Optimal Vehicle Maneuvers — Lecture 1

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Lecture 1

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Outline

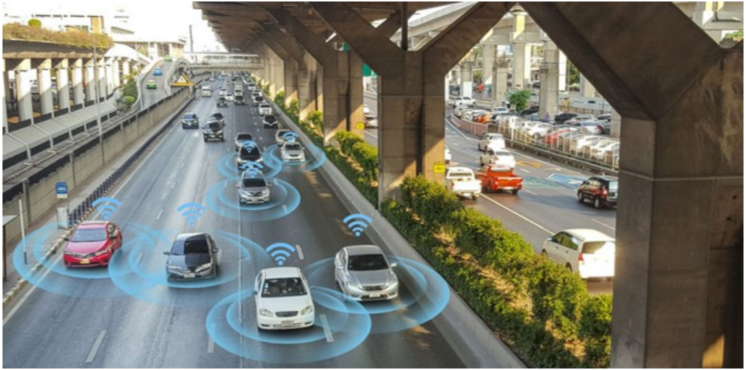
1. Autonomous driving in general
2. Course outline
3. Optimal Vehicle Motion Control - Introduction

Autonomous driving in general

Topics:

- Levels of autonomy (SAE definitions)
- Scenarios of autonomous driving in general
  - Comments on modeling

Autonomous driving in general



There are different levels of automation for vehicles

- Driver assistance systems
- Highly automated vehicles
- Fully autonomous vehicles

SAE's automation level definitions

In SAE's automation level definitions, "driving mode" means "a type of driving scenario with characteristic dynamic driving task requirements (e.g., expressway merging, high speed cruising, low speed traffic jam, closed-campus operations, etc.)"

- Level 0: The automated system issues warnings and may momentarily intervene but has no sustained vehicle control.

SAE's automation level definitions

- Level 1 ("hands on"): The driver and the automated system share control of the vehicle.
  Examples are systems where the driver controls steering and the automated system controls engine power to maintain a set speed (Cruise Control) or engine and brake power to maintain and vary speed (Adaptive Cruise Control or ACC); and Parking Assistance, where steering is automated while speed is under manual control. The driver must be ready to retake full control at any time. Lane Keeping Assistance (LKA) Type II is a further example of Level 1 self-driving. Automatic emergency braking which alerts the driver to a crash and permits full braking capacity is also a Level 1 feature, according to Autopilot Review magazine.

- Level 2 ("hands off"): The automated system takes full control of the vehicle: accelerating, braking, and steering. The driver must monitor the driving and be prepared to intervene immediately at any time if the automated system fails to respond properly.
  The shorthand "hands off" is not meant to be taken literally – contact between hand and wheel is often mandatory during SAE 2 driving, to confirm that the driver is ready to intervene. The eyes of the driver might be monitored by cameras to confirm that the driver is keeping their attention to traffic. A common example is adaptive cruise control which also utilizes lane keeping assist technology so that the driver simply monitors the vehicle, such as "Super-Cruise" in the Cadillac CT6 by General Motors.

SAE's automation level definitions

- Level 3 ("eyes off"): The driver can safely turn their attention away from the driving tasks, e.g. the driver can text or watch a movie. The vehicle will handle situations that call for an immediate response, like emergency braking. The driver must still be prepared to intervene within some limited time, specified by the manufacturer, when called upon by the vehicle to do so. You can think of the automated system as a co-driver that will alert you in an orderly fashion when it is your turn to drive. An example would be a Traffic Jam Chauffeur, another example would be a car satisfying the international Automated Lane Keeping System (ALKS) regulations.

SAE's automation level definitions

- Level 4 ("mind off"): As level 3, but no driver attention is ever required for safety, e.g. the driver may safely go to sleep or leave the driver's seat. However, self-driving is supported only in limited spatial areas (geofenced) or under special circumstances. Outside of these areas or circumstances, the vehicle must be able to safely abort the trip, e.g. slow down and park the car, if the driver does not retake control. An example would be a robotic taxi or a robotic delivery service that covers selected locations in an area, at a specific time and quantities.

