

# Electric Power System tutorial 2. Solutions

## Powergrid and technology for renewable production

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### Problems from Electric Power Systems, by Ned Mohan

**MATLAB files are available in the LISAM course room related to the solution of the tutorials. You are encouraged to make your own Matlab files, but the files here can be used for reference. Simulink files are given both for versions R2017b and R2016a.**

## Simscape power flow setup

Simscape is an extension of Simulink to provide circuit simulation models in for example power systems. In Simscape we can build up circuits using models for the voltage sources, transmission lines and loads.

In this task you shall setup up the same 3-bus transmission system as in the previous task.

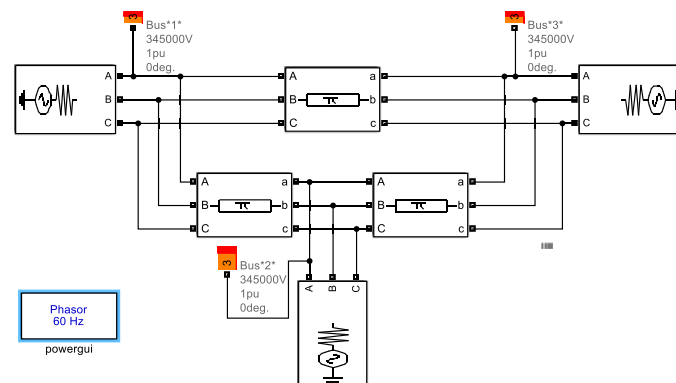


Figure 1 Three-bus transmission system example in Simscape

To invoke Simscape type **simscape** in the command window. The following window will appear, where **Power Systems** shall be selected.

