Optimal Vehicle Maneuvers — Lectures 3-4

Lars Nielsen[1] & Björn Olofsson[2]

[1]Division of Vehicular Systems, Linköping University, Sweden
[2]Department of Automatic Control, Lund University, Sweden

Lectures 3-4

Outline

Two main blocks:

- Block 1: Meeting May 9-10
  Introduction to optimization (optimal control). A goal is to provide a general tool with wide applicability, even though all examples are automotive aiming for emergency maneuvers at the limit of friction.
- Block 2: Meeting May 23-25
  Application of optimization (optimal control), analysis, crash databases, attainable forces, finding control principles, and developing on-line control schemes.

Examination:

- Meeting May 23-25 and June 19 (or other day)
  Discussion and presentation of homework assignments.
  On June 19 oral presentation and discussion of anything interesting you have found. You will have somewhat different assignments, so all aids are allowed including discussion with other persons (including course participants and teachers). The objective is that you say something interesting and can discuss it.

Course structure

Lectures in Block 1:

- Introduction (Lecture 1)
  - Autonomous driving at large
  - Course outline
  - Introduction to Optimal Vehicle Motion Control
- Optimization framework (Lecture 2)
  - Solving the Optimal Control Problem
  - Introduction, numerical tools, modeling issues
- Computing optimal maneuvers (Lectures 3-4)
  - Maneuvers: hairpin, curve, moose test (ISO-3888-2), ...
  - Vehicle models (particle, ST, DT)
  - Minimum time
  - Entry speed, v_0, and exit speed v_f
  - Recovery
  - Racing
  - Narrow lane and path tolerance

SAE's automation level definitions

SAE level	Name	Narrative definition	Execution of steering and acceleration/ deceleration	*Monitoring* of driving environment	Fallback performance of *dynamic driving task*	System capability *(driving modes)*
***Human driver* monitors the driving environment**						
0	**No automation**	the full-time performance by the *human driver* of all aspects of the *dynamic driving task*, even when enhanced by warning or intervention systems	Human driver	Human driver	Human driver	n/a
1	**Driver assistance**	the *driving mode*-specific execution by a driver assistance system of either steering or acceleration/deceleration using information about the driving environment and with the expectation that the *human driver* perform all remaining aspects of the *dynamic driving task*	Human driver and system	Human driver	Human driver	Some driving modes
2	**Partial automation**	the *driving mode*-specific execution by one or more driver assistance systems of both steering and acceleration/deceleration using information about the driving environment and with the expectation that the *human driver* perform all remaining aspects of the *dynamic driving task*	**System**	Human driver	Human driver	Some driving modes
***Automated driving system* ("system") monitors the driving environment**						
3	**Conditional automation**	the *driving mode*-specific performance by an *automated driving system* of all aspects of the dynamic driving task with the expectation that the *human driver* will respond appropriately to a *request to intervene*	System	**System**	Human driver	Some driving modes
4	**High automation**	the *driving mode*-specific performance by an automated driving system of all aspects of the *dynamic driving task*, even if a *human driver* does not respond appropriately to a *request to intervene*	System	System	**System**	Some driving modes
5	**Full automation**	the full-time performance by an *automated driving system* of all aspects of the *dynamic driving task* under all roadway and environmental conditions that can be managed by a *human driver*	System	System	System	**All driving modes**

- Critical situations
- but also other situations when dynamic effects my be important
  - High performance mining
  - Highway driving

Optimization Formulation

- The optimal control problem for minimization of a criterion J for a maneuver is formulated as:

$$\begin{aligned} \text{minimize} \quad & J \\ \text{subject to} \quad & A(u, \dot{u}, \ldots) \leq 0 \\ & F_c x(0) = \tilde{x}_0, \; G_c x(t_f) = \tilde{x}_f, \\ & f(X_p, Y_p) \leq 0 \\ & \dot{x} = G(x, y, u), \; h(x, y, u) = 0 \end{aligned}$$

- Criterion and Situation awareness: J and f.
- Actuator constraints: $A(u, \dot{u}, \ldots)$ are constraints on e.g. steering angle δ (or $\dot{\delta}$), and wheel torques T_i and $\dot{T}_i$.
- Vehicle model: Chassis dynamics, wheel dynamics, and tire model in $\dot{x} = G(x, y, u)$ and $h(x, y, u) = 0$.

One example was:
The friction limited particle avoiding an obstacle centered at $X = X_a$

$$
\begin{aligned}
\text{minimize} \quad & -v_0 \\
\text{subject to} \quad & u_1^2 + u_2^2 \leq (m\mu g)^2, \\
& x(0) = 0, \quad y(0) = 1, \quad x_f = 2X_a, \quad y_f = 1, \\
& \left(\frac{(X_P - X_a)}{R_1}\right)^6 + \left(\frac{Y_P}{R_2}\right)^6 \geq 1 \\
& m\dot{v}_x = u_1, \\
& m\dot{v}_y = u_2, \\
& \dot{x} = v_x, \\
& \dot{y} = v_y
\end{aligned}
$$

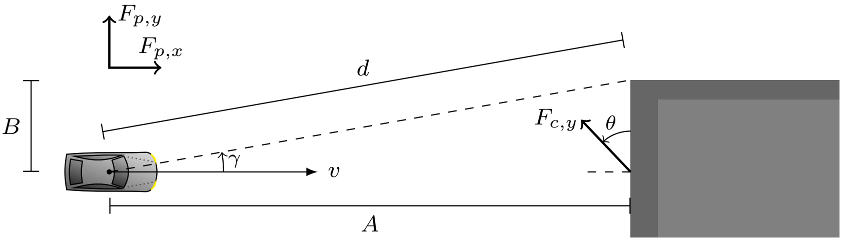
Program for coming lectures

The progarm now is to study and draw conclusions from different versions of the optimal control formulation.

- Other maneuvers, $f(X_P, Y_P)$
- Other models, $\dot{x} = G(x, y, u)$, $h(x, y, u) = 0$, and/or $A(u, \dot{u}, ...)$
- Other criteria, J.

Description of vehicle and obstacle(s)? Typically

- Represent vehicle by a point (in CoG)
- Extend size of obstacle accordingly



PEP: vehicle/obstacle representation

Represent vehicle by corners (two). Truck-trailer with more corners. Are the computation times much worse? (Single track but even particle.)

Maneuvers

Requirements on $f(X_P, Y_P) \leq 0$?

- Currently no improvements in computation with smoothness of f?
- Reason: it is just a comparison of positions (Oral communication with a researcher in the field)
- Counter argument: "experience" that it can be tricky to just pass by very close to a sharp corner (Oral communication with another researcher in the field)

Maneuvers

Maneuvers to be described by $f(X_P, Y_P) \leq 0$ are

- Avoidance
- Double lane change, ISO-3888-2 (“älgtest”, moose test, elk test)
- Curve (90 degrees with constant radius]
- Hairpin curve
- Moving moose
- (Race track)

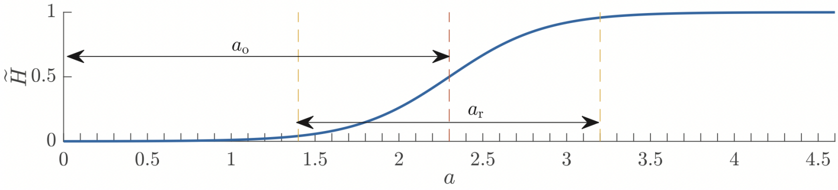
Maneuvers

Avoidance, Double lane change, ISO-3888-2 ("älgtest", moose test, elk test) and similar

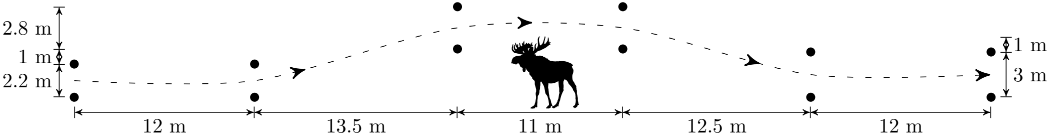
- Just define using cases (switching functions)
- Super ellipses
- Smooth switching functions