SAE's automation level definitions

- Level 5 ("steering wheel optional"): No human intervention is required at all. An example would be a robotic vehicle that works on all kinds of surfaces, all over the world, all year around, in all weather conditions.

SAE's automation level definitions

Handling of critical situations needed for

- Level 3, 4, and 5, of course
- Level 2. It is not realistic that a non-alert driver can be expected to take over control if there suddenly is a critical moment.
- Level 1. For critical situations same as Level 2.

SAE's automation level definitions

SAE level	Name	Narrative definition	Execution of steering and acceleration/ deceleration	*Monitoring* of driving environment	Fallback performance of *dynamic driving task*	System capability *(driving modes)*
***Human driver* monitors the driving environment**						
0	**No automation**	the full-time performance by the *human driver* of all aspects of the *dynamic driving task*, even when enhanced by warning or intervention systems	Human driver	Human driver	Human driver	n/a
1	**Driver assistance**	the *driving mode*-specific execution by a driver assistance system of either steering or acceleration/deceleration using information about the driving environment and with the expectation that the *human driver* perform all remaining aspects of the *dynamic driving task*	Human driver and system	Human driver	Human driver	Some driving modes
2	**Partial automation**	the *driving mode*-specific execution by one or more driver assistance systems of both steering and acceleration/deceleration using information about the driving environment and with the expectation that the *human driver* perform all remaining aspects of the *dynamic driving task*	**System**	Human driver	Human driver	Some driving modes
***Automated driving system* ("system") monitors the driving environment**						
3	**Conditional automation**	the *driving mode*-specific performance by an *automated driving system* of all aspects of the dynamic driving task with the expectation that the *human driver* will respond appropriately to a *request to intervene*	System	**System**	Human driver	Some driving modes
4	**High automation**	the *driving mode*-specific performance by an automated driving system of all aspects of the *dynamic driving task*, even if a *human driver* does not respond appropriately to a *request to intervene*	System	System	**System**	Some driving modes
5	**Full automation**	the full-time performance by an *automated driving system* of all aspects of the *dynamic driving task* under all roadway and environmental conditions that can be managed by a *human driver*	System	System	System	**All driving modes**

Autonomous driving at large - some major examples

Maneuvering in confined areas like harbors or mines.

- Low speed and very low lateral acceleration
- Kinematic vehicle models

Autonomous driving at large - some major examples

Traffic or Highway driving

- Medium or high speed, but limits on lateral acceleration (say less than 0.3G)
- Kinematic models or simple dynamic vehicle models

Critical situations

- At-the-limit acceleration (in order of 1G)
- Dynamic vehicle models

1 Autonomous driving in general

2 Course outline

3 Optimal Vehicle Motion Control - Introduction

Course structure

Remark: Compared to tentative course program in December, there are some changes in course structure.

The following slides give the actual course plan (as for this moment).

Two main blocks:

- Block 1: Meeting May 9-10
  Introduction to optimization (optimal control). A goal is to provide a general tool with wide applicability, even though all examples are automotive aiming for emergency maneuvers at the limit of friction.
- Block 2: Meeting May 23-25
  Application of optimization (optimal control), analysis, crash databases, attainable forces, finding control principles, and developing on-line control schemes.

Examination:

- Meeting May 23-25 and June 19 (or other day)
  Discussion and presentation of homework assignments.
  On June 19 oral presentation and discussion of anything interesting you have found. You will have somewhat different assignments, so all aids are allowed including discussion with other persons (including course participants and teachers). The objective is that you say something interesting and can discuss it.

Course structure

Examination

- Overall rule: Decided individually in consultation with examiners
- Nominally 6 credits, but increased or decreased ambition can be discussed.
- For example, for Block 1 there are two mandatory assignments (with individual twists) and one or more to be chosen from a set of about ten.

Within frame of course or as Increased ambition

- Some low hanging fruits possible for joint or own papers (conference or more).
- Final assignment could be from your own research field.
- Possibilities: Increased credits for this course or separate course (in consultation with your supervisor).

