low mechanical quality factor • Large electric loss • Poor stability	• Low piezoelectric coefficient • Low dielectric constant • Shapes are restricted because of the difficulty in machining these crystals.	• Relatively low piezoelectric strain constant	
Applications	• High-power transducers • Wide-band filters	• Vibrators • High-selectivity filters • High-temperature ultrasonic transducers	• Underwater ultrasonic measuring • Pressure sensing • Explosion igniting	• Hydroacoustic • Electroacoustic • Ultrasonic applications • Medical applications
Examples	$BaTiO_3$	PMN-PT, PZN-PT	Polyvinylidene fluoride (PVDF)	PZT5A1, PZT5H1, and PZT401

non-magnetic piezoelectric materials - used for applications where you can't have metal i.e. MRI

2.3 Polarization and poling process

Ceramics before poling do not exhibit piezoelectric properties. This is because the electric dipoles in the grains of the materials are randomly oriented which makes the net electric charge zero. To make the ceramics piezoelectric, a poling process is needed. In poling, a strong DC field is applied to the ceramic material while heated. This field orients the electric dipoles to be in the field direction, and thus produces a net polarization, which is known as the ferroelectricity. After the poling process, a remnant polarization is created in the piezoceramic and this is called polarization **P**. The poling process is shown in **Figure 2**.

Polarized piezoceramics expand along and contract perpendicular to the poling axis if a voltage with the same polarity as the poling voltage is applied. The inverse piezoelectric effect works by the generation of voltage when compressive or tensile forces are applied to the piezoceramic element.



(a) (b)

Figure 2. Poling process, (a) piezoceramic element before poling showing that the electric dipoles are randomly oriented, and the net electric charge is zero, (b) piezoceramic during the poling process with the electric dipoles aligned along the poling axis.

According to the relationship between the applied electric field **E** and the induced polarization **P**, there are three different kinds of polarizations: dielectric, paraelectric, and ferroelectric polarization (as shown in **Figure 3**). In dielectric polarization, the relationship between **P** and **E** is strictly linear. In paraelectric polarization, the relationship becomes nonlinear, as the slope of the curve, known as electric permittivity, is not constant in this case. In ferroelectric materials, the polarization curve shows a spontaneous nonzero polarization, which means that the polarization is at $\mathbf{P_R}$ when the applied electric field is zero. If a strong electric field is applied in the reverse direction, the spontaneous polarization is reversed and a hysteresis loop is formed analogous to phenomena in ferromagnetic materials. This is why this phenomenon is named ferroelectricity although that most of the materials that exhibit this kind of polarization do not contain ferro (or iron) as an ingredient.

When the ferroelectric material is heated beyond a certain temperature, a phase transition takes place, the spontaneous polarization vanishes, and the material transforms to the paraelectric phase. This temperature is called the Curie Temperature $\mathbf{T_c}$. The piezoelectric properties of ferroelectric materials vanish above $\mathbf{T_c}$ because of the centrosymmetric crystal structure of the paraelectric phase.

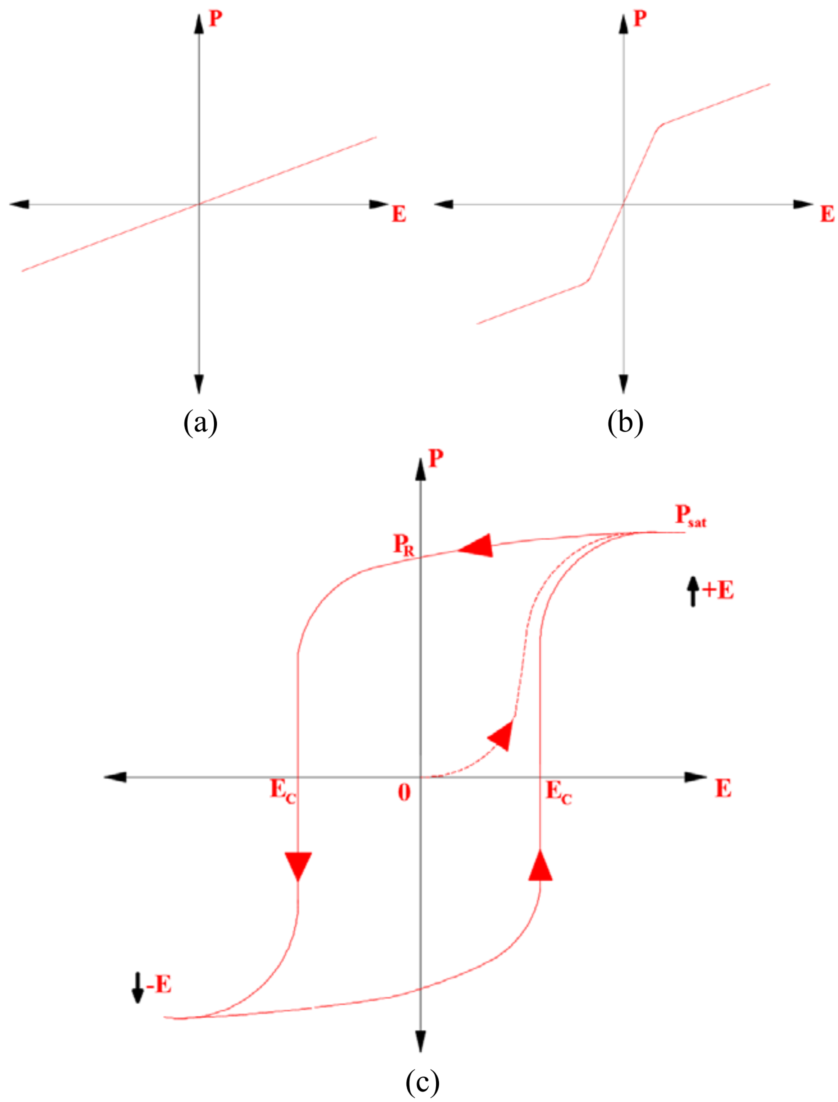


Figure 3. Different forms of polarization. (a) Dielectric polarization, (b) paraelectric polarization, (c) ferroelectric polarization, showing the hysteresis loop of a poled piezoceramic.

The P-E (Polarization vs Electric Field) and S-E (Strain vs Electric Field) hysteresis loops shown in **Figures 3 (c)** and **4**, respectively, are among the main characteristics and attributes of ferroelectric ceramics. When the material is being polarized for the first time, it follows the dashed curve that starts at the origin, which is called the virgin curve. Once the ferroelectric material is polarized, it follows the P-E hysteresis loop. The two main characteristics of this curve are the remnant polarization P_R and the coercive field E_C. The former is the polarization value at zero electric field, while the latter is the field at which the polarization is zero. The maximum polarization value is called P_{sat}, which refers to the polarization saturation at the top and the bottom of the hysteresis loop. The hysteresis loop is both temperature and frequency dependent [6]. In the strain-field (S-E) hysteresis loop, also known as the butterfly curve, polarization switching leads to strain-electric field hysteresis. The curve (**Figure 4**) shows dielectric strain characteristics for a PZT piezoceramic, and how the strains direction varies with the change in the direction of the applied electric field. The strain results from the application of the electric field is dictated by the converse piezoelectric effect. S^R is the remnant strain, or the strain value at zero electric field. As the field increases, the strain is no longer linear with the field as domain walls start switching. It is evident from the curve that at the S^R point, the domains are completely reversed when reversing the direction of the applied electric field.