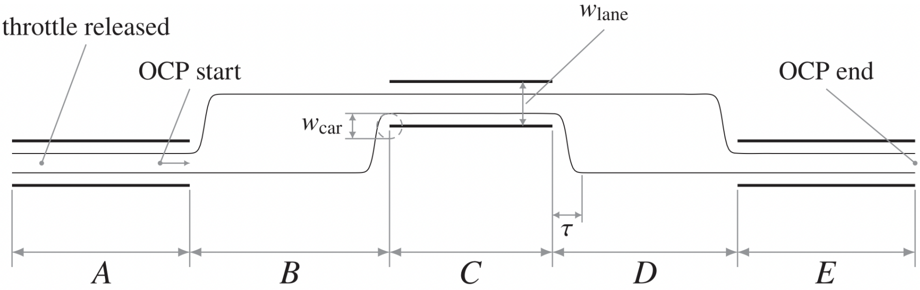
Different names like Sigmoid or Smooth Heaviside

$$H_{\text{sigmoid}}(x) = 0.5\left(1 + \tanh\left(2\pi x/\tau\right)\right)$$

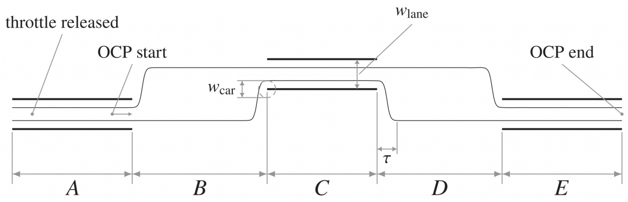


Double lane change, ISO-3888-2 ("älgtest", moose test, elk test)





Maneuvers



$$Y_p \leq a + c(H(X_p - X_{t1}) - H(X_p - X_{t2})),$$
$$Y_p \geq -a + c(H(X_p - X_{b1}) - H(X_p - X_{b2})),$$

Notation	Value [m]
A	12
B	13.5
C	11
D	12.5
E	12
w_{lane}	3
w_{car}	1.7
τ	2
a	$(w_{\text{lane}} - w_{\text{car}})/2$
c	$w_{\text{lane}} + 1$
X_{t1}	$A + \tau/2$
X_{t2}	$A + B + C + D - \tau/2$
X_{b1}	$A + B - \tau/2$
X_{b2}	$A + B + C + \tau/2$

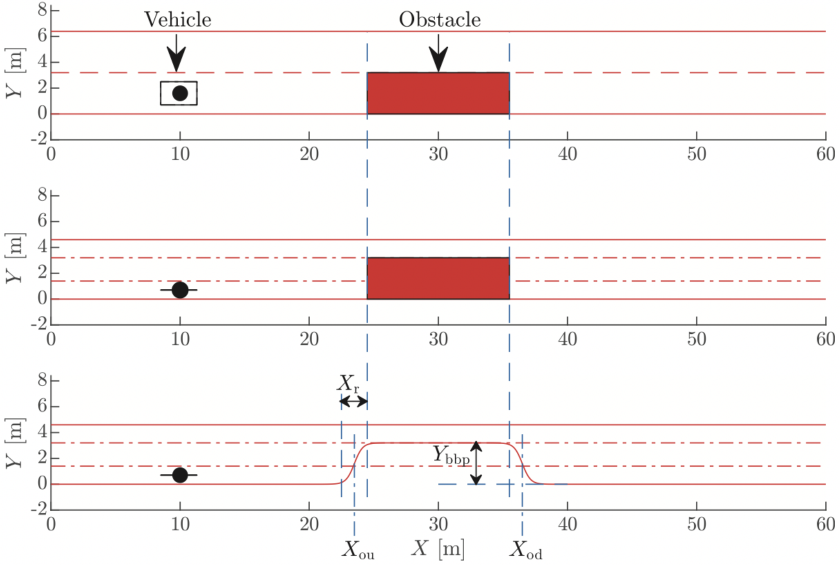
One more example of obstacle description by sigmoids (smooth Heavisides)

$$\widetilde{H}_{a_\mathrm{o}}^{a_\mathrm{r}}(a) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\pi}{a_\mathrm{r}}\left(a - a_\mathrm{o}\right)\right).$$

$$Y_\mathrm{bb}(X(t)) = Y_\mathrm{bbp}(\widetilde{H}_{X_\mathrm{ou}}^{X_\mathrm{r}}(X(t)) - \widetilde{H}_{X_\mathrm{od}}^{X_\mathrm{r}}(X(t))),$$

Notation	Value (unit)	Notation	Value (unit)
X_r	1.8 m	X_0	0 m
Y_{bbp}	3.2 m	Y_0	0.7 m
X_{ou}	23.5 m	X_{t_f}	100 m
X_{od}	36.5 m	Y_{t_f}	1.4 m
		v_0	70 km/h

Maneuvers



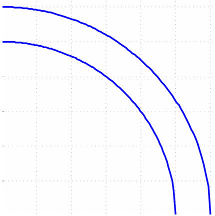
Maneuvers

Curve and Hairpin (typically much simpler to describe)

Sectors of circles, say curve 90 degrees of

$$\left(\frac{X_P}{R^i}\right)^2 + \left(\frac{Y_P}{R^i}\right)^2 \geq 1$$

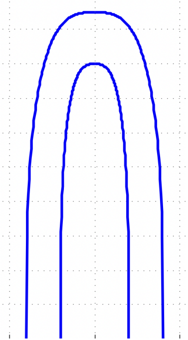
$$\left(\frac{X_P}{R^o}\right)^2 + \left(\frac{Y_P}{R^o}\right)^2 \leq 1$$



or sectors of super ellipses, say hairpin as 180 degrees of

$$\left(\frac{X_P}{R_1^i}\right)^6 + \left(\frac{Y_P}{R_2^i}\right)^6 \geq 1$$

$$\left(\frac{X_P}{R_1^o}\right)^6 + \left(\frac{Y_P}{R_2^o}\right)^6 \leq 1$$



Models

Choice of model depends on purpose

- Simulation
- Optimization
- Verification (needs to be able to add own functions to realistic model)

Detailed models for realistic simulation and verification

- CarMaker
- TruckMaker

Less complex models suitable for optimization

- Often more close to reality than expected
- Need to avoid artefacts explored by optimization algorithms

Models

Main model types for optimization are

- Friction limited particle (PM).
- Kinematic models (KM)
- Single track model (ST)
- Double track model (DT)

A number of subcathegories exist

- For ST and DT: direct force or wheel torque (always latter for us)
- Rate limits for direction control
- Models of road-tire interaction
  - linear vs non-linear
  - Circular or elliptical friction limits
  - Combined longitudinal and lateral forces
- For ST and DT: different versions of load transfer

Models

This course is based on the description in Chapter 2 of the PhD Thesis of Victor Fors.

- As declared before the course: this not a course in vehicle dynamics, i.e. derivation of the equations is not a topic here.
- Remember that the basic idea in the standard formulation of the optimization is to include these equations as is.

Course requirements

- You should read Chapter 2 of the PhD Thesis of Victor Fors.
- You should know the types of models used in optimization and be able to find all equations needed.

Models

The rest of this lecture part treating Models is aimed as help to read and be able to use Chapter 2 of the PhD Thesis of Victor Fors.