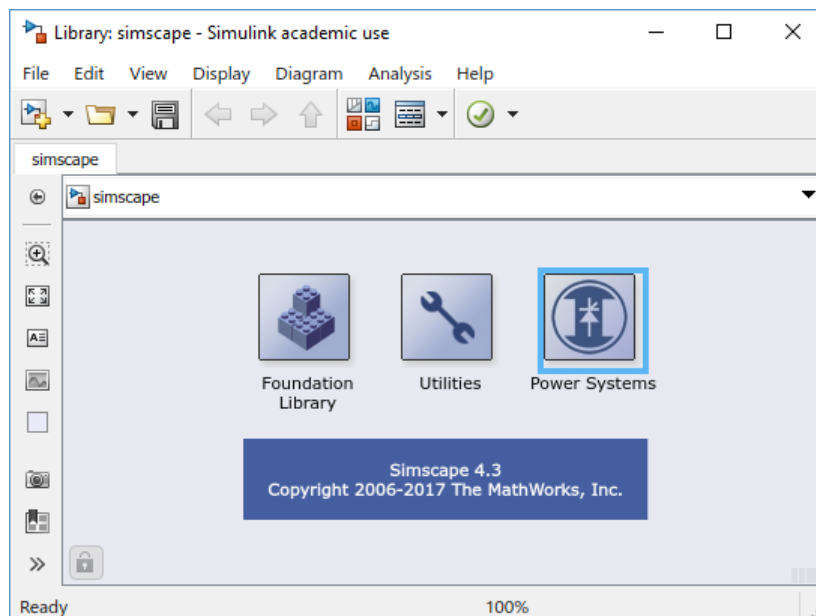


Figure 2

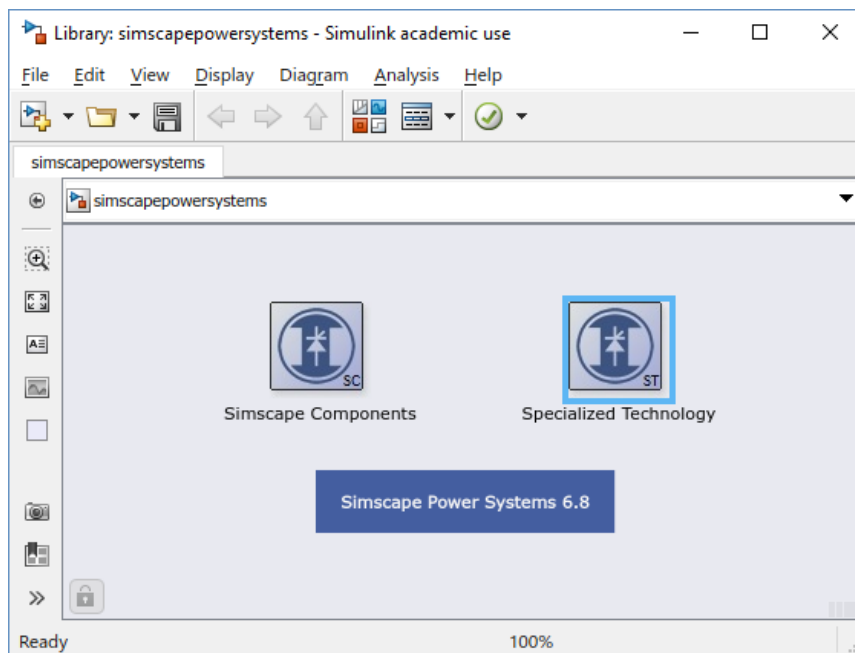


Figure 3

Thereafter, **Specialized technology** shall be selected, and in the next step the power flow components are found under **Fundamental blocks**.

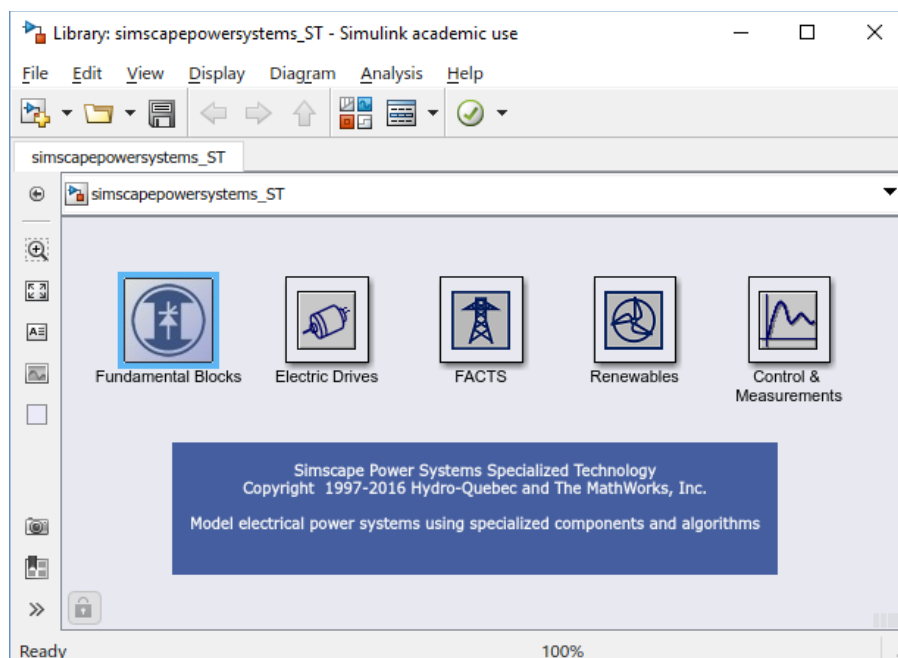




Figure 4

1. Create a blank model through the button .
2. Under the menu Model Configuration Parameters  and the tab Solver, set solver options to **variable-step** and **ode23tb(stiff/TR-BDF2)**.
3. Add the powergui block for simulation control and to obtain load flow calculation functionality.
  - a. In the block parameters, set Simulation type to Phasor, 60 Hz.

Now you shall find components for the slack bus, the PV-bus, the PQ-bus and transmission lines.

- Slack-bus, PV-bus and PQ-bus (PQ-load) can all be modelled using the **Three-phase Source** (Figure 6) found among **Electrical Sources**.
- Transmission lines are modelled by **Three-phase PI-section Line** (Figure 7) also found among **Elements**.

Note: A quick way of adding new components is to after pointing at the model area and typing the name of the component: e.g. Three-phase source. All models with the same first words will appear for selection.