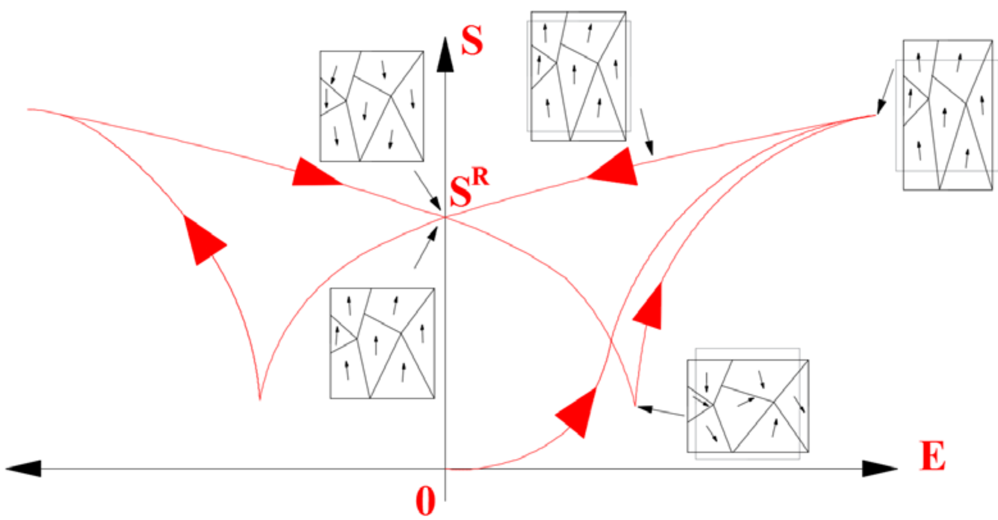


Figure 4. A typical butterfly curve for piezoceramics (Strain-Electric field hysteresis loop).

To further describe what happens in the ferroelectric materials after polarization, see **Figure 5**. The symmetrical cubic crystal structure is referred to as the Perovskite structure. Proveskite is a name of a mineral that was discovered by L.A. Perovski. The general formula for Proveskite compounds is ABX_3, such as, $BaTiO_3$, where in this case A is Barium (Ba) (the blue atom), B is Titanium (Ti) (the black atom), and X is Oxygen (O) (the red atom). When a strong DC electric field is applied, the black atom (Titanium in this case) leaves its location at the center of the cell and moves along the poling axis. This causes the cubic structure to change to a tetragonal/rhombohedral structure and makes the material polarized because the positive and negative charge sites no longer coincide. The tetragonal shape of the crystal structure remains as long as the temperature is below the Curie Temperature $\mathbf{T_c}$. Once the temperature of the material goes above $\mathbf{T_c}$, the black atom returns to the center of the cell, the crystal structure returns to a cubic and centrosymmetric form with the positive and negative charge sites coinciding, and the material transforms into the paraelectric phase, and thus loses its piezoelectric properties.

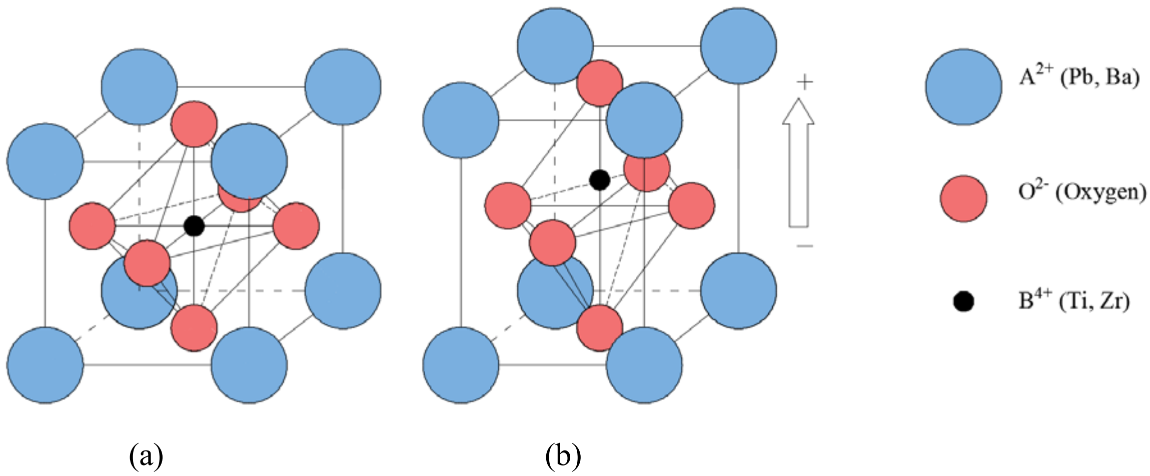


Figure 5. Piezoceramic elementary cell, (a) cubic structure ($T>T_c$), (b) tetragonal structure ($T<T_c$).

Dielectric polarization P, or the dipole moment per unit volume, is a function of the electric field strength E, the permittivity of free space ε_o, and the susceptibility of the medium χ (as shown in **Eq. 1**). The electric displacement D can be defined by **Eq. 2**, which is function of the dielectric polarization P, the field strength E, and the permittivity of free space ε_o (equal to 8.854×10^{-12} F m^{-1}). Therefore, the electric displacement D is proportional to the electric field strength E. The slope of the curve is defined by the dielectric constant of the material ε, which is equal to the permittivity of free space times the relative permittivity ε_r.

$$P = \varepsilon_o \chi E \qquad \textbf{Eq. 1}$$

$$D = \varepsilon_o E + P = \varepsilon_o (1+\chi)E = \varepsilon_o \varepsilon_r E \qquad \textbf{Eq. 2}$$

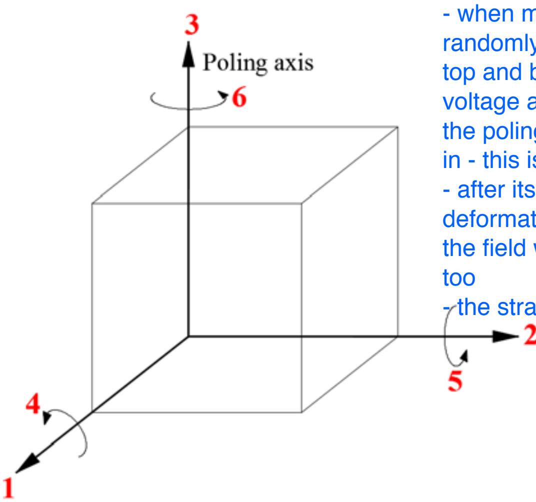
$$D = \varepsilon E \qquad \textbf{Eq. 3}$$

2.4 Piezoelectric material characteristics

The material characteristics are anisotropic in piezoceramics. That means that the characteristics of the material depend on the direction of the applied mechanical stress, strain, electric field, and electric displacement. To define the directions, the axes are represented in numbers. The z-axis (usually the poling axis) is represented as 3, while the x and y axes are represented as 1 and 2, respectively. The shear about the axes 1, 2, and 3, is represented by 4, 5, and 6, respectively (see **Figure 6**).

To completely define the piezoelectric material constants, subscripts and superscripts are added. Subscripts include two numbers. The first number represents the direction of applied electrical field, while the second number represents the direction of the stress or strain. Superscripts are used to indicate the constant electrical or mechanical boundary conditions, as follows; mechanically free, mechanically clamped, electrical short circuit, and electrical open circuit, are represented by the following superscripts ***T***, ***S***, ***E***, and ***D***, respectively. This means if E is constant (or zero) and D is changing, then this is a short-circuit boundary condition, and so on.

Naming conventions:



- when material is made - heated up, dipoles are randomly oriented, add electrodes to the material at top and bottom as poling axis, orient dipoles by adding voltage along the poling axis
the poling axis is whatever axis the voltage is applied in - this is the axis labeling convention.
- after its cooled down, apply voltage for actuation, deformation occurs along 1, 2 and at 3 even though the field was applied along axis 3, sometimes rotation too
- the strain has to do with the d coefficient

Figure 6. Directions used in representing the piezoelectric material properties.