We have already introduced

- Friction limited particle (PM).
- Kinematic models (KM)

Will now continue with

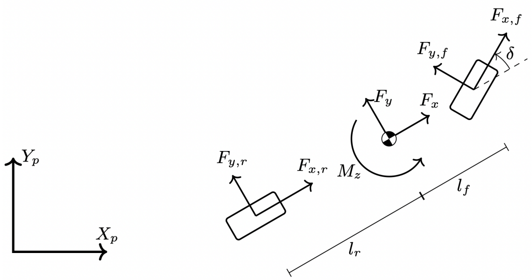
- Single track model (ST)
- Double track model (DT)

looking at the main parts

- Chassis modelling
- Tire and Wheel modelling
- Actuator modelling

ST - Single Track model

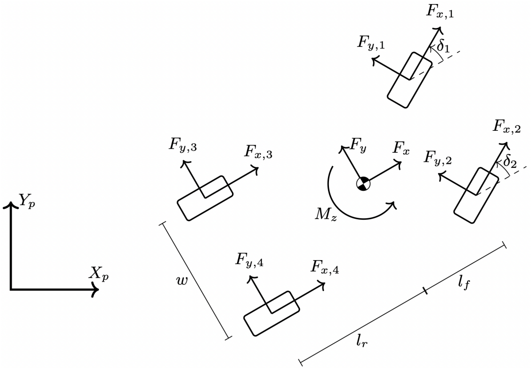
$$m(\dot{v}_x - v_y\dot{\Psi}) = F_{x,f}\cos(\delta) + F_{x,r} - F_{y,f}\sin(\delta)$$
$$m(\dot{v}_y + v_x\dot{\Psi}) = F_{y,f}\cos(\delta) + F_{y,r} + F_{x,f}\sin(\delta)$$
$$I_z\ddot{\Psi} = l_f F_{y,f}\cos(\delta) - l_r F_{y,r} + l_f F_{x,f} + \sin(\delta)$$



DT - Double Track model

Exactly the same as ST but now sum over four wheels instead of just two.

$$
\begin{aligned}
m(\dot{v}_x - v_y\dot{\Psi}) &= F_x \\
m(\dot{v}_y + v_x\dot{\Psi}) &= F_y \\
I_z\ddot{\Psi} &= M_z
\end{aligned}
$$



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\end{aligned}
$$

From the figure (on previous slide) we have
$l_{x,1} = l_{x,2} = l_f$, $l_{x,3} = l_{x,4} = -l_r$, $l_{y,1} = l_{y,3} = w/2$, and $l_{y,2} = l_{y,4} = -w/2$
giving the total forces as a sum over the four wheels

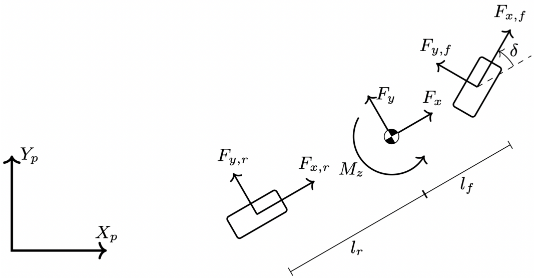
$$
\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -l_{y,i} & l_{x,i} \end{bmatrix} \begin{bmatrix} \cos(\delta_i) & -\sin(\delta_i) \\ \sin(\delta_i) & \cos(\delta_i) \end{bmatrix} \begin{bmatrix} F_{x,i} \\ F_{y,i} \end{bmatrix}.
$$

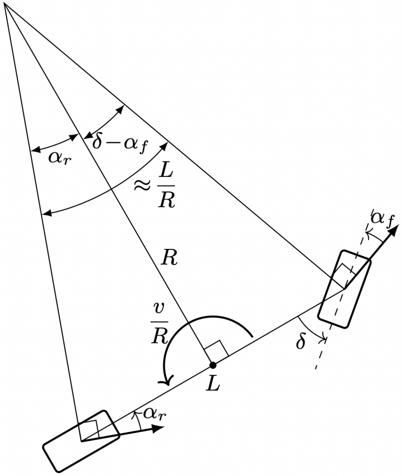
Models

How to describe the forces $F_{x,f}$, etc?

The longitudinal forces $F_{x,f}$ and $F_{x,r}$ are functions of longitudinal acceleration and braking.

The lateral forces $F_{y,f}$ and $F_{y,r}$ are functions of slip angles α_f, α_r.



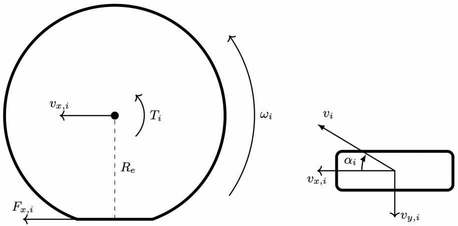


Models

How to describe the forces $F_{x,f}$, etc?

The longitudinal forces $F_{x,f}$ and $F_{x,r}$ are functions of longitudinal acceleration and braking expressed in longitudinal slip, κ.

The lateral forces $F_{y,f}$ and $F_{y,r}$ are functions of slip angles α_f, α_r.



(a) Side view. (b) Top view.

Figure 2.9: Wheel model.

Simplest force-slip relation - linear

$$F_{x,i} = k_i \kappa_i$$
$$F_{y,i} = C_i \alpha_i$$

Combined, simplest - Friction Ellipse (FE)

$$(\frac{F_{x,i}}{F_{x,i,max}})^2 + (\frac{F_{y,i}}{F_{y,i,max}})^2 \leq 1$$

- What is the physical reason for combined slip?
- Why ellipse and not circle?

Model by Pacejka (Magic formula)

- Better description of both longitudinal and lateral force-slip characteristics
- Can be combined with friction ellipse (FE)
- Better combined with weighting functions (WF)

Just static equations but with many parameters

$$
\begin{aligned}
F_{x,i} = {} & D_{x,i} \sin(C_{x,i} \arctan(B_{x,i}\kappa_i \\
& - E_{x,i}(B_{x,i}\kappa_i - \arctan(B_{x,i}\kappa_i)))), \\
F_{y,i} = {} & D_{y,i} \sin(C_{y,i} \arctan(B_{y,i}\alpha_i \\
& - E_{y,i}(B_{y,i}\alpha_i - \arctan(B_{y,i}\alpha_i)))),
\end{aligned}
$$

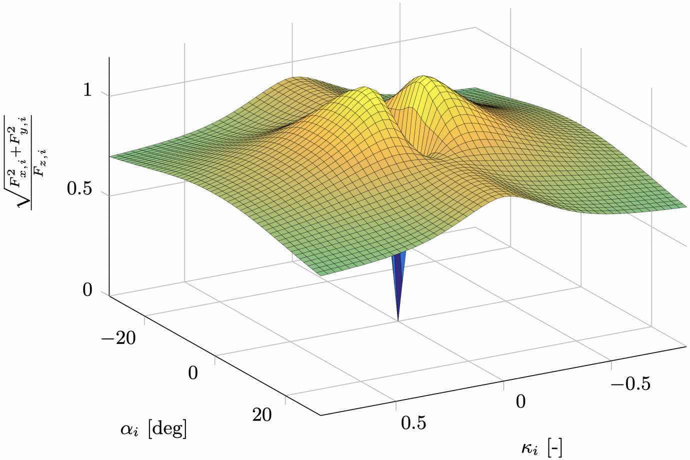
and even more equations for combined slip.
Assume linear dependence on friction, μ, and normal force, $F_{z,i}$, i.e. that

$$
D_{x,i} = \mu_{x,i} F_{z,i}, \quad D_{y,i} = \mu_{y,i} F_{z,i}.
$$

Typical force-slip diagram

WF which is short for Pacejka with Weighting Functions also implying that the non-linear force-slip functions are used,

- Localize linear regime
- Notice typical difference in longitudinal and lateral characteristics



Models

Tire modeling

- Previous slide is still normalized with the normal force, $F_{z,i}$,
- We need equations for $F_{z,i}$.

Two different concepts

- Load transfer (still assumption of rigid body; no added dynamics)
- Weight transfer (psrung mass)

Principle of Load transfer

Tire modeling

- We need equations for $F_{z,i}$.