Course structure

Lectures in Block 1:

- Introduction (Lecture 1)
  - Autonomous driving at large
  - Course outline
  - Introduction to Optimal Vehicle Motion Control
- Optimization framework (Lecture 2)
  - Solving the Optimal Control Problem
  - Introduction, numerical tools, modeling issues
- Computing optimal maneuvers (Lectures 3-4)
  - Maneuvers: hairpin, curve, moose test (ISO-3888-2), ...
  - Vehicle models (particle, ST, DT)
  - Minimum time
  - Entry speed, v_0, and exit speed v_f
  - Recovery
  - Racing
  - Narrow lane and path tolerance

Schedule

Schedule for Block 1:

- Tuesday May 9 Systemet
  - 09.30 Lecture 1
  - 10.45 Lecture 2
  - 13.15 Exercises
- Wednesday May 10 L-huset
  - 08.30 Lecture 3
  - 09.30 Lecture 4
  - 13.15 Exercises
- Wednesday May 17
  - 08.30 Tentative follow-up on Zoom

Schedule

Schedule for Block 2:

- Tuesday May 23 Visionen
  - 09.30 Lecture 5
  - 10.30 Exercises/presentations/discussions
- Wednesday May 24 Visionen
  - 08.30 Lecture 6
  - 09.30 Lecture 7
- Thursday May 25 L-huset
  - 08.30 Lecture 8
  - 09.30 Exercises/assignments/discussions/etc
- Some day June 7-9
  - Some time: Tentative follow-up on Zoom
- Monday June 19
  - Examination

Course readings

1. **Models and methodology for optimal trajectory generation in safety-critical road-vehicle manoeuvres.** Karl Berntorp, Björn Olofsson, Kristoffer Lundahl, and Lars Nielsen (2014). Vehicle System Dynamics, 52(10), 1304–1332.
2. **Formulation and interpretation of optimal braking and steering patterns towards autonomous safety-critical manoeuvres.** Fors, V., Olofsson, B., and Nielsen, L. (2019). Vehicle System Dynamics, 57(8), 1206–1223.
3. **Using Crash Databases to Predict Effectiveness of New Autonomous Vehicle Maneuvers for Lane-Departure Injury Reduction.** Björn Olofsson, and Lars Nielsen (2021). IEEE Transactions on Intelligent Transportation Systems, 22(6), 3479–3490.
4. **Attainable force volumes of optimal autonomous at-the-limit vehicle manoeuvres.** Victor Fors, Björn Olofsson, and Lars Nielsen (2020). Vehicle System Dynamics, 58(7), 1101–1122.
5. **Autonomous Wary Collision Avoidance.** Victor Fors, Björn Olofsson, and Lars Nielsen (2021). IEEE Transactions on Intelligent Vehicles, 6(2), 353–365.
6. **Predictive force-centric emergency collision avoidance.** Fors, V., Anistratov, P., Olofsson, B., and Nielsen, L. (2021). ASME Journal of Dynamic Systems, Measurement and Control, 143(8), 081005.

Course readings

Needed background (to be able to use the results, not to derive)

1. **Autonomous Vehicle Maneuvering at the Limit of Friction.** Victor Fors (2020). PhD thesis, Chapter 2.

Course readings

Optional readings (explained in lectures)

1. **An Investigation of Optimal Vehicle Maneuvers for Different Road Conditions** B. Olofsson, K. Lundahl, K. Berntorp, & L. Nielsen (2013). In *Proc. 7th IFAC Symposium on Advances in Automotive Control*, Tokyo, Japan.
2. **Analysis and design of recovery behaviour of autonomous-vehicle avoidance manoeuvres.** P Anistratov, B Olofsson, L Nielsen (2022). Vehicle system dynamics 60 (7), 2231-2254
3. **Implications of path tolerance and path characteristics on critical vehicle manoeuvres.** K Lundahl, E Frisk, L Nielsen Vehicle system dynamics 55 (12), 1909-1945

Very optional (for those with an extra interest in high performance path following)

1. **Path-tracking velocity control for robot manipulators with actuator constraints** B Olofsson, L Nielsen (2017). Mechatronics 45, 82-99.