2.4.1 Piezoelectric coupling coefficient (k):

The piezoelectric coupling coefficient, also called the electromechanical coupling coefficient, is the ratio of the stored electrical energy in response to a mechanical stress, or the accumulated mechanical energy to an applied electrical input [7]. The coupling coefficient is dependent on the vibration mode.

$$k = \sqrt{\frac{Stored\ mechanical\ energy}{Applied\ electrical\ energy}} = \sqrt{\frac{Stored\ electrical\ energy}{Applied\ mechanical\ energy}}$$

According to [8], the internal energy for a piezoelectric material U has three energy components, elastic energy, electric energy, and mutual energy. The internal energy can be determined by the following equations:

$$U = U_e + 2U_m + U_d$$

where,
U_e is the elastic energy,
U_d is the electric (or dielectric) energy,
U_m is the mutual energy, also known as the elastic-dielectric interaction energy [3]

These energy components can be calculated as functions of the applied stress *T*, induced strain *S*, electric field *E*, and electric displacement *D*, as in [8], and then the coupling factor can be determined as follow

$$k = \frac{U_m}{\sqrt{U_e U_d}}$$

The equation above is the general formula that is suitable for both static and dynamic conditions. While the formula at resonance frequencies is

$$k_d^2 = \frac{f_r^2 - f_o^2}{f_r^2}$$

where k_d is the electromechanical coupling coefficient at resonance, f_r is the antiresonant frequency, and f_o is the resonant frequency.

2.4.2 Piezoelectric voltage coefficient (g):

The piezoelectric voltage coefficient, also called the piezoelectric stress constant, is the ratio of the resulting strain to the electric charge generated per unit area, or the ratio of the electric field developed to the applied mechanical stress (Vm/N) [7].

$$g = \frac{Strain\ developed}{Applied\ charge\ density} = \frac{Field\ developed}{Applied\ mechanical\ stress}$$

$$g = \frac{d}{\varepsilon^T}\ (\frac{Vm}{N})$$

where the coefficient *d* is defined in the next subsection. As an example, in the 31 mode,

$$g_{31} = \frac{d_{31}}{\varepsilon_{33}^T}$$

with ε_{33}^T representing the piezoelectric material permittivity along the polarization axis *3* (parallel to direction in which ceramic element is polarized), and measured at constant stress *T*, i.e. mechanically-free boundary condition.

2.4.3 Piezoelectric charge coefficient (d):

The piezoelectric charge coefficient, also called the piezoelectric strain constant, is the ratio of the strain produced to the applied electric field (m/V), or the ratio of electric charge generated per unit area to an applied force (C/N) [7].

$$d = \frac{Strain\ developed}{Applied\ field} = \frac{Charge\ density}{Applied\ stress}$$

$$d = k\sqrt{\varepsilon^T s^E}\ (\frac{C}{N})$$

As an example, in the 31 mode,

$$d_{31} = k_{31}\sqrt{\varepsilon_{33}^{T} s_{11}^{E}}$$

with s_{11}^{E} representing the piezoelectric material compliance along the *1* axis (i.e. *x*-axis, or the axis perpendicular to the poling axis) measured at constant electric field *E*, i.e. short-circuit boundary condition.

2.4.4 Dielectric constant (ε_r):

The dielectric constant, called also relative dielectric constant, is the ratio of the permittivity of the material ε to the permittivity of the free space ε_o. It is typically calculated by measuring the static capacitance at 1 kHz using a standard impedance bridge. Dielectric constant, at constant stress, of a piezoelectric material can be calculated as follows [9], [6]:

$$\varepsilon_r^T = \frac{\varepsilon^T}{\varepsilon_0} = \frac{C^T t}{\varepsilon_0 A}$$

where,
C^T is the capacitance (in farads) of the piezoelectric material measured at constant stress
t is the thickness of the piezoelectric material, i.e. the distance between electrodes in meters
A is the area of an electrode in square meters

2.4.5 Dielectric loss factor (tan δ):

The dielectric loss factor is the tangent of the loss angle δ. The loss angle of a dielectric is complementary to the lead angle φ. In ideal dielectrics, the lead angle is 90^o without dielectric loss.

$$\varphi = 90 - \delta$$

The dielectric loss occurs when an electric field is applied to a dielectric and is mainly due to two phenomena: the polarization relaxation and the current leakage [3]. The dielectric polarization relaxation is the time required for the polarization of a dielectric material to go up from zero and reach the final value. The dielectric loss is sensitive to the field strength, the temperature, and the frequency of the electric field. The dielectric loss can be measured directly using an impedance bridge. It can be estimated as the ration of active power *P* to reactive power *Q* in a dielectric [3], or the ratio of the leakage current I_R to the charging current I_C [10].

$$\tan\delta = \frac{Active\ power\ (P)}{Reactive\ power\ (Q)} = \frac{I_R}{I_C}$$

2.4.6 Electrical quality factor (Q_e):

The electrical quality factor is defined as the reciprocal of the dielectric loss factor.

$$Q_e = \frac{1}{\tan \delta}$$

2.4.7 Mechanical quality factor (Q_m):

The mechanical quality factor is the energy consumed by the piezo element to overcome the inner friction during a period at its resonance [3]. The mechanical quality factor is dependent on the vibration mode.

$$Q_m = 2\pi \frac{Mechanical\ energy\ stored\ during\ a\ period\ at\ resonance}{Loss\ of\ mechanical\ energy\ during\ a\ period\ at\ resonance}$$

In reading the piezoelectric materials characteristics, and as demonstrated through the few examples mentioned already in the text, the first digit of the subscript represents the electrical condition, i.e. poling direction or electrodes direction, and the second digit of the subscript represents the mechanical condition, either as input (i.e. applied stress) or output (i.e. induced strain). Some examples of how to read the piezoelectric materials characteristics are shown in **Figure 7**.

2.5 Piezoelectric relations and constitutive equations

The linear electrical behavior of an unstressed medium under the influence of an electric field is defined by

$$D = \varepsilon E$$

where

D is the dielectric displacement (or electric flux density)

E is the field strength

ε is the permittivity of the medium

d31, d33 (common constants) electrodes perpendicular to the 3rd axis (poling axis) and applied stress is along the 3 axis

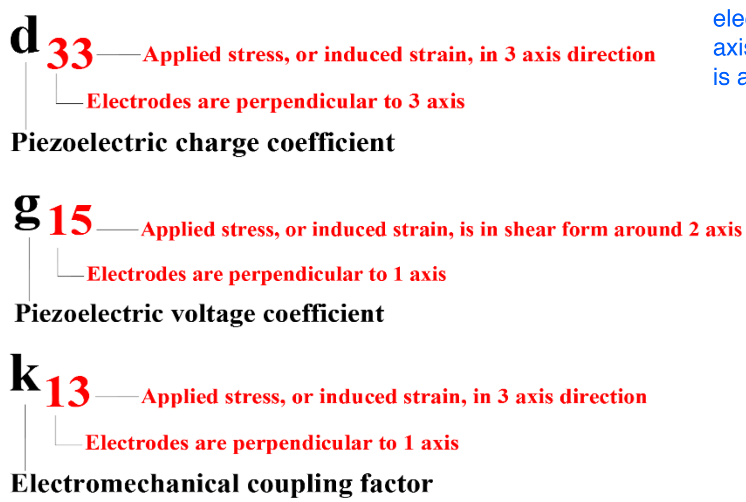


Figure 7. Reading the piezoelectric material characteristics.