Determined by vertical force and torque balances. See VF Chap 2 for derivation of

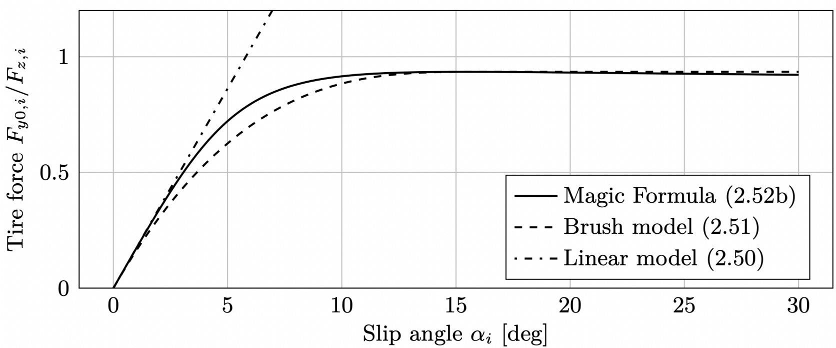
$$
\begin{aligned}
F_{z,1} &= \frac{mgwl_r - hwF_x - hLF_y}{2wL}, \\
F_{z,2} &= \frac{mgwl_r - hwF_x + hLF_y}{2wL}, \\
F_{z,3} &= \frac{mgwl_f + hwF_x - hLF_y}{2wL}, \\
F_{z,4} &= \frac{mgwl_f + hwF_x + hLF_y}{2wL}.
\end{aligned}
$$

Models

Tire modeling

The tire model is what changes when the road surface changes to where snow, gravel, or ice (recall that this is where the interaction betweeen road and tire is modeled).

Also simpler models work well. See comparison in Victor Fors Chapter 2:



Wheel modeling

$$I_w \dot{\omega}_i = T_i - R_e F_{x,i},$$

where T_i is the applied driving or braking torque on wheel i, I_w is the rotational wheel inertia, and R_e is the effective wheel radius.

To model the dynamics of the brake system, a first-order system with time constant γ_T is used.
Each wheel is individually actuated by the commanded torque $T_{u,i}$:

$$\gamma_T \dot{T}_i + T_i = T_{u,i}.$$

Actuator constraints

- It takes some time for a control command, like steering or braking, to develop.

Examples

- Brake actuator

$$T_{u,i,min} \leq T_{u,i} \leq T_{u,i,max}$$

where $T_{u,i,max} = 0$ if only braking is considered.

- Steering

$$|\delta| \leq \delta_{max}, \quad |\dot{\delta}| \leq \dot{\delta}_{max}$$

Typically $\dot{\delta} = u$.

ST with wheel dynamics and Pacejka WF.

- Three degrees of freedom for chassis, two for wheels
- Five states
- Typically works well

DT with wheel dynamics and Pacejka WF

- Three degrees of freedom for chassis, four for wheels
- Seven states
- Should work well, but little used (unfortunately too little?)
- Compared to ST differential braking possible to study since four wheels.

Now add weight transfer to DT, i.e. add sprung mass for pitch and roll

DT WF - Double Track model with weight transfer (WT) i.e. pitch and roll

Simple in principle but much book keeping of equations

For example pitch looks like

$$\begin{aligned}
\ddot{\theta} I_\theta + \nu_\theta &= -(D_\theta \dot{\theta} + K_\theta \theta) + h\tau_\theta, \\
I_\theta &= I_{yy}\cos^2(\phi) + I_{zz}\sin^2(\phi), \\
\nu_\theta &= r\Big(r\sin(\theta)\cos(\theta)(\Delta I_{xy} + \cos^2(\phi)\Delta I_{yz}) \\
&\quad - \dot{\phi}\cos^2(\theta) I_{xx} + \sin^2(\phi)\sin^2(\theta) I_{yy} \\
&\quad + \sin^2(\theta)\cos^2(\phi) I_{zz} - \dot{\theta}(\sin(\theta)\sin(\phi)\cos(\phi)\Delta I_{yz})\Big), \\
\tau_\theta &= mg\sin(\theta)\cos(\phi) - F_x\cos(\theta)\cos(\phi),
\end{aligned}$$

Models

DT WF

- Five degrees of freedom for chassis, four for wheels
- Eleven states (why not nine?)
- Needs new equations for the normal force on each tire, $F_{z,i}$.
- A very good model capturing all important aspects (?)
- The latter item is a main advantage of DT WF!

Minimum time

To large extent based on

1. **Models and methodology for optimal trajectory generation in safety-critical road-vehicle manoeuvres.** Karl Berntorp, Björn Olofsson, Kristoffer Lundahl, and Lars Nielsen (2014). Vehicle System Dynamics, 52(10), 1304–1332.
2. **An Investigation of Optimal Vehicle Maneuvers for Different Road Conditions** B. Olofsson, K. Lundahl, K. Berntorp, & L. Nielsen (2013). In *Proc. 7th IFAC Symposium on Advances in Automotive Control*, Tokyo, Japan.

Plan

- "'Models and methodology for ..." covered by the lectures, exercises, and course material.
- Now focus on some results and findings.

Minimum time

Study of 6 models of different complexity

- Criterion is minimum time
- Maneuvers are left turn and hairpin
- Wheel dynamics is included
- "Standard" actuator constraints are included

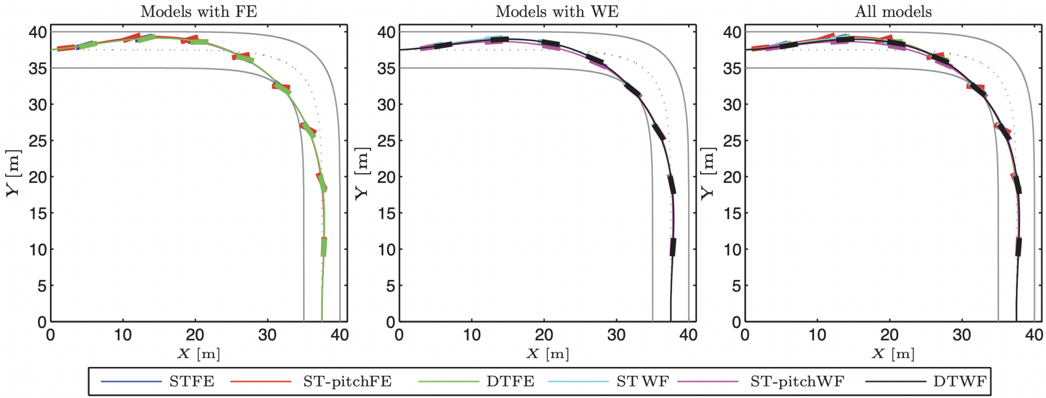
Minimum time

Study of 6 models of different complexity

- ST FE
- ST WF
- ST-pitch FE
- ST-pitch WF
- DT WT FE
- DT WT WF

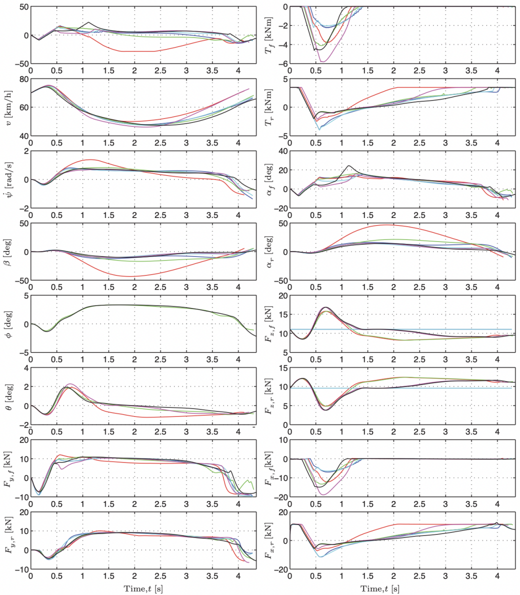
Minimum time

A main (and slightly surprising) finding is that the optimal paths are so similar.

