Performance of Modulation Schemes using Monte Carlo Simulation

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***Abstract*—This report documents the task that is required in the final project of the Graduate Course ECE 535A in the context of analyzing the performance of modulation schemes, typically M-ary Phase Shift Keying. The binary bits from transmitter are mapped to the constellation space which is dependent on M. Subsequently, these modulated bits are passed through AWGN channel and then is decoded using optimal detector. Monte Carlo Simulation is performed to estimate the bit error rate, symbol error rate with varying signal to noise ratio.**

***Index Terms*—AWGN, Monte Carlo, SNR, BER, SER, M-ary PSK, PAM.**

I. Introduction

IN Digital Modulation, one or more properties of the carrier signal, for example, amplitude, phase, frequency is varied according to the base-band signal. Base-band signal is a user defined message which is to be transmitted. The carrier signal is typically a high frequency signal containing no user related information. The purpose of the modulation is to transmit the base-band signal to farther location by impressing the information on carrier wave. Modulation of base-band signal is performed by modulator which maps the base-band signal to signal constellation. When this modulated signal is passed through the channel, bit errors are introduced due to the presence of noise, interference, synchronization errors or distortion from external sources. At this stage, the demodulator comes into the picture which uses optimal detector to decode into one of the possible transmitted base-band signals. In this case study the source transmits binary signal which is modulated using M-ary Phase Shift Keying (PSK) and is passed through Additive White Gaussian Noise (AWGN) channel wherein a noise is added by simulating different Signal to Noise Ratio (SNR) values. The receiver then decodes the received signal following the maximum a posteriori probability criterion. The probability of error or Bit Error Rate (BER) is computed by taking the ratio of number of bit errors divided by the total number of transmitted bits. The Symbol Error Rate (SER) is computed by taking the ratio of number of symbol errors divided by the total number of transmitted symbols. The source code is written in C++ programming language without using any predefined functions or libraries intended for M-ary modulation/demodulation techniques. In this simulation, the source sends approximately 800,000 binary bits to destination in total. Monte Carlo simulation is then performed 1000 times with different time-based seed to have more randomness and the data is then averaged. The rest of the section is as follows.

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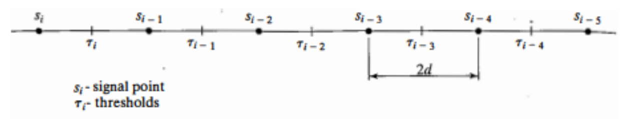

Fig. 1. Signal constellation in PAM

Section II derives the theoretical bit error probability for the M-ary Pulse Amplitude Modulation (PAM). Section III describes the performance of M-ary PSK schemes in the context of BER, SER with varying SNR which is achieved by performing Monte Carlo simulation. Section IV compares the performance of M-ary PSK schemes obtained through simulation with the theoretical expressions of BER, SER. Section V summarizes the task(s) involved in the project.

II. Probability of Error for M-ary PAM

In PAM, the amplitude of carrier signal is varied according to the base-band signal. Hence they are geometrically represented as M one-dimensional signal points as shown in the Figure 1. The demodulation is achieved by either correlator or matched filter. In the case of M-ary PAM, when the modulated signal is passed through AWGN channel, the input to the detector is

$$y = s_m + n \tag{1}$$

where n is a Gaussian random variable typically a noise with zero mean and variance $N_o/2$ and s_m denotes the mth transmitted amplitude level defined as

$$s_m = (2m - 1 - M)d, \quad m = 1, 2, ..., M \tag{2}$$

where d is the distance factor.

The detector bases its decision on the distance metrics as follows:

$$D(y, s_m) = (y - s_m)^2, \quad m = 1, 2, ..., M \tag{3}$$

The detector compares the input y with a set of $M - 1$ thresholds which are placed at the midpoints of successive amplitude levels, as shown in Figure 1. The decision rule favours that signal s_m that results in least distance metric when compared with the received signal y. Equivalently the above expression can be written as the correlation metric as follows:

$$C(y, s_m) = 2ys_m - s_m^2 \tag{4}$$

The average probability of error is simply the probability that the noise variable n exceeds in magnitude one-half of the distance between levels. However an error occurs when either one of the two outside levels $\pm(M-1)$ is transmitted [1]. Thus the error probability can be written as:

$$P_m = \frac{M-2}{M} P(|y - s_m| > d) + \frac{2}{M} P(y - s_m > d), \quad (5)$$

$$P_m = \frac{M-1}{M} P(|y - s_m| > d), \quad (6)$$

Since linear combination of Gaussian random variable is Gaussian random variable, we get,

$$P_m = \frac{M-1}{M} \frac{2}{\sqrt{2N_o}} \int_d^\infty e^{-x^2/N_o} dx \quad (7)$$