Connection of blocks is done by tapping and dragging between terminals to join. Also, when placing a new block in line with another, light blue lines will appear as suggested connections. Tapping these will make them connect. Change of connections is done by dragging the end to a new point.

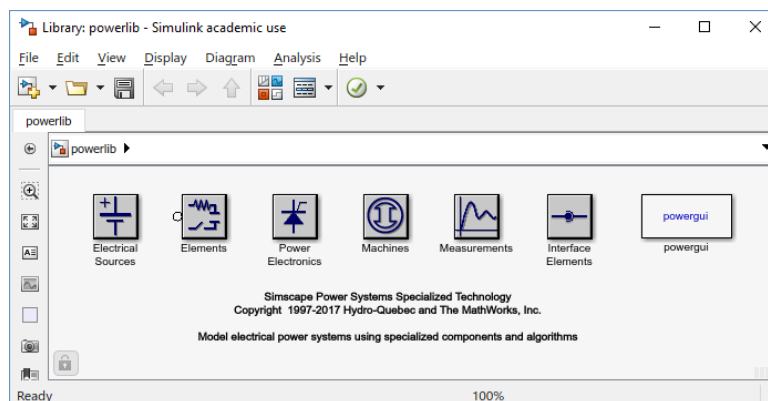


Figure 5

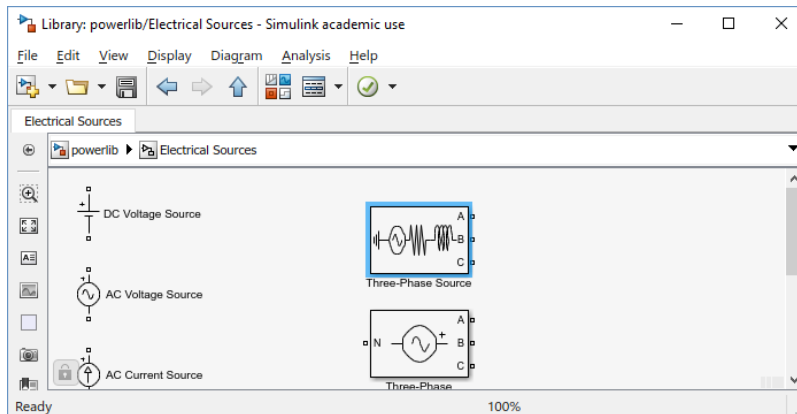


Figure 6

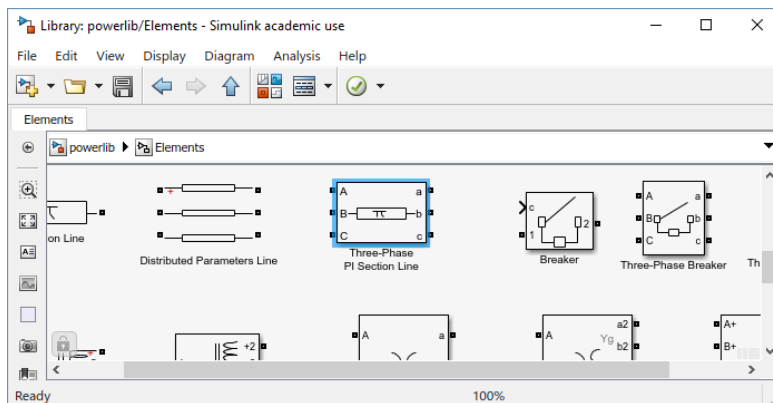


Figure 7

### Setting of slack bus (swing bus)

The following parameters are defined for the slack bus in Figure 8:

1. Configuration: Yg (Star connected to ground)
2. Ph-ph voltage: 345 kV
3. Ph angle: 0 deg
4. Freq: 60 Hz
5. Impedance internal: 0.01 ohm, L=0, Vbase=345 kV
6. Load flow: swing

Block Parameters: Three-Phase Source

Three-Phase Source (mask) (link)

Three-phase voltage source in series with RL branch.