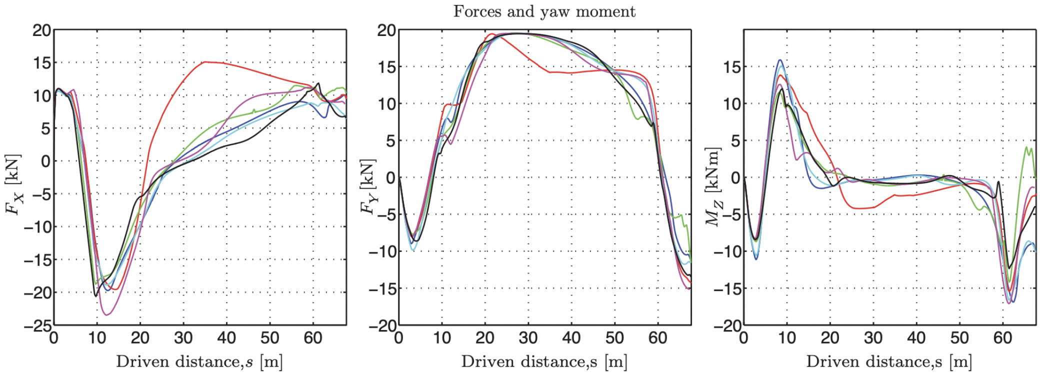


Minimum time

Looking at all variables gives detailed insight but is also rather complex to interpret.

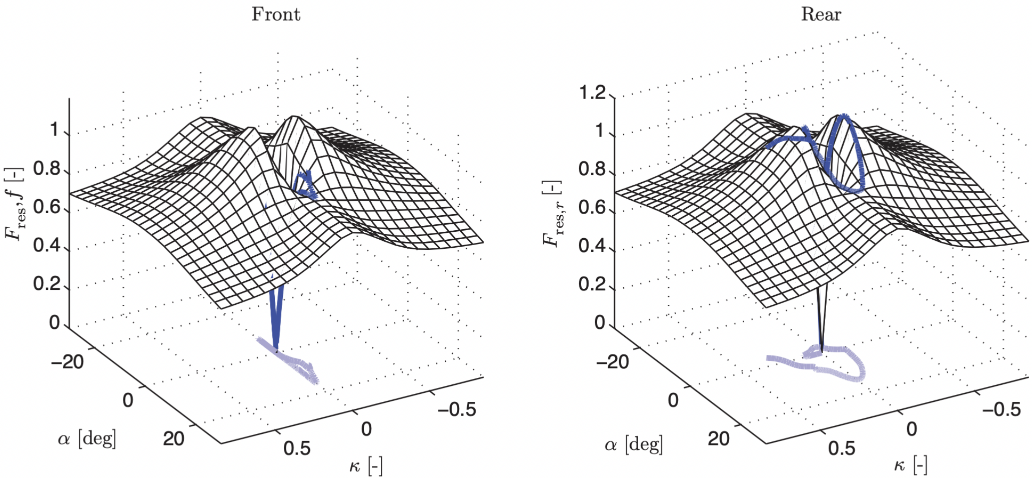


An easier, and many times fruitful, way of looking at, and comparing, solutions is to look at the TOTAL forces on the vehicle, F_X, F_Y, M_Z.



Force-slip diagrams (FS-diagrams or by some called Nielsen-diagrams), WF below

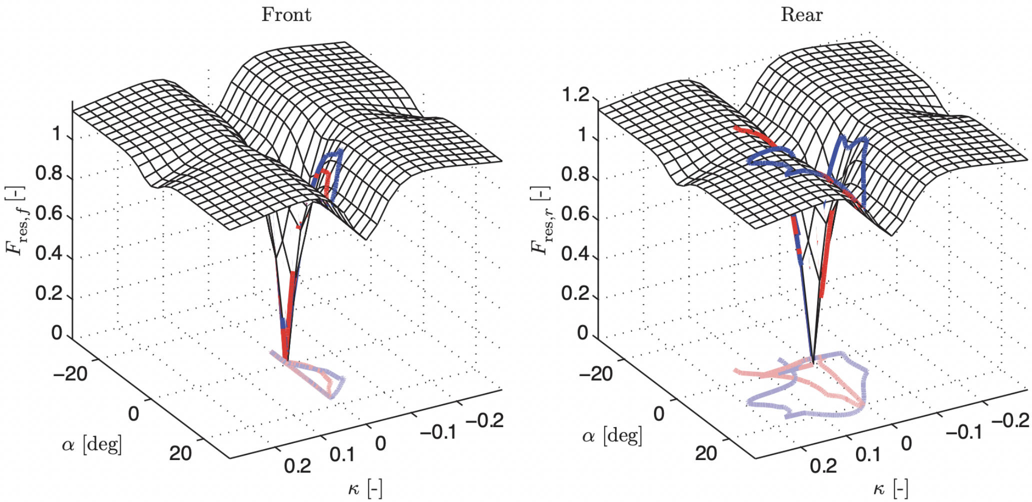
- Very useful especially during development
- But also to show that solution seems reasonable



Minimum time

and this one is FE

- $\sqrt{F_x^2 + F_y^2}$ as before
- Also plot solution in the diagram



Minimum time

Just a quick look at the case of a hairpin

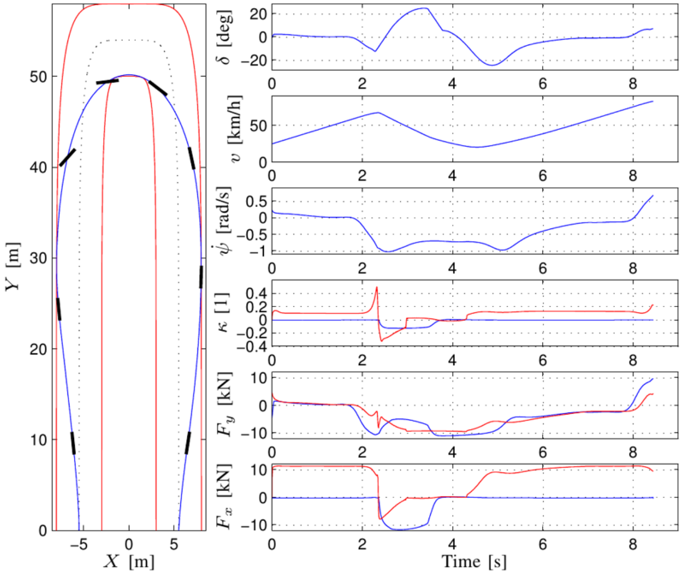
Overall conclusions are the same

We did not see a pendulum turn

PEP: Pendulum turn?

Experiment with vehicle parameters (especially inertia) to see if it occurs. (Are many rally drivers wrong?)

Note relation m, I_z.



What changes when road conditions are different?

Dry or wet asphalt, snow, or ice

Minimum time

Not changing only friction, μ, i.e. not just changing one variable.

The force-slip curves change in shape

Remember: A tire model is not a model of a tire, it is a model of tire-road interaction.