Hooke's law describes the mechanical behavior of the same medium at zero electric field strength as follows:

$$S = sT$$

where
S is the strain
T is the applied stress
s is the compliance of the medium

Piezoelectricity involves the interaction between the electrical and mechanical behaviors of the medium, which can be described by the following equations [11]:

$$\{S\} = [s^E]\{T\} + [\boldsymbol{d}^t]\{E\}$$
$$\{D\} = [\boldsymbol{d}]\{T\} + [\varepsilon^T]\{E\}$$

and in matrix form, the constitutive equations can be written as follows:

linear model (not realistic)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\ s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

E1, E2 are zero, E3 is the electric field along the 3 axis when you are doing the standard way -> 3 axis is the only one that's polarized

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

where
$\{S\}$ is the strain vector,
$\{D\}$ is the electric displacement vector,
$\{T\}$ is the stress vector,
$\{E\}$ is the electric field vector,
$[s^E]$ is the matrix for compliance at constant (or zero) electric field strength,
$[\varepsilon^T]$ is the matrix for dielectric constant (or permittivity) at constant (or zero) stress,
$[d]$ is the matrix for direct piezoelectric effect (or piezoelectric strain constant),
$[d^t]$ is the matrix for the converse piezoelectric effect (or the transpose of the direct effect).

2.6 Piezoelectric material types

2.6.1 Soft and hard PZT

Based on the characteristics of the piezoelectric materials that were discussed in the last section, PZT materials can be classified into soft and hard materials. PZT is a solid solution of lead zirconate ($PbZrO_3$) and lead titanate ($PbTiO_3$) materials. The difference between soft and hard PZT is the titanium zirconium ratio [12]. A comparison of the characteristics of soft and hard PZT materials is given in **Table 4**. Detailed characteristics of commercial soft and hard PZT materials by SensorTech are given in Appendix (A) [13].

Table 4: Comparison of soft and hard PZT ceramic properties

Properties	Soft PZT	Hard PZT
Piezoelectric constant (*d*)	Higher	Lower
Dielectric constant	Higher	Lower
Dielectric loss	Higher	Lower
Hysteresis	Higher	Lower
Mechanical quality factor (Q_m)	Lower	Higher
Coupling factor	Higher	Lower
Resistivity	Higher	Lower
Coercive field	Lower	Higher
Elastic compliance	Higher	Lower
Aging effects	Lower	Higher

2.6.2 Lead free piezoelectric materials

For many decades, lead-based piezoceramics have been used widely in a wide range of products such as sensors, actuators, and transducers. This is mainly because their large piezoelectric and coupling coefficients and high Curie temperature. They also have other several advantages including ease of manufacturing and shaping, a cheap price, and their high solubility with various elements which makes their composition easily modified for desirable properties [14]. However, lead Pb, and lead oxide (PbO) in particular, are toxic and hazardous materials. Therefore, many research efforts have been dedicated to develop lead free piezoelectric materials.

Two main lead-free piezoelectric materials with perovskite structure have been found as potential alternatives to PZT ceramics: (1) Potassium sodium niobate, $K_{0.5}Na_{0.5}NbO_3$ (abbreviated as KNN), and (2) bismuth sodium titanate, $Na_{0.5}Bi_{0.5}TiO_3$ (abbreviated as NBT) [15]. Both families have the advantages of low dielectric constant, low density, high coupling coefficient, and higher mechanical strength as compared to PZT counterparts [16]. Piezoelectric constants d_{33} greater than 150 pC/N and Curie temperatures higher than 200 °C were obtained.

Moreover, bismuth-layered compounds have good stability at temperatures higher than 600 °C [17].

2.6.3 Thin film piezoelectric materials

Piezoelectric materials have several attributes that qualifies them to be used in micro-electromechanical systems (MEMS) in different applications. Among these attributes are the good scaling ability and high energy density of the piezoelectric materials, the ability to perform large amplitude actuation with low drive voltages and low hysteresis, and the fact that piezoelectric sensors do not require power to operate [18]. Those characteristics led to the evolution of thin film piezoelectric materials. The difference between bulk and thin film piezoceramics is in size and the production method. The thickness of thin film is from hundreds of micro meters to less than hundreds of nano meters. Fabrication of piezoelectric thin films can be classified into two main production methods: (1) liquid phase deposition, and (2) vapour phase deposition [19].

2.7 Nonlinearities in piezoelectric materials

2.7.1 Hysteresis

In open-loop operation of piezoactuators, the state of the material is determined by its previous history. This means that when the same driving voltage is applied, two different actuator expansions can be obtained depending on the direction and history of the driving voltage. This is referred to as hysteresis. To avoid or minimize the hysteresis nonlinearity, two solutions can be applied: using a charge control instead of voltage control, or using the piezoactuator in closed-loop operation. The amount of hysteresis increases with an increase in the field strength (driving voltage). The width of the voltage/displacement curve (shown in **Figure 8**) reaches up to 15% of the actuator stroke [20] at quasi-static excitation conditions.

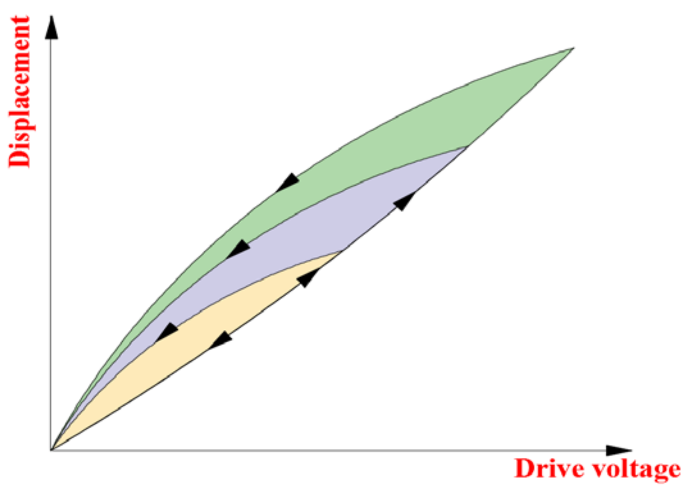


Figure 8. Hysteresis curves of an open-loop operated piezoceramic actuator at various peak voltages. The curves show the hysteresis behavior below the coercive field [10].

2.7.2 Creep

When piezoelectric transducers are used in open-loop slow applications, the creep problem appears [21], [22]. The creep is a slow, continuous, unwanted movement, or drift, over time. Creep can continue for few hours [21]. **Figure 9** shows a creep curve for a PZT actuator after a sudden change of 60 µm. The curve shows that the creep is on the order of 1% per time decade.

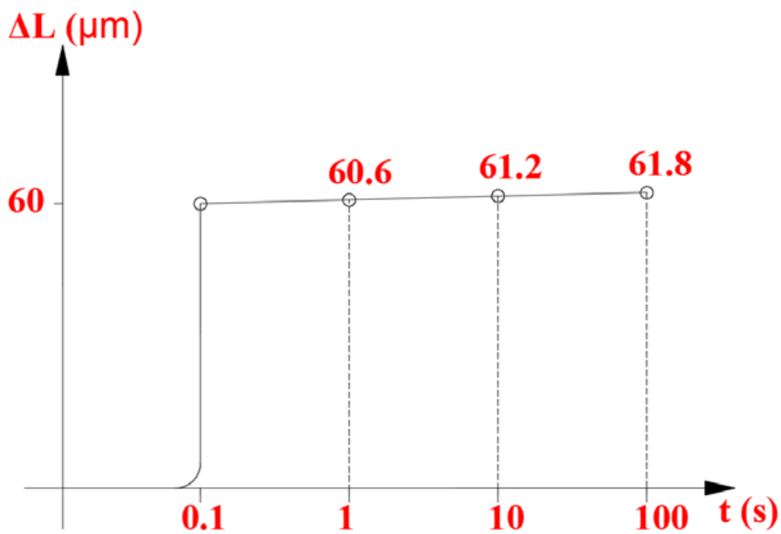


Figure 9. Creep in a PZT element.