1 Autonomous driving in general

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Vehicle Motion Control

Typical layers in Vehicle Motion Control:

- Basic operations (autonomous or not): steering, braking, accelarating, . . . , ABS, . . .
- Vehicle maneuvers (autonomous or not): obstacle avoidance, double lane change, cornering, . . . (typically open free space or simple obstacles)
- Driving (autonomous or not): composed maneuvers, handling traffic situatins, going with the flow, . . . (simplified compared to open free space driving thanks to lane markers etc, but more complex due to more interactions)
- Drive planning (autonomous or not): route planning,. . .

In this course we focus on the second layer "Vehicle Maneuvers", assuming the first layer of basic operations given.

Optimal Control

What do we mean by Optimal Vehicle Motion Control?

Optimal control can mean at least two things

- Finding optimal open-loop control. Gives an optimal solution, for example as function of time, t, or as function of road coordinate, s.
- Finding an optimal controller for closed-loop control in the sense of optimized control parameters. An example is LQG design.

In this course, in this first Block 1, we are going to look at the first item above.
Two questions:

- Why?
- How? What is assumed known and what is solved for?

Optimal Vehicle Motion Control

Optimal control: Why?

Prof RS Sharp 2014: Most often, the optimal control itself will be interesting mainly insofar as it enables the discovery of the best possible system performance. Occasionally, the optimal control will provide a basis for the design and operation of practical systems.

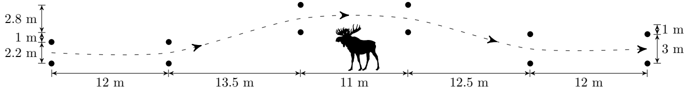
Inspiration:
Chris Gerdes: "Race and rally drivers can save critical situations."
Larger set of maneuvers at hand
New advanced safety systems?

Optimal Vehicle Motion Control

Why?:
An optimal control solution can be used directly, for example, as reference values for a controller.

At least three more uses of optimal control

- Understanding at-the-limit driving
- Analyzing crash databases
- Obtaining control principles



Optimal Vehicle Motion Control

Having discussed "Why optimal control?" we are now entering "How?"

The formulation of an optimal vehicle motion control problem has a number of components:

- Situation awareness, basically describing free space to maneuver in.
- Model of vehicle dynamics. This includes the interface between road and tire which is included in the tire model.
- Actuator constraints, typically physical limitations of actuator behavior.
- Initial and final conditions
- A criterion to minimize

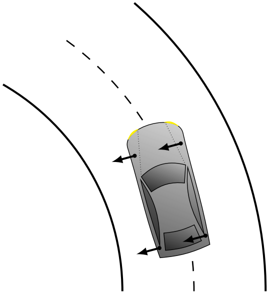
Now come slides that treat the formulation of the items above.

Situation Awareness - Position Constraints

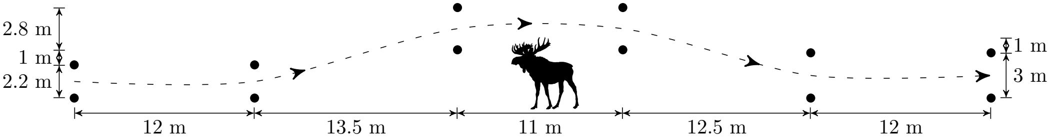
A modern vehicle knows its surroundings, lane limits, obstacles,...

Industrial trend for (at least some) vehicle manufacturers

- Inherit platforms for situation awareness from Nvidia and other providers.
- Motion control is expertise and core business for these OEMs



In new cars a view from above is presented!
Even though cameras are onboard (Tesla, Volvo, ...).