Parameter	Dry	Wet	Snow	Ice
$\mu_{x,f}$	1.20	1.06	0.407	0.172
$\mu_{x,r}$	1.20	1.07	0.409	0.173
$B_{x,f}$	11.7	12.0	10.2	31.1
$B_{x,r}$	11.1	11.5	9.71	29.5
$C_{x,f}, C_{x,r}$	1.69	1.80	1.96	1.77
$E_{x,f}$	0.377	0.313	0.651	0.710
$E_{x,r}$	0.362	0.300	0.624	0.681
$\mu_{y,f}$	0.935	0.885	0.383	0.162
$\mu_{y,r}$	0.961	0.911	0.394	0.167
$B_{y,f}$	8.86	10.7	19.1	28.4
$B_{y,r}$	9.30	11.3	20.0	30.0
$C_{y,f}, C_{y,r}$	1.19	1.07	0.550	1.48
$E_{y,f}$	-1.21	-2.14	-2.10	-1.18
$E_{y,r}$	-1.11	-1.97	-1.93	-1.08
$C_{x\alpha,f}, C_{x\alpha,r}$	1.09	1.09	1.09	1.02
$B_{x1,f}, B_{x1,r}$	12.4	13.0	15.4	75.4
$B_{x2,f}, B_{x2,r}$	-10.8	-10.8	-10.8	-43.1
$C_{y\kappa,f}, C_{y\kappa,r}$	1.08	1.08	1.08	0.984
$B_{y1,f}, B_{y1,r}$	6.46	6.78	4.19	33.8
$B_{y2,f}, B_{y2,r}$	4.20	4.20	4.20	42.0

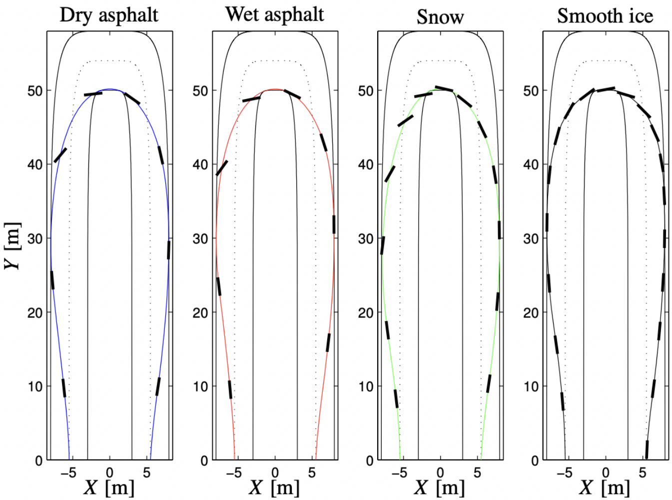
Minimum time

Once again:
Overall observation that the optimal paths are quite similar

However, speed is very different (look at number of vehicle indications in the case of smooth ice compared to dry asphalt).

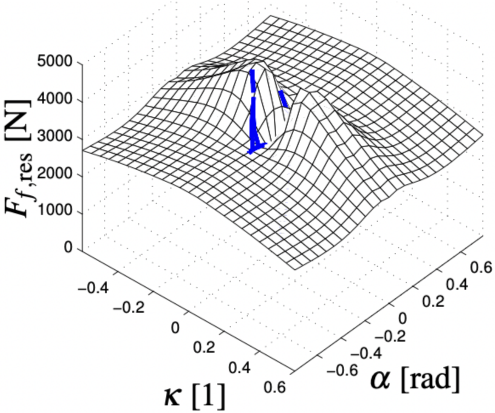
PEP: Velocity control

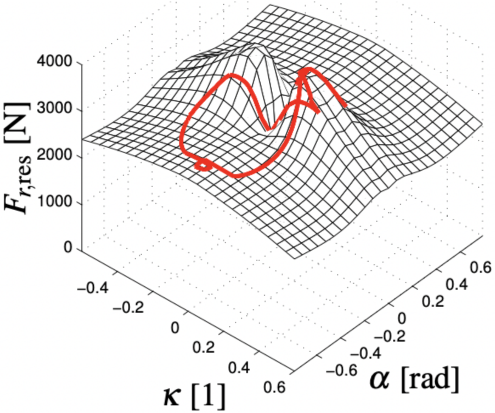
Philosphically, do you believe that velocity control is a viable control principle (compared to more common ideas)?



Also here Nielsen-diagrams are interesting

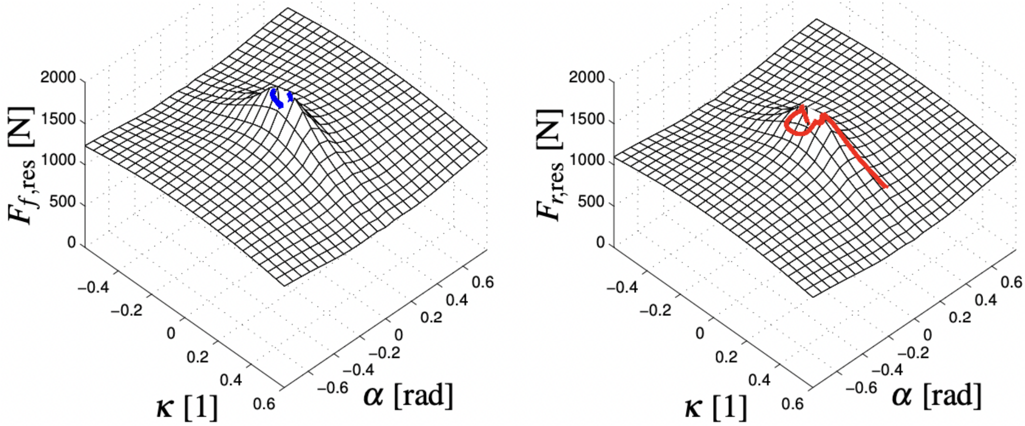
This one for snow





This one for ice

Notice how careful the trajectories are



max v_0 and v_f

To large extent based on

1. **Formulation and interpretation of optimal braking and steering patterns towards autonomous safety-critical manoeuvres.** Fors, V., Olofsson, B., and Nielsen, L. (2019). Vehicle System Dynamics, 57(8), 1206–1223.

Key ideas

- Criterion J as maximum entry epeed and maximum exit speed
- and interpolation between them
- Has been very fruitful with many interesting observations and consequences

max v_0 and v_f

The optimization problem studied:

- Criterion is interpolation between maximum entry epeed and maximum exit speed
- Maneuvers are left turn, avoidance, and double lane change.
- Wheel dynamics is included
- "Standard" actuator constraints are included

Thus

$$
\begin{aligned}
\text{minimize} \quad & J \\
\text{subject to} \quad & T_{i,\min} \leq T_i \leq T_{i,\max}, \;\; i \in \{f, r\} \text{ or } \{1, 2, 3, 4\} \\
& |\dot{T}_i| \leq \dot{T}_{i,\max}, \;\; i \in \{f, r\} \text{ or } \{1, 2, 3, 4\} \\
& |\delta| \leq \delta_{\max}, \;\; |\dot{\delta}| \leq \dot{\delta}_{\max} \\
& F_c x(0) = \tilde{x}_0, \;\; G_c x(t_f) = \tilde{x}_f, \\
& f(X_p, Y_p) \leq 0 \\
& \dot{x} = G(x, y, u), \;\; h(x, y, u) = 0
\end{aligned}
$$

with

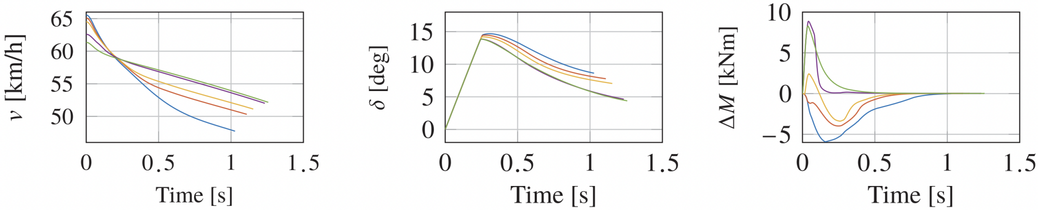
$$
J = -\eta v_0 - (1 - \eta) v_f
$$

max v_0 and v_f

Maneuver is left turn

Some typical solutions for different η in $J = -\eta v_0 - (1-\eta) v_f$

- Which one is $\eta = 1$ i.e. maximum entry speed?
- Which one is $\eta = 0$ i.e. maximum exit speed?