3. Piezoelectric actuators

3.1 Longitudinal actuators vs transversal actuator

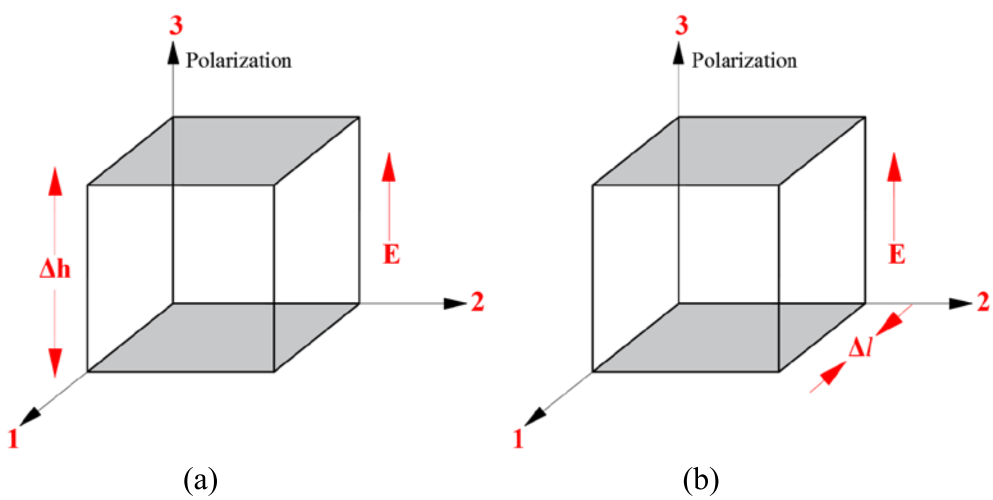


Figure 10. Deformation of piezoceramics (a) longitudinal, (b) transverse.

When the resulting deformation is parallel to the applied electric field, this type of deformation is called longitudinal, or axial, effect (see **Figure 10 (a)**). When the deformation is perpendicular to the electric field, this is called the transverse effect (see **Figure 10 (b)**). Both effects take place at the same time, but based on the desired direction of motion, the actuator is referred to as either longitudinal or transverse.

3.2 Modeling of piezoelectric actuators

An accurate model for piezoelectrics is needed to predict and understand the system response to external inputs. Three types of models of piezoelectric actuators are found in the literature: finite element models (FEM), equivalent circuit models (ECM), and energy conversion models [23], [24]. The focus here is on the ECM modeling method (An example of modeling and basic analysis made for a bulk piezoelectric actuator and comparing the analytical modeling solutions to the solutions based on the ANSYS FEM software model is given in Appendix (B)).

Piezoceramic actuators are simply considered as capacitances if the energy dissipation is ignored. However, for power requirement analysis, the energy dissipation needs to be considered. Different approaches to account for energy dissipation are found in the literature. Goldfarb et al. [25], [26] used a Maxwell Resistive Capacitance (MRC) model that considers energy dissipation and represents the hysteresis in the material as well. Park [27] compared two simple models: a capacitor and an internal resistor in series and in parallel. He found that the series capacitor resistor model represents the piezoceramic characteristics better than the parallel capacitor resistor model. Guan and Liao [28] investigated ECM models for loaded and unloaded piezoelectric actuators as well as while working at a resonant frequency and at a non-resonant frequency.

Generally, piezoelectric actuators are considered as electromechanical transducers. The electric part represents the dielectric constant, or the capacitance of the piezoceramic material; and the mechanical part represents the actuator mass, stiffness, and damping of the piezo material. The main input to the system is the input voltage, while the main output is the mechanical force or torque. The relationship between these two constituents of the system, the mechanical and the electrical, can be represented by a constant, which is commonly known as the force factor, or the electromechanical coupling factor. The equivalent circuit of an electromechanical transducer, representing a piezoelectric actuator in this case, is shown in **Figure 11** [29].

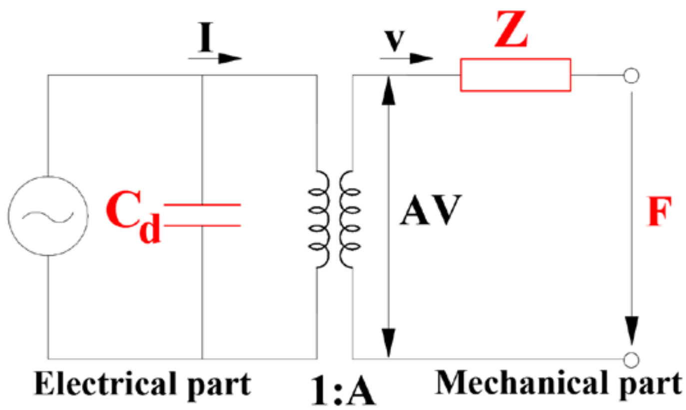


Figure 11. An electromechanical transducer circuit representing a piezoelectric actuator.

The behavior of this electromechanical transducer can be captured by the following equations. These two equations are analogous to the two piezoelectric constitutive equations [29].

$$F = AV - Zv$$
$$I = Y_d V + Av$$

where,
I is the electric current at the electrical terminal
V is the input voltage
A is the force factor
v is the velocity at the mechanical terminal
Z is the mechanical impedance of piezoceramic material
F is the output force at the mechanical terminal
Y_d is the blocking admittance

$$Y_d = \frac{1}{j\omega C_d}$$

The force factor A is a function of the dimensions and the properties of the piezoceramic material. It can be calculated as follows [29]:

$$A = bd_{31}Y_{11}$$

where,
b is the width of the piezoceramic
d_{31} is piezoelectric strain constant
Y_{11} is the Young's modulus of the piezoceramic

The mechanical impedance Z, which is shown in **Figure 11,** also needs to be determined. Sashida [29] represented this impedance by an equivalent electrical R-L-C branch as shown in **Figure 12**. R_d is introduced here to represent the dielectric loss of the piezoceramic material. This circuit has two impedances; the first one is called the blocking impedance, and it represents the non-resonant part [28], and the second part is called the motional impedance, and it represents the resonant part [28], which is responsible of the peaks f_o and f_r shown in **Figure 13**. These two peaks are called the resonant and the antiresonant frequencies of a piezoelectric actuator, respectively. The resonant frequency is the frequency at which the impedance of the actuator is minimum, and the antiresonant frequency is that at which the impedance is maximum, or the admittance is minimum. The piezoceramic behaves capacitively below f_o and above f_r, and inductively between them [6]. For piezoelectric ultrasonic transducers and motors, the resonant mode is the operating mode, while for other transducers and actuators the non-resonant mode is typically the working mode. Piezoelectric actuators that operate at non-resonant frequencies that are below the ultrasonic range are often referred to as quasi-static piezoelectric actuators [30].