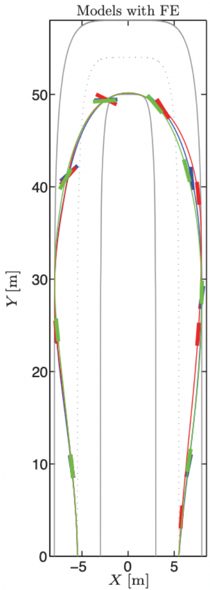
Situation Awareness - Position Constraints

Situation awareness is assumed. Lane limits, obstacles, etc, are formulated as constraints on vehicle position. Mathematically:

$$f(X_p, Y_p) \leq 0$$

- Available free space described by set of $\leq$-statements.
- Sometimes simplifying "tricks" e.g. superellipses

$$\left(\frac{X_P}{R_1^i}\right)^6 + \left(\frac{Y_P}{R_2^i}\right)^6 \geq 1$$
$$\left(\frac{X_P}{R_1^o}\right)^6 + \left(\frac{Y_P}{R_2^o}\right)^6 \leq 1$$



Optimal Vehicle Motion Control

Vehicle dynamics model

- This is not a course in vehicle dynamics. Models will be given. Some very easy to develop, some not so easy. (Easy in principle but many terms to keep track of.)

What does dynamics mean?

- In mathematics: any differential equation. This includes kinematic models with velocity as input.
- In mechanics: Newton's equations, i.e., a differential equation relating forces to motion. This does not include kinematic models with velocity as input.

Both types are fine to use in the optimization framework

Vehicle dynamics model

Two introductory examples:

- Point mass (dynamic model in the sense of mechanics)
- Kinematic model (not a dynamic model in the sense of mechanics)

A friction limited particle is a 2-D particle

$$m\dot{v}_x = ma_x = u_1,$$
$$m\dot{v}_y = ma_y = u_2.$$

with actuator constraint (corresponding to the friction ellips of road-tire interaction)

$$a_x^2 + a_y^2 \leq (\mu g)^2,$$

or equivalently

$$u_1^2 + u_2^2 \leq (m\mu g)^2,$$

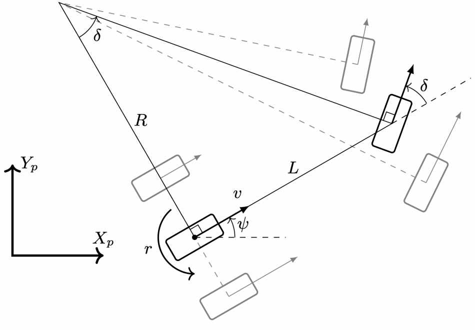
A typical common kinematic model is

$$\begin{aligned}
\dot{X}_p &= v\cos(\psi), \\
\dot{Y}_p &= v\sin(\psi), \\
\dot{\psi} &= v\frac{\tan(\delta)}{L}, \\
u_1 &= \delta, \\
u_2 &= v
\end{aligned}$$

A steering constraint can be added by

$$\begin{aligned}
|\delta| &\leq \delta_{max}, \\
|\dot{\delta}| &\leq \dot{\delta}_{max},
\end{aligned}$$

and then using $\dot{\delta}$ as input u_1.



Kinematic model with illustration of Ackermann steering.

Vehicle dynamics model

We have seen two introductory examples, both important and much used:

- Point mass (dynamic model in the sense of mechanics)
- Kinematic model (not a dynamic model in the sense of mechanics)

Introducing PEP as short notation for either "Possible Exercise Problem" or "Possible Exam Problem".

PEP: Kinematic model vs Point mass model

Which one is simplest? Which is best?

Actuator constraints

- It takes some time for a control command, like steering or braking, to develop.

For example

- Force limited point mass. With or without rate limit.
- Wheel dynamics. The propulsion torque or braking torque is input to the wheel rotational dynamics.

We have already seen some examples

$$u_1^2 + u_2^2 \leq (m\mu g)^2,$$

and

$$|\delta| \leq \delta_{max},$$
$$|\dot{\delta}| \leq \dot{\delta}_{max}$$

More generally

$$A(u, \dot{u}, \ldots) \leq 0$$

Optimal Vehicle Motion Control

Criterion J to optimize (minimize)

- Any criterion like energy or comfort can be used.
- However, aiming at studies of at-the-limit maneuvers typically a criterion requiring high performance is used. Examples are short time or high speed.

Major examples

- Minimum time (minimize t_f)
- Maximum entry speed (minimize $-v_0$)
- Maximum exit speed (minimize $-v_f$)

PEP: Time difference between these three criteria

Compute and compareare time used for these criteria. You can use avoidance for kinematic model or point mass model, or some other scenario.