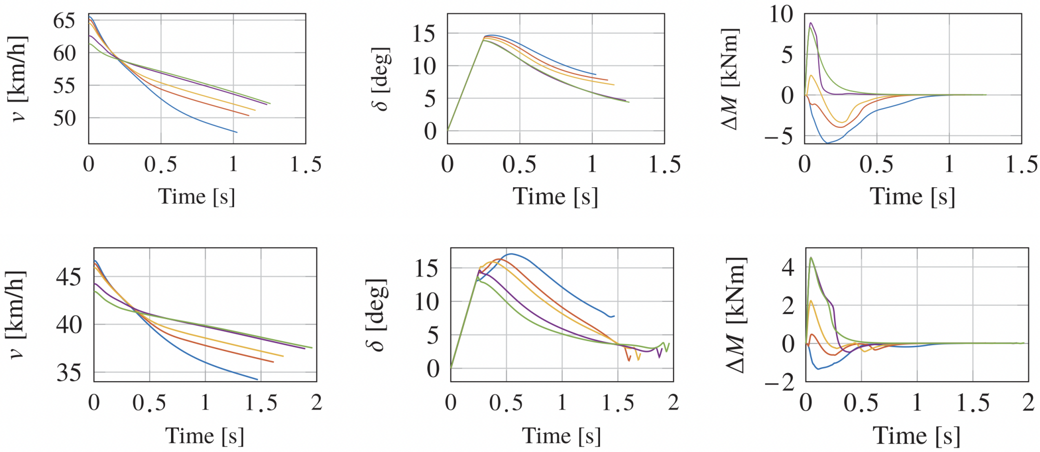


max v_0 and v_f

Maneuver is left turn

Another case with different road-tire friction

- Is the road-tire friction lower or higher in the bottom figure?

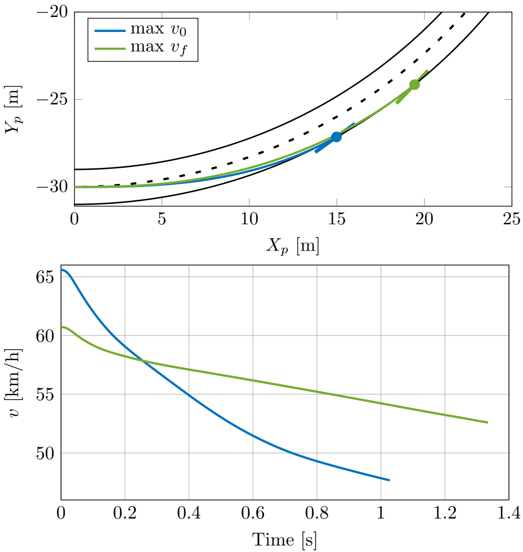


max v_0 and v_f

Now focusing on the two main cases

- Maximum entry speed, max v_0, is Optimal lane-keeping by design. Characterized by heavy braking.
- Maximum exit speed, max v_f, AYC, see next slide.

- Typical solution paths in upper plot (interpolations between).
- Typical braking curves in lower plot.

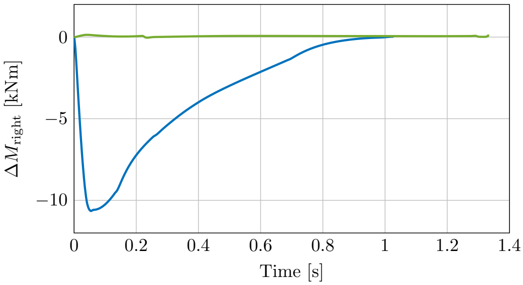


max v_0 and v_f

For the maneuver of maximum exit speed, max v_f, in a curve, the sum of the inner wheel torques (blue curve) and the sum of the outer wheel torques (green curve) are plotted. Discovery that green is zero.

Thus (important and useful):

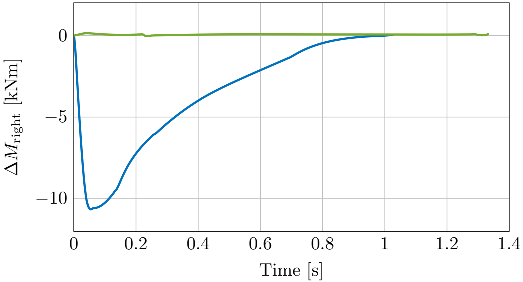
- Discovery:
  No braking on outer wheels for max v_f.
- Thus, a model for AYC behavior without having to implement a controller.



PEP: max v_f property

Prove that the torque contribution from outer wheels is zero.

$$
\begin{aligned}
\text{minimize} \quad & J \\
\text{subject to} \quad & A(u, \dot{u}, \ldots) \leq 0 \\
& F_c x(0) = \tilde{x}_0, \;\; G_c x(t_f) = \tilde{x}_f, \\
& f(X_p, Y_p) \leq 0 \\
& \dot{x} = G(x, y, u), \;\; h(x, y, u) = 0
\end{aligned}
$$



max v_0 and v_f

So far, the steering, δ, has been part of the optimization.

- Requires a steering servo (with some bandwidth).
- max v_0 directly gives OLKC (Optimal Lane Keeping Control).
- max v_f gives AYC/ESC with optimal steering.

How about a simpler steering principle (requiring less bandwidth in the steering servo)?
or a well behaved driver that steers in the direction of the road?

max v_0 and v_f

A beauty of the optimization formulation:

- Just add one single equation for the steering, δ (instead of having it as an optimization variable).

One can use nominal steering in a curve

$$\delta = \frac{L}{R}$$

- max v_0 gives optimal braking for this steering principle
- max v_f gives optimal AYC/ESC braking for this steering principle

max v_0 and v_f

Another (new) idea for a formulation is to steer in the direction of the road even if vehicle orientation has drifted a bit:

$$\delta = \theta - \psi$$

- max v_0 gives optimal braking for this steering principle
- max v_f gives optimal AYC/ESC braking for this steering principle

PEP: steering in direction of the road

Check that equation above is correct and solve optimization.
(Automatically a paper (cheaply?).)

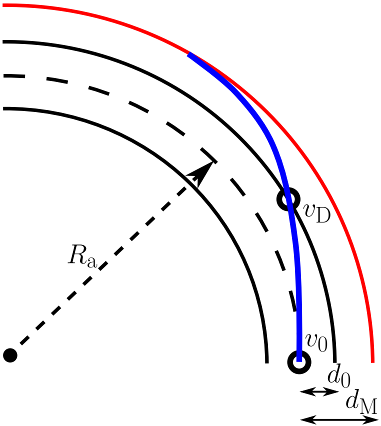
Is there significant potential in high performance automated steering?

Computing max v_0 given curve radius R, i.e., computing maximum manageable velocity in a curve is useful in crash database analysis.

Yet another use of optimization is to estimate Departure velocity, v_D, that is useful in risk analysis.

PEP: Determine departure velocity

Solve for v_D by including min R_o in the optimization (instead of iteration as in paper)



Lane Deviation Penalty (LDP)

Major examples of criteria, J, are as said before

- Minimum time (minimize t_f)
- Maximum entry speed (minimize $-v_0$)
- Maximum exit speed (minimize $-v_f$)

but other exist.

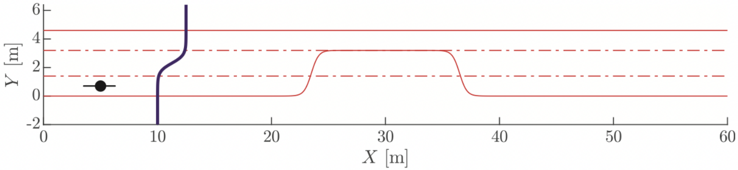
Pavel studied LDP (lane-deviation penalty)

Optional reading, not included in the course:

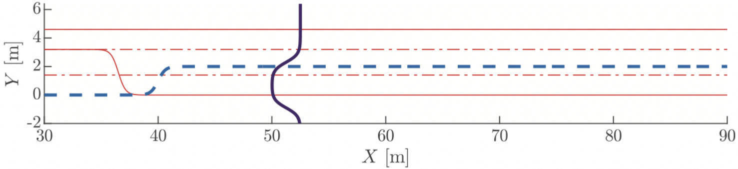
Lane-deviation penalty formulation and analysis for autonomous vehicle avoidance maneuvers. P Anistratov, B Olofsson, L Nielsen (2021) Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering Vol. 235, pp. 3036-3050

Lane Deviation Penalty (LDP)

- Main idea: it is dangerous to be in opposing lane
- Used smooth Heaviside to formulate criterion, $H_{Y_\mathrm{o}} = \widetilde{H}_{Y_\mathrm{o}}^{Y_\mathrm{r}}(Y(t))$
- Main takeaway: less time in opposing lane than minimum-time solution



(a)



(b)

Recovery

Recovery: After an avoidance maneuver you would like to go back to normal driving.