Parameters Load Flow

Configuration: Yg

Source

☐ Specify internal voltages for each phase

Phase-to-phase voltage (Vrms): 345e3

Phase angle of phase A (degrees): 0

Frequency (Hz): 60

Impedance

☒ Internal ☐ Specify short-circuit level parameters

Source resistance (Ohms): 0.01

Source inductance (H): 0

Base voltage (Vrms ph-ph): 345e3

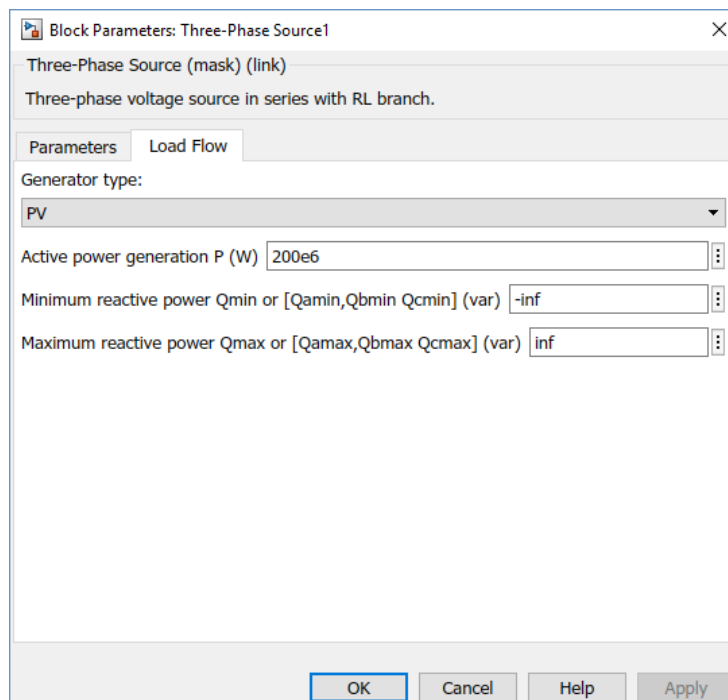
OK Cancel Help Apply

Figure 8

### Setting of PV-bus

The PV-bus (2) is setup same as the slackbus except for the Load flow Tab (Figure 9):

1. Configuration: Yg (Star connected to ground)
2. Ph-ph voltage: 345 kV
3. Ph angle: 0 deg
4. Freq: 60 Hz
5. Impedance internal: 0.01 ohm, L=0, Vbase=345 kV
6. Load flow: PV, P=200 MW, Qmin=-inf, Qmax=inf



*Figure 9 Load flow settings for the PV-bus*

### Setting of PQ-bus

The PQ-load is setup for 500 MW, 100 MVar loading by negative numbers for the Three-phase source component as shown by Figure 10. Other parameters of the Three-phase source are set same as for the slack-bus.