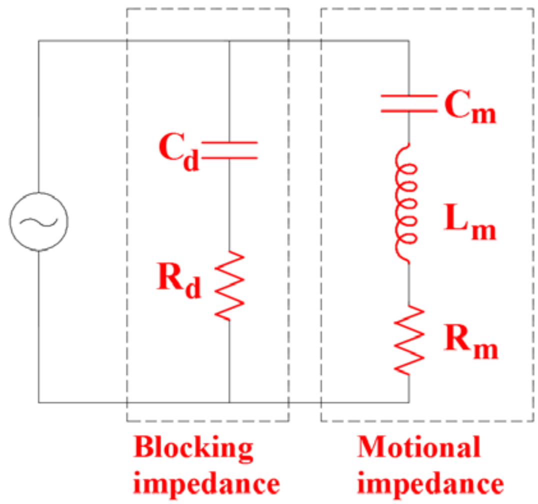


Figure 12. Basic equivalent circuit of a piezoelectric actuator.

Guan and Liao [28] proved that a slight change in the circuit arrangement can enhance the accuracy of the model. They investigated three different models for the non-resonant part of the circuit: the series circuit model, which consists of a capacitance and a resistance in series; a parallel circuit model, in which the capacitance and the resistance are in parallel; and a hybrid model, which consists of RC elements in series and a resistor in parallel. The hybrid model leads to better results as far as matching empirical impedance/frequency curve.

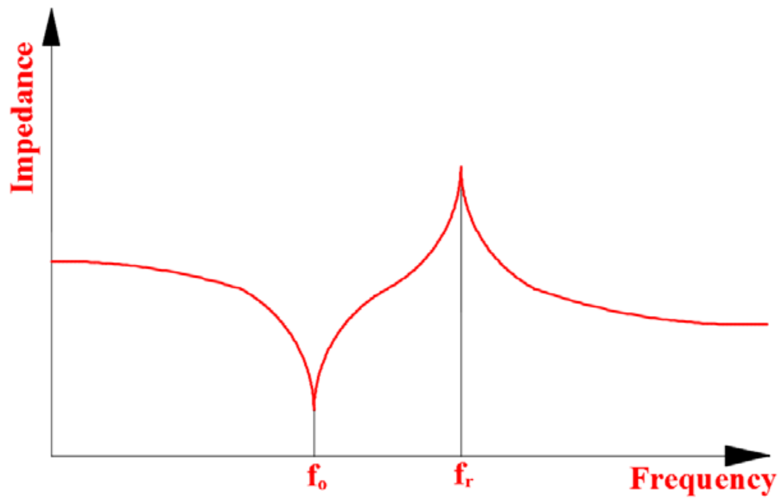


Figure 13. Typical resonant (f_o) and antiresonant (f_r) frequencies of a piezoelectric actuator.

At resonance, Sashida [29] stated that R_d can be neglected for frequencies below the ultrasonic range, i.e. 25 kHz. This was also recommended by the IEEE standard of piezoelectricity [11] that present an ECM model for piezoelectric vibrators that work at resonance and has the form shown in **Figure 14**. This model is valid as long as the circuit parameters are constant and independent of frequency. The parameters are considered independent of frequency near a resonance frequency and if the operating resonant mode is pure, i.e. isolated from other resonant modes. This ECM model is also known as the Van Dyke model [6], [31]. C_d is referred to as the damped, or blocking, capacitance. It is the capacitance of the piezo transducer below the resonant frequency, or simply the capacitance of a regular dielectric. R_m-L_m-C_m represents the equivalent circuit of the vibrating stator. R_m is the resistance caused by mechanical loss, or the equivalent mechanical loss, and it is also known as the internal resistance. C_m is capacitance of the mechanical circuit, or the equivalent stiffness. It represents the spring effect of the ceramic body and the associate metal ring if one is attached. L_m is the inductance of the mechanical circuit, or the equivalent moment of inertia, and it represents the mass effect of the ceramic body and the associated metal ring.

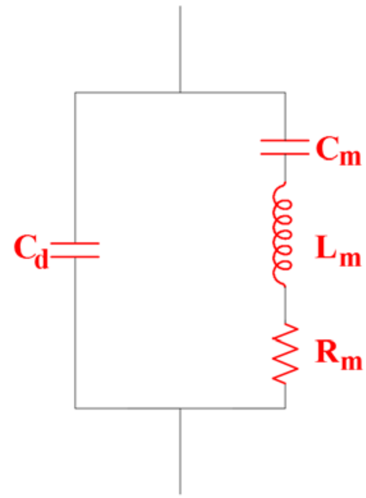


Figure 14. Equivalent circuit of a piezo transducer.

To identify the circuit parameters, the measured impedance spectrum of the piezoceramic element may be used. The resulting figure is similar to **Figure 13**. The mechanical resonance is visible on the spectrum because of the electromechanical coupling. Once the impedance versus frequency curve is obtained, the resonance and the antiresonance frequencies can be easily identified. Then the coupling coefficient, the quality factor, and the equivalent circuit parameters can all be found by using the equations below [29].

The electromechanical coupling coefficient *k* can be estimated as follows:

$$k = \sqrt{\frac{C_m}{C_d + C_m}} = \sqrt{1 - \left(\frac{f_o}{f_r}\right)^2}$$

Or k can be approximated by

$$k \approx \sqrt{\frac{C_m}{C_d}}$$

Since C_d is generally about two orders of magnitude larger than C_m. To represent the efficiency of the piezoceramic actuator, the quality factor can be estimated as:

$$Q = \frac{\omega_o m}{r_o}$$

where

$$\omega_o = \sqrt{\frac{1}{C_m L_m}} = \sqrt{\frac{K}{m}}$$

where,
m is the lumped mass of the piezoceramic and any attached metal piece
K is the lumped stiffness of the piezoceramic and metal

To determine C_d as a function of the admittance at a given frequency, the following equation is used

$$C_d = \frac{Y(@f)}{2\pi f}$$

where, $Y(@f)$ is the admittance at the frequency f. The frequency f is chosen to be in the lower frequencies range, (e.g. 1 kHz).

C_m and L_m can be calculated if C_d, and, the resonant and antiresonant frequencies f_o and f_r are determined. This is done as follows

$$C_m = C_d \frac{{f_r}^2 - {f_o}^2}{{f_o}^2}$$

$$L_m = \frac{1}{C_d (2\pi)^2 \left({f_r}^2 - {f_o}^2\right)}$$

R_m is the impedance value at resonance ($f = f_o$). The mechanical quality factor Q_m can be determined by the following equation [3]:

$$Q_m = \frac{1}{4\pi (C_d + C_m) r_o (f_r - f_o)}$$

3.3 Equivalent circuits for longitudinal and transverse effects

Sashida [29] mentioned that the ECM models presented above are for piezoelectric vibrators working in the transverse mode. Zhao [3] stated that the same circuit can account for the longitudinal effect by simply adding another capacitor with negative C_d value to the circuit as shown in **Figure 15**. The various RLC branches in the circuits represent the multiple resonance modes of the piezoelectric vibrator. Thus, for n resonance modes of the piezoelectric vibrator, there are *n* RLC branches in the equivalent circuit. However, if the analysis is made for a specific resonance frequency, e.g. the second one, then only the $R_2L_2C_2$ branch of the circuit may be used.