Initial and final conditions

$$F_c x(0) = \tilde{x}_0, \; G_c x(t_f) = \tilde{x}_f$$

- Why the tilde, i.e. why $\tilde{x}_0$ instead of x_0?
- Used to indicate that not all state variables need to be assigned initial values (or final values). (Sometimes an initial (or final) value may even be the criterion, e.g. when searching for maximum entry speed (or maximum exit speed).)
- The matrices F_c, G_c pick out the components that are assigned values.

Optimal Vehicle Motion Control

The components below can now be collected in a mathematical optimzation problem:

- Situation awareness, basically free space to maneuver in.
- Model of vehicle dynamics. This includes the interface betewwen road and tire which is included in the tire model.
- Actuator constraints, typically physical limitations of actuator behavior.
- Initial and final conditions
- A criterion to minimize

This is done on next slide

Optimization Formulation

- The optimal control problem for minimization of a criterion J for a maneuver is formulated as:

$$\begin{aligned} \text{minimize} \quad & J \\ \text{subject to} \quad & A(u, \dot{u}, ...) \leq 0 \\ & F_c x(0) = \tilde{x}_0, \;\; G_c x(t_f) = \tilde{x}_f, \\ & f(X_p, Y_p) \leq 0 \\ & \dot{x} = G(x, y, u), \;\; h(x, y, u) = 0 \end{aligned}$$

- Criterion and Situation awareness: J and f.
- Actuator constraints: $A(u, \dot{u}, ...)$ are constraints on e.g. steering angle δ (or $\dot{\delta}$), and wheel torques T_i and $\dot{T}_i$.
- Vehicle model: Chassis dynamics, wheel dynamics, and tire model in $\dot{x} = G(x, y, u)$ and $h(x, y, u) = 0$.

Finally an example:
The friction limited particle avoiding an obstacle centered at $X = X_a$

$$\begin{aligned}
\text{minimize} \quad & -v_0 \\
\text{subject to} \quad & u_1^2 + u_2^2 \leq (m\mu g)^2, \\
& x(0) = 0, \quad y(0) = 1, \\
& -\left(\frac{(X_P - X_a)}{R_1}\right)^6 - \left(\frac{Y_P}{R_2}\right)^6 + 1 \leq 0 \\
& m\dot{v}_x = u_1, \\
& m\dot{v}_y = u_2, \\
& \dot{x} = v_x, \\
& \dot{y} = v_y
\end{aligned}$$

Note the minus signs and the two last equations.

Vehicle Motion Control

Next, a closely related example:

The friction limited particle with rate limited direction control where the direction, δ of the force is controlled by :

$$|\delta| \leq \delta_{max}, \quad |\dot{\delta}| \leq \dot{\delta}_{max}$$

where

$$\dot{\delta} = u.$$

The friction limited particle with rate limited direction control:

$$
\begin{aligned}
\text{minimize} \quad & -v_0 \\
\text{subject to} \quad & u_1^2 \leq (m\mu g)^2, \\
& x(0) = 0, \quad y(0) = 1, \\
& -\left(\frac{(X_P - X_a)}{R_1}\right)^6 - \left(\frac{Y_P}{R_2}\right)^6 + 1 \leq 0 \\
& m\dot{v}_x = u_1 \cos(\delta), \\
& m\dot{v}_y = u_1 \sin(\delta), \\
& \dot{\delta} = u_2 \\
& |\delta| \leq \delta_{max}, \quad |\dot{\delta}| \leq \dot{\delta}_{max} \\
& \dot{x} = v_x, \\
& \dot{y} = v_y
\end{aligned}
$$

Vehicle Motion Control

This lecture: Formulation of optimal control problem.
Next lecture (Lecture 2): How to solve the optimization problem.

Two lectures after that (Lectures 3-4): More formulations and analysis of results, i.e. first item below:

- Understanding at-the-limit driving
- Analyzing crash databases
- Obtaining control principles

The two last items in Lectures 5-8 in next block on May 23-25.