Isn't that trivial?

Illustration by a simple problem:
For an ST (single track model) solve minimum time respecting a velocity limit:

- Trivial solution: accelerate to the velocity limit, and then keep constant speed.
- Why has an optimization algortihm large problems, and may not converge at all?

Recovery: After an avoidance maneuver you would like to go back to normal driving.

Isn't that trivial?

Illustration by a simple problem:
For an ST (single track model) solve minimum time respecting a velocity limit:

- Trivial solution: accelerate to the velocity limit, and then keep constant speed.
- Why has an optimization algorthm large problems, and may not converge at all?

Because: at constant speed only the sum of the two wheel forces is determined

Mathematically: The wheel forces F_{xr}, F_{xf} can vary freely in the manifold
$F_{xr} + F_{xf} = const.$

Recovery

Optional reading, not included in the course:

Analysis and design of recovery behaviour of autonomous-vehicle avoidance manoeuvres. P Anistratov, B Olofsson, L Nielsen (2022). Vehicle system dynamics 60 (7), 2231-2254

However, central concepts should be known

- Adding an innocent looking requirement, like $v \leq v_{max}$, to the optimization problem can change it from well posed to ill posed.
- Singularity study (computing a determinant to see when it is zero) as a way of finding manifolds the solution can wander around without converging.
- Resolvning the problem by regularization.

Resolving actuator singularities by regularization:

Modify the criterion, J, by adding terms valid in the recovery zone (or wherever there is a velocity boundary).

Define recovery zone by smooth Heaviside in longitudinal direction

$$H_{X_1} = \widetilde{H}_{X_1}^{X_{\mathrm{r}}}(X(t)).$$

Extend J with something like (here for minimum time):

$$\begin{aligned} R_{X_1} = H_{X_1}(1 - H_{Y_1}) + H_{X_1}(p_v(v - v_{\mathrm{ref}})^2 + \\ \gamma + p_T(T_1^2 + T_2^2 + T_3^2 + T_4^2) + p_\delta \delta^2), \end{aligned}$$

Racetracks and racing line

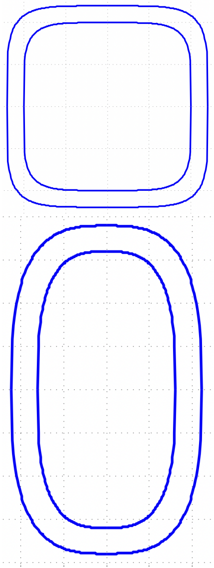
Racing line: The resulting path from solving the standard problem with minimum time as criterion

Considerations

- How to parameterize the track is a concern
- Initialization may be a problem. May need to do a simulation at lower speed to get an initial path and an initial velocity profile.

Simplest for first try: Circles or super ellipses, now not sectors but all the way around.

Call the Super ellipse "oval" a "Soval" (Stanford has a "Joval".)



Variable friction (rain)

After some rain the racing line will be very slippery compared to less used parts of the race track.

A friction model in corners that captures the basics is

$$\mu = \mu_i + (\mu_o - \mu_i)S(r)$$

where $S(r)$ is a (monotonic) function with $S(R_i) = 0$ and $S(R_o) = 1$.

It can be a smooth Heaviside or a polynomial:

- $S(r) = \widetilde{H}_{R_i}^{R_o}(r))$
- $S(r) = \left(\frac{r - R^i}{R^o - R^i}\right)^n$

Variable friction (rain)

After some time the track will dry up. Adding a model for this

$$\mu = \mu_i + (\mu_o - \mu_i)S(r)$$
$$\mu_i = \mu_{i0} + (\mu_o - \mu_{i0})\left(1 - e^{-t/\tau}\right)$$

Racing - Possible studies

PEP: Constant friction

Compute the racing line for Soval. Choose k, n for race track shape. Compare different values of constant friction μ. (May extend and vary model including steering constraints.)

PEP: Friction after rain

Compute the racing line for circular race track. Analytically when not time varying.

PEP: Extension

Try any extension of above.

PEP: Paper or thesis

Formulate a synopsis of an investigation or a definition of a Master thesis. Same amount of text as usual for a Master thesis proposal.

Narrow lane and path tolerance

Intuition

- Driving with CoG-tolerance 80 cm is easy
- Driving with CoG-tolerance 1 cm is difficult
- Driving with CoG-tolerance 1 cm at high speed is very difficult

To large extent based on

Implications of path tolerance and path characteristics on critical vehicle manoeuvres. K Lundahl, E Frisk, L Nielsen (2017). Vehicle system dynamics 55 (12), 1909-1945

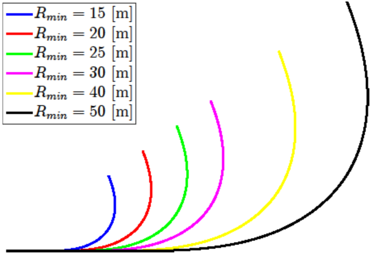
Narrow lane and path tolerance

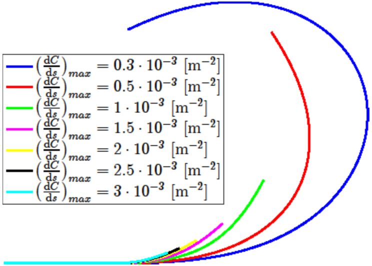
Study maneuvers having different path tolereances

- Vary tolerance (1, 5, 10, 20, 40, 80 cm)
- Maneuvers are Clothoids of varying aggressiveness
- Different model complexity (the usual suspects)

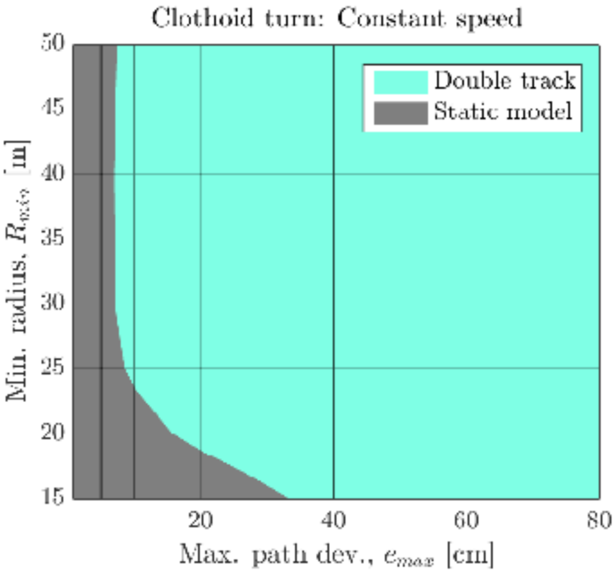
For the clothoids vary

- Minimum curvature
- Rate of curvature change





Narrow lane and path tolerance



Results (or rather indications)

- Simple model works surprisingly well down to small tolerances (like 10 cm)

Would have liked

- Comparison with TruckMaker
- More insight into roll-over
  - Indices or wheel lift (normal force zero)

Vehicle Motion Control

Course objectives

This first block of lectures has covered

Computation of optimal vehicle maneuvers and the first item below:

- Understanding at-the-limit driving
- Analyzing crash databases
- Obtaining control principles

The two last items in Lectures 5-8 in next block on May 23-25.