The above equation is modified as

$$P_m = \frac{M-1}{M} \frac{2}{\sqrt{2\pi}} \int_{\sqrt{2d^2/N_o}}^\infty e^{-x^2/2} dx \quad (8)$$

The above integral can be solved using Q-function. Hence the expression gets reduced as follows:

$$P_m = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{2d^2}{N_o}}\right) \quad (9)$$

III. Performance of M-ary PSK with varying SNR

The software is designed for general purpose M-ary PSK schemes. The user is asked to enter the integer value that indicates the modulation scheme. If the entered value is not valid, for example, if M happens not to be a power of 2, the user is prompted again until the valid value of M is entered. Subsequently the user is asked to enter the number of bits that source wishes to transmit. The software checks whether the number of bits, let's say K is an integer multiple of $log_2 M$. If not then the software sequentially increases the K until it becomes integer multiple of $log_2 M$. Grey coding is then performed such that the consecutive bits differ in Hamming distance of 1. This is important because if the presence of noise makes the bit go to either side of the decision boundary, it results in 1 bit error. Signal constellation is then carried out as a part of the modulation where each of the symbols out of $logM/log2$, is mapped on a unit circle which are equally spaced from each other. This is achieved by computing their angle, $2i\pi/M$ where $i = 0, 1, 2, ..., M-1$. Noise is then added to the In-phase and Quadrature-phase of the symbol. The decoder then calculates the distance between the received noisy symbol and all possible signal constellations and favors that constellation that has the least distance with the received noisy symbol. The simulation records the BER, SER with SNR beginning from -50 dB to +45 dB with increments of 5. In the Figure 2, it can be seen that the BER performance of all the considered modulation schemes (BPSK, QPSK, 8-ary PSK) improves as the SNR increases. The unnecessary details from -500 dB to -25 dB and from 15 dB to 45 dB are omitted in the figure. The probability of error was close to 0.5 when the SNR was -50 dB and this was consistent until the SNR reached around -25 dB. The probability of error then exponentially reduces with gradual increase in SNR. Onwards from 15 dB SNR, the probability of error is very much close to 0.

The same holds true for Figure 3. This time instead of bits, symbols are considered and probability of symbol error

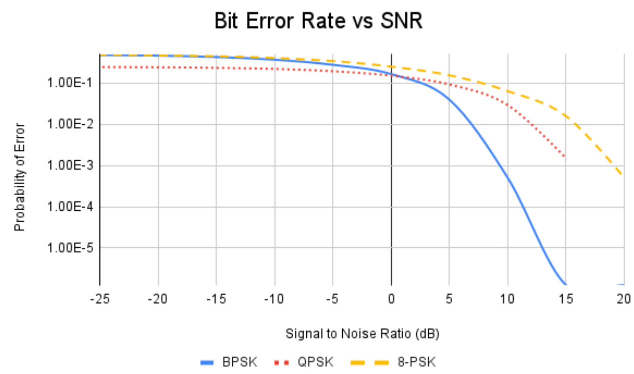

Fig. 2. BER with varying SNR

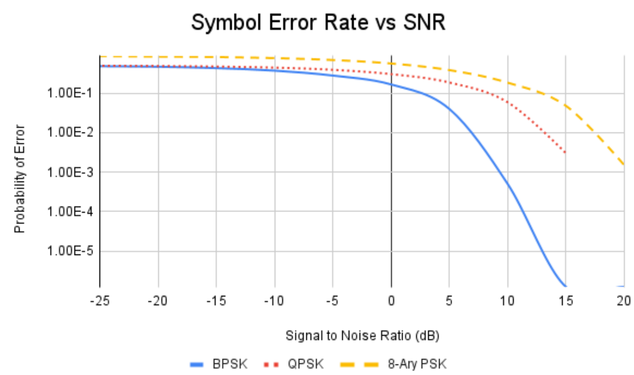

Fig. 3. SER with varying SNR

is calculated. As expected, for higher modulation schemes, high SNR is required to achieve same Probability of Error since the symbols are packed with more number of bits that require high SNR to achieve same performance.

IV. Comparison of M-ary PSK schemes obtained through Monte Carlo simulation with the theoretical expression

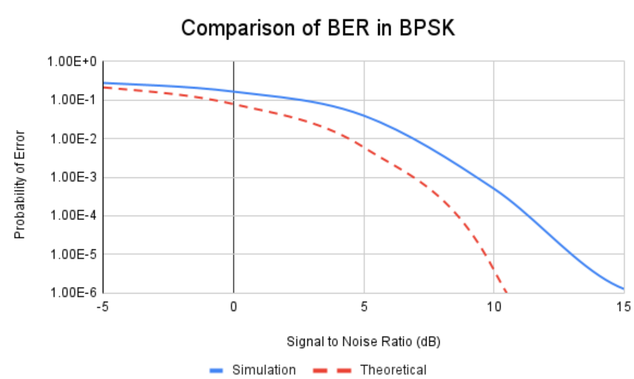

Fig. 4. BER comparison in BPSK

In this section, the performance of M-ary modulation schemes obtained through Monte Carlo simulation is compared with the theoretical expression [1]. The math library in C programming language cannot compute the Q-function directly but it can compute the complimentary error function value. The relationship between Q-function and complimentary error function is as follows:

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \tag{10}$$

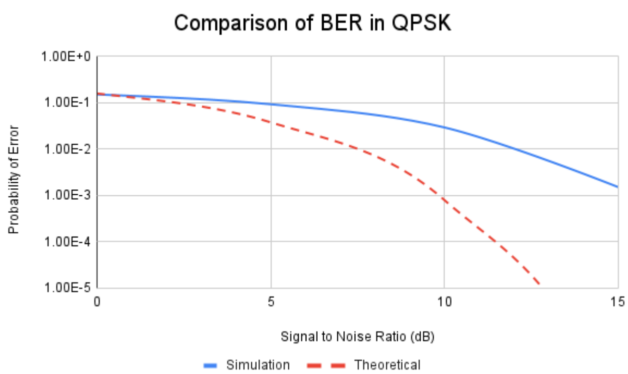

Fig. 5. BER Comparison in QPSK

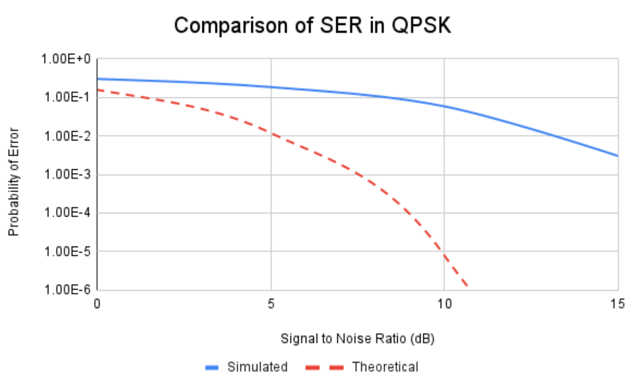

Fig. 6. SER Comparison in QPSK

It can be observed from Figures 4, 5, 6, 7, 8 that the Monte Carlo simulation has a little deviation from the theoretical expression when the SNR crosses a particular value. This little deviation is intuitively expected as there are some randomness generated when dealing with simulations. Overall, Monte Carlo simulation follows the similar trends which is decrease in error probability with increase in SNR with that of theoretical expression but not close.

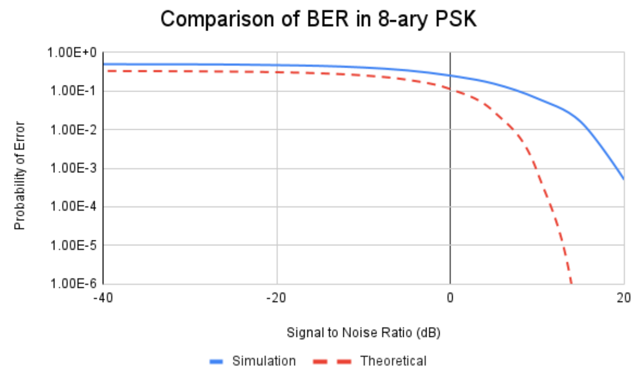

Fig. 7. BER Comparison in 8-ary PSK

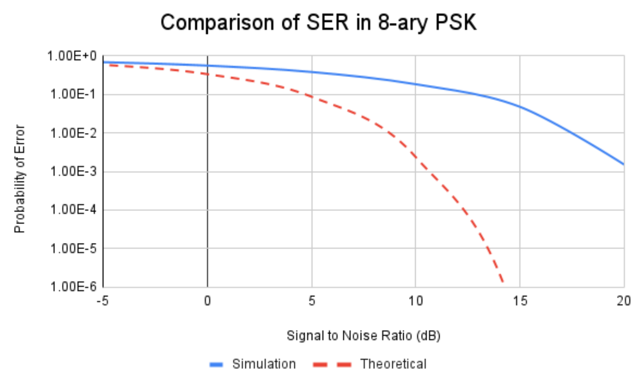

Fig. 8. SER Comparison in 8-ary PSK

V. Conclusion

Approx 800,000 binary bits are transmitted by source to the destination which are modulated using M-ary PSK schemes following Grey-code technique. Channel noise is introduced by using AWGN channel and the decoder detects the transmitted bits using MAP criteria. The simulation is run 1000 times. Performance of the M-ary PSK modulation is analyzed in terms of BER, SER with varying SNR using Monte Carlo simulation written in C++ programming language. Finally the performance of Monte Carlo simulation is compared with theoretical expression with varying SNR.

References

[1] J. G. Proakis and M. Salehi, *Digital communications*. McGraw-hill New York, 2001, vol. 4.