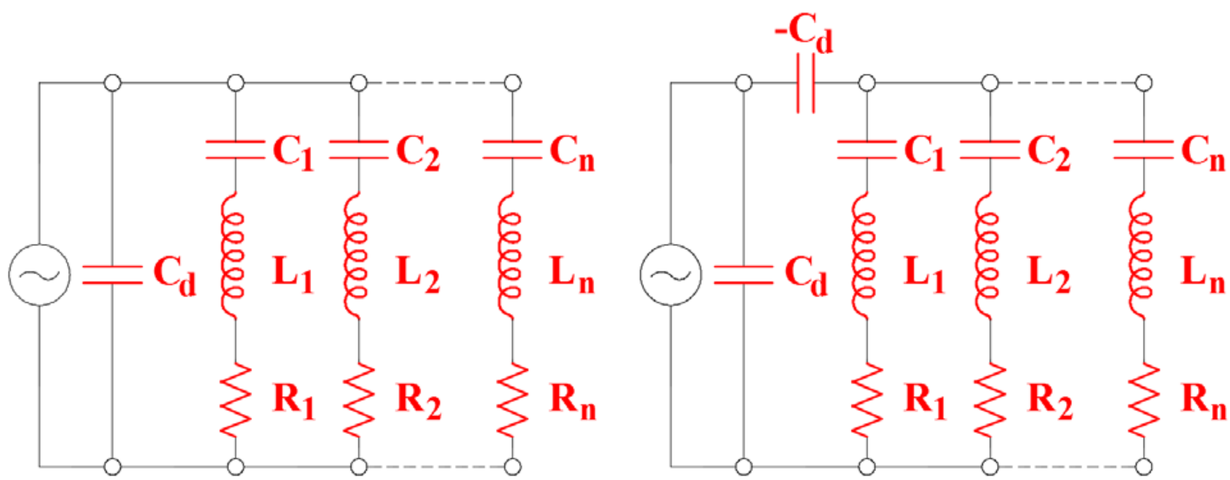


Figure 15. ECM models for piezoelectric vibrators, (a) the circuit for transverse effect, (b) the circuit for longitudinal effect.

3.4 Power requirements

The electrical power consumption of a piezoelectric actuator can be simply derived from the basic equation of power as follows:

$$P = I \times V$$

where,
I is the required current
V is the driving voltage
P is the power consumption

By assuming that the piezoelectric material can be represented as a capacitor only, then the relationship between current and voltage is

$$I = C\frac{dV}{dt} \ or \ I(s) = \ CsV(s)$$

For a sinusoidal input voltage, s=jω, therefore,

$$I(j\omega) = jC\omega V(j\omega)$$

By using the latter equation of current in the basic power equation, adding the loss tangent to consider the losses, and using the root mean square value for the AC voltage, the power required to drive a piezoceramic can be determined by the following equation [6]

$$P = 2\pi f \times C \times \tan\delta \times V_{rms}^2$$

where,
f is the driving frequency in Hz
C is the capacitance
$\tan \delta$ is the loss tangent
V_{rms} is the root mean square of the excitation voltage

References

[1] A. Preumont, *Mechatronics: Dynamics of Electromechanical and Piezoelectric Systems*. Springer Science & Business Media, 2006.

[2] S. Katzir, "Who knew piezoelectricity? Rutherford and Langevin on submarine detection and the invention of sonar," pp. 1–17, 2012.

[3] C. Zhao, *Ultrasonic motors: technologies and applications*. Springer Science & Business Media, 2011.

[4] S. Bauer and F. Bauer, "Piezoelectric Polymers and their Applications," in *Piezoelectricity*, 2013, pp. 157–177.

[5] W. Heywang, K. Lubitz, and W. Wersing, *Piezoelectricity*, vol. 114. 2008.

[6] T. L. Jordan and Z. Ounaies, "Piezoelectric Ceramics Characterization," 2001.

[7] B. Jaffe, W. R. Cook, and H. Jaffe, *Piezoelectric ceramics*. London, New York: Academic Press, 1971.

[8] S. H. Chang, N. N. Rogacheva, and C. C. Chou, "Analysis of methods for determining electromechanical coupling coefficients of piezoelectric elements," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 42, no. 4, pp. 630–640, 1995.

[9] Ferroperm Piezoeceramics, "High quality components and materials for the elctronic industry." p. 24.

[10] S. Sherrit and B. K. B. K. Mukherjee, "Characterization of Piezoelectric Materials for Transducers," *arXiv*, vol. arXiv:0711, no. Nye, p. 45, 2007.

[11] F. S. Meitzler, A., Tiersten, H. F., Warner, A. W., Berlincourt, D., Couqin, G. A., & Welsh III, "An American National Standard IEEE Standard on Piezoelectricity," 1988.

[12] S. E. Prasad, D. F. Waechter, R. G. Blacow, H. W. King, and Y. Yaman, "Application of piezoelectrics to smart structures," in *Il eccomas thematic conference on smart structures and materials*, 2005, pp. 1–16.

[13] "Sensor Technology (LTD)," 2016. [Online]. Available: http://sensortech.ca/.

[14] A. Safari and M. Abazari, "Lead-free piezoelectric ceramics and thin films," *Ultrason. Ferroelectr. Freq. …*, vol. 57, no. 10, 2010.

[15] S. Zhang, R. Xia, and T. R. Shrout, "Lead-free piezoelectric ceramics vs. PZT?," *J. Electroceramics*, vol. 19, no. 4, pp. 251–257, 2007.

[16] M. D. Maeder, D. Damjanovic, and N. Setter, "Lead free piezoelectric materials," *J. Electroceramics*, vol. 13, no. 1–3, pp. 385–392, 2004.

[17] P. K. Panda, "Review: Environmental friendly lead-free piezoelectric materials," *J. Mater. Sci.*, vol. 44, no. 19, pp. 5049–5062, 2009.

[18] P. Muralt, "Thin film piezoelectrics for MEMS," pp. 7–17, 2004.

[19] A. Mansingh, "Fabrication and applications of piezo-and ferroelectric films," *Ferroelectrics*, vol. 102, no. 1, pp. 69–84, 1990.

[20] Physik Instrumente (PI) GmbH & Co. KG, "Piezoelectrics in Positioning. Tutorial on

Piezotechnology in Nanopositioning Applications," 2008.

[21] H. Kaizuka and B. Siu, "A Simple Way to Reduce Hysteresis and Creep When Using Piezoelectric Actuators," *Japanese J. Appl. Phys. Part 2-Letters*, vol. 27, no. 5, pp. L773–L776, 1988.

[22] R. Changhai and S. Lining, "Hysteresis and creep compensation for piezoelectric actuator in open-loop operation," *Sensors Actuators, A Phys.*, vol. 122, no. 1 SPEC. ISS., pp. 124–130, 2005.

[23] D. P. Murphy, "Analysis of a Rotary Ultrasonic Motor for Application in Force-Feel Systems," 2008.

[24] G. Bal and E. Bekiro, "Characteristic estimation of travelling wave ultrasonic motor using equivalent circuit model," in *International conference on electrical and electronics engineering*, 2001, pp. 7–11.

[25] M. Goldfarb and N. Celanovjc, "A Lumped Parameter Electromechanical Model for Describing the Nonlinear Behavior of Piezoelectric Actuators," *Trans. ASME*, vol. 119, no. September 1997, pp. 478–485, 1997.

[26] M. Goldfarb and L. D. Jones, "On the Efficiency of Electric Power Generation with Piezoelectric Ceramic," *J. Dyn. Syst. Meas. Control*, vol. 121, no. September 1999, pp. 566–571, 1999.

[27] C. H. Park, "On the Circuit Model of Piezoceramics," *J. Intell. Mater. Syst. Struct.*, vol. 12, no. 7, pp. 515–522, 2001.

[28] M. Guan and W.-H. Liao, "Studies on the circuit models of piezoelectric ceramics," in *IEEE International Conference on Information Acquisition, 2004. Proceedings.*, 2004, pp. 26–31.

[29] T. Sashida and T. Kenjo, "Introduction to ultrasonic motors," 1993.

[30] K. Spanner, "Survey of the Various Operating Principles of Ultrasonic Piezomotors," *White Pap. Actuator 2006*, 2006.

