The Pendulum Turn: Are Rally Drivers Wrong?

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Problem Statement

PEP: Pendulum Turn

Experiment with vehicle parameters (especially inertia) to see if it occurs. (Are many rally drivers wrong?)

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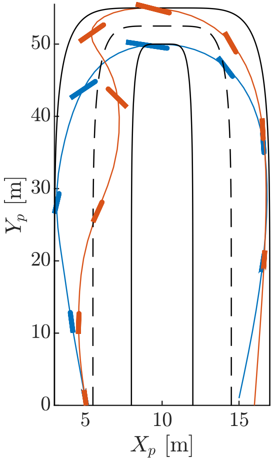
Cost functions:

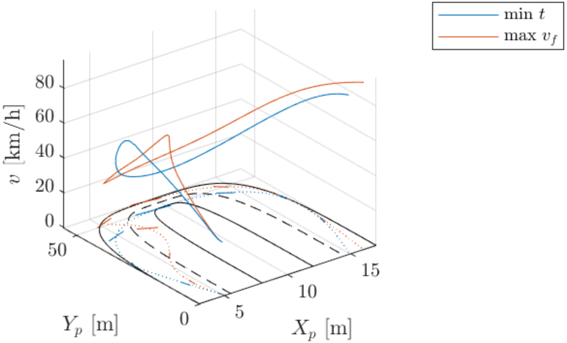
$$J = \begin{cases} \text{Min} \quad t + 0.1\,(\beta_x + \beta_y) & \text{min } t \\ \text{Max} \quad v_x(t_f) + 0.1\,(\beta_x + \beta_y) & \text{max } v_f \end{cases}.$$

subject to	$\dot{x} = f(x, u),$	ODE
	$f_u(u) \leq 0$	Constraints
	$f_o(x, u) \leq 0$	Path
	$x_0,\ x_f,$	Initial values

Some Results



Always interesting to see Arvind diagrams.



Seems like the pendulum turn is the optimal solution when the goal is to maximize exit velocity.

Let's talk numbers

	t_f	v_f
(topt) Min t_f	7.83 s	25.11 m/s (90.4 km/h)
(vopt) Max v_f	8.84 s	26.68 m/s (96 km/h)

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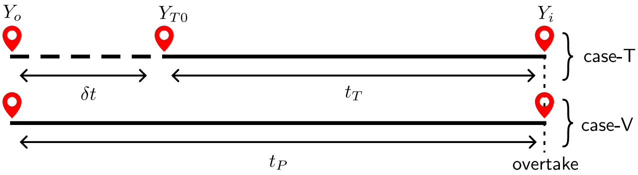

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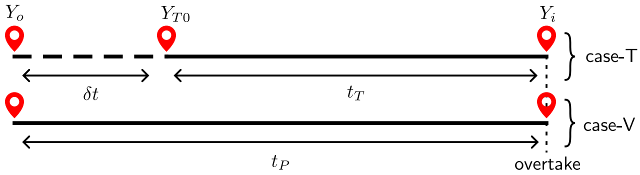
Assuming a point mass model.



Yes! If $V_{T0} < V_{yo(V)}$, i.e., $\mu < 0.158$.
So icy conditions at the end of the hairpin?

For the pendulum term to be "*viable*", $V_{T0} < V_{yo(V)}$. In other words, the topt solution?

Houston, we have a problem

The hairpin turn maneuver is difficult to find a solution.

400 Bad Request

- ☠ `Restoration Failed.`
- ☠ `Solver encountered NaN.`
- ☠ `Problem may be infeasable.`

Challenging to re-create the results.
Very sensitive to the initial conditions (initial guesses).

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A right-hand turn maneuver easier to find a solution.

So what's good

- No need for homotopic.
- Fast computation time.

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The investigation hereafter considers a right-hand turn maneuver for the ST-model with load transfer.

Some results

Still talking numbers.

		Dry Asphalt		Wet Asphalt		Snow	
		t_f	v_f	t_f	v_f	t_f	v_f
topt	Min t_f	6.64 s	29.1 m/s 104.77 km/h	6.86 s	27.68 m/s 99.65 km/h	10.9 s	17.64 m/s 63.49 km/h
vopt	Max v_f	8.15 s	29.66 m/s 106.78 km/h	7.94 s	28.23 m/s 101.69 km/h	11.77 s	18.18 m/s 65.43 km/h

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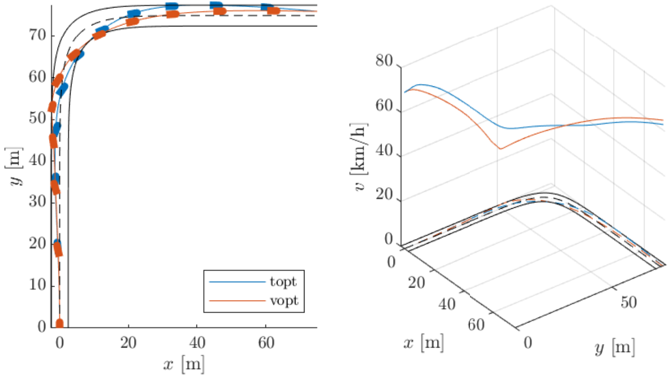
The velocity optimized is still "*slower*" than the time optimized

However. . .

We Have Hope

Similar time- and velocity-optimized trajectories for snow conditions.



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- Improving the model to include the benefits of "pendulum turn".
  Incorporate a DT-model.
  Norminal forces at steady state on the wheel:

$$F_{z(i)} = \frac{\mu m g \pm f(F_x, F_y)}{L}, \qquad \text{where } f(F_x, F_y) \text{ should be a non-linear function.}$$

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  Altering position of the center of gravity (affects LT).

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- Rally cars are not the same as stock cars.
  Altering position of the center of gravity (affects LT).
- Pendulum turn is more probably at low frictions (snow, gravel, etc.).

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