[31] J. K. J. Kim, B. L. Grisso, J. K. Kim, D. S. H. D. S. Ha, and D. J. Inman, "Electrical modeling of Piezoelectric ceramics for analysis and evaluation of sensory systems," in *2008 IEEE Sensors Applications Symposium*, 2008, pp. 122–127.

Appendix (A)
Material properties for soft and hard PZT by Sensor Technology Ltd. (www.sensortech.ca)

Soft PZT Materials

	Symbols	Units	BM500	BM527	BM532
Electrical			Navy Type II	Navy Type V	Navy Type VI
Relative Dielectric Constant	εr^{T}_{33}	---	1750	2750	3250
Dissipation Factor	Tan δ	%	1.6	2.0	2.0
Piezoelectric					
Coupling Factor	kp	---	0.62	0.62	0.65
	k_{31}	---	0.37	0.37	0.39
	k_{33}	---	0.72	0.72	0.75
Charge Constant	d_{31}	10^{-12} C/N	-175	-215	-270
	d_{33}	10^{-12} C/N	365	500	590
Voltage Constant	g_{31}	10^{-3} V·m/N	-11.5	-9.5	-9.0
	g_{33}	10^{-3} V·m/N	25	22	20
Mechanical Quality Factor	Q_M	---	80	70	70
Frequency Constants	N_p	Hz·m	2050	2050	2000
	N_1	Hz·m	1400	1400	1425
	N_4	Hz·m	1800	1850	1850
Mechanical					
Compliance	S^{E}_{11}	10^{-12} m^2/N	15.5	14.5	14.0
	S^{E}_{33}	10^{-12} m^2/N	19.0	19.5	20.0
Density	ρ	g/cm3	7.65	7.6	7.65
Curie Temperature	Tc	^{o}C	360	225	210
Ageing Characteristics (% change/time decade)					
Coupling Factor	k_p	---	-0.5	-1.0	-1.0
Relative Dielectric Constant	εr^{T}_{33}	---	-1.0	-1.0	-1.0
Frequency Constant	N_4	Hz·m	0.5	1.0	1.0

Hard PZT Materials

	Symbols	Units	BM400	BM800	BM200
Electrical			Navy Type I	Navy Type III	
Relative Dielectric Constant	εr^T_{33}	---	1350	1000	1080
Dissipation Factor	Tan δ	%	0.4	0.3	0.3
Piezoelectric					
Coupling Factor	kp	---	0.60	0.50	0.60
	k_{31}	---	0.35	0.30	0.31
	k_{33}	---	0.70	0.64	0.64
Charge Constant	d_{31}	10^{-12} C/N	-125	-85	-100
	d_{33}	10^{-12} C/N	300	225	250
Voltage Constant	g_{31}	10^{-3} V·m/N	-10.5	-10.5	-10.0
	g_{33}	10^{-3} V·m/N	25	26	26
Mechanical Quality Factor	Q_M	---	500	1000	1000
Frequency Constants	N_p	Hz·m	2150	2350	2350
	N_1	Hz·m	1650	1700	1770
	N_4	Hz·m	1900	2000	1900
Mechanical					
Compliance	S^E_{11}	10^{-12} m^2/N	12.5	11.0	10.8
	S^E_{33}	10^{-12} m^2/N	15.0	13.5	15.4
Density	ρ	g/cm3	7.6	7.6	7.6
Curie Temperature	Tc	^oC	350	325	330
Ageing Characteristics (% change/time decade)					
Coupling Factor	k_p	---	-2.5	-2.5	-2.5
Relative Dielectric Constant	εr^T_{33}	---	-6.0	-6.0	-6.0
Frequency Constant	N_4	Hz·m	1.5	1.5	1.5

Appendix (B)
Modeling by ANSYS

Example:
Given that a piezoelectric bulk cylinder has 0.05 m radius and 0.1 m height. The piezoelectric material used is Pz26.

Tutorial 1: Apply electric field 200 V to the upper surface in 3-axis (**Z**-direction). Determine the strain ε in 3-axis (Z-direction). The boundary condition is that the lower surface has zero displacement and is at zero potential.

Answer:

$$\varepsilon = sT + d\bar{E}$$
$$d_{33} = 3.28\times10^{-10}\ \mathrm{m/V}$$
$$\varepsilon_3 = sT + d_{33}\bar{E}_3$$
$$\bar{E}_3 = \frac{V}{t} = \frac{200\ V}{0.1\ m} = 2000\ \frac{V}{m}$$
$$\varepsilon_3 = d_{33}\bar{E}_3 = 6.56\times10^{-7}\ \ or\ \ 0.656\times10^{-6}$$
$$\Delta L = L\,\varepsilon_3 = 0.1\times\ 0.656\times10^{-6} =\ 6.56\times10^{-8} = 0.0656\ \mu m \qquad \textbf{Calculated}$$
$$\Delta L\ (z - direction) = 0.061\ \mu m \qquad \textbf{ANSYS}$$

Tutorial 2: If the displacement in 3-axis (**Z**-direction) is 0.061µm, determine the input voltage V in 3-axis (Z-direction). The boundary condition is that the lower surface has zero displacement and is at zero potential.

Answer:

Uz (or ΔL) =0.061 µm

$$\varepsilon_3 = \frac{\Delta L}{L} = \frac{0.061\times10^{-6}}{0.1\ m} = 6.1\times10^{-7}$$
$$\varepsilon_3 = d_{33}\bar{E}_3$$
$$\bar{E}_3 = \frac{\varepsilon_3}{d_{33}} = \frac{6.1\times10^{-7}}{3.28\times10^{-10}} = 1859.7\ \approx 1860\ \frac{V}{m} \qquad \textbf{Calculated}$$
$$\bar{E}_3 = 1856.4\ \frac{V}{m} \qquad \textbf{ANSYS}$$

The calculated and the voltage by ANSYS are ≈ 186 V

Tutorial 3: Apply an electric field 200 V to the upper surface in 3-axis (**Z**-direction). Determine the strain ε in 1-axis (X-direction). The boundary condition is that the lower surface has zero displacement and is at zero potential.

Answer:

$$d_{31} = -1.28\times10^{-10}\ \mathrm{m/V}$$
$$\varepsilon_1 = d_{31}\bar{E}_3 = -1.28\times10^{-10}\times2000 =\ -2.56\ \times10^{-7} \qquad \textbf{Calculated}$$
$$\varepsilon_1 = 2.42\ \times10^{-7} \qquad \textbf{ANSYS}$$
$$\Delta L = L\,\varepsilon_1 = 0.1\times\ 2.56\times10^{-7} =\ -2.56\ \times10^{-8}\ m \qquad \textbf{Calculated}$$
$$\Delta L\ (x - direction) = -1.24\ \times10^{-8}\ m \qquad \textbf{ANSYS}$$

Tutorial 4: Apply a 100 N force to the upper surface in 3-axis (**Z**-direction). Determine the strain ε in 3-axis (Z-direction). The boundary condition is that the lower surface has zero displacement and is at zero potential.

Answer:

$$A=1.9635\times10^{-3}\ \mathrm{m^2}$$
$$T=50929.58\ \mathrm{N/m^2}$$
$$V=200\ \mathrm{V}$$
$$s_{33}=1.96\times10^{-11}\ \mathrm{m^2/N}$$

$$\varepsilon_3 = s_{33}T_3 + d_{33}\bar{E}_3$$
$$\varepsilon_3 = 1.96\times10^{-11}\times 50929.58 + 3.28\times10^{-10}\ \times 2000$$
$$\varepsilon_3 = 1.654\ \times 10^{-6}$$ **Calculated**
$$\varepsilon_3 = 1.62\ \times 10^{-6}$$ **ANSYS**