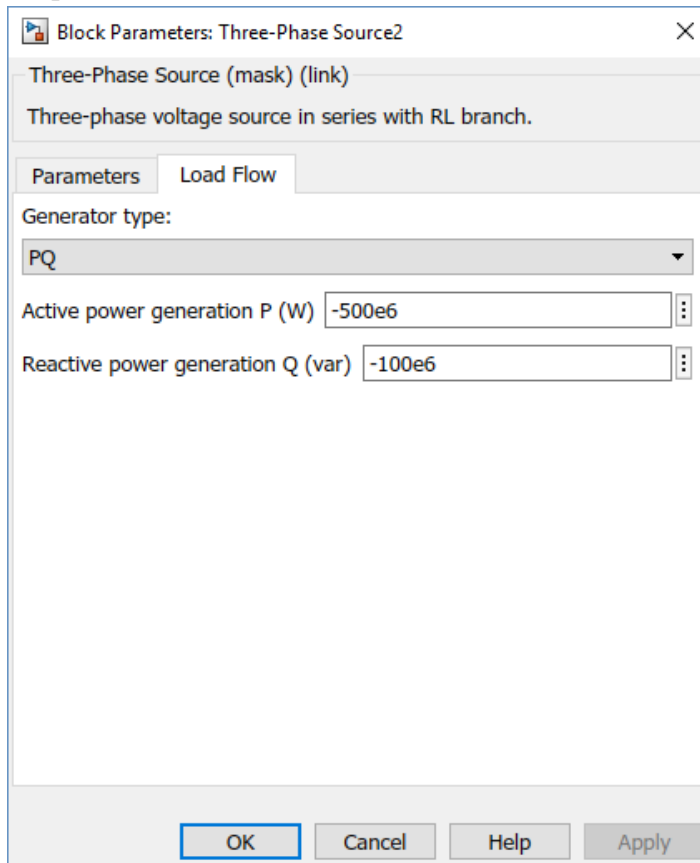


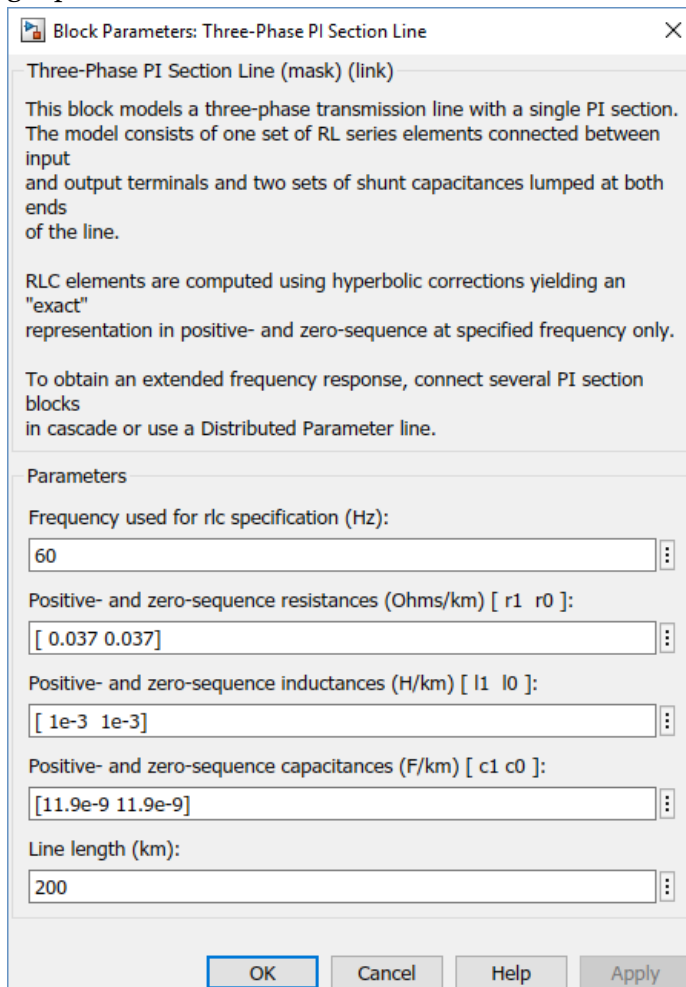
Figure 10 PQ-bus load flow settings



## Setting of transmission lines

The transmission line data in this model is given as resistance, inductance and capacitance per km. Consequently, the series reactance and shunt susceptance values defined in example 5.4 above shall be converted to inductance and capacitance for a 60 Hz system frequency. The data in the model is given as positive and zero sequence values, which could be set equal as shown by Figure 11.

Since all the lines have the same line parameters except for the length, setup one line first and copy the whole line block to the other locations. Thereafter setting the length parameters.



**Block Parameters: Three-Phase PI Section Line**

Three-Phase PI Section Line (mask) (link)

This block models a three-phase transmission line with a single PI section. The model consists of one set of RL series elements connected between input and output terminals and two sets of shunt capacitances lumped at both ends of the line.

RLC elements are computed using hyperbolic corrections yielding an "exact" representation in positive- and zero-sequence at specified frequency only.

To obtain an extended frequency response, connect several PI section blocks in cascade or use a Distributed Parameter line.

**Parameters**

Frequency used for rlc specification (Hz):  
60

Positive- and zero-sequence resistances (Ohms/km) [ r1 r0 ]:  
[ 0.037 0.037]

Positive- and zero-sequence inductances (H/km) [ l1 l0 ]:  
[ 1e-3 1e-3]

Positive- and zero-sequence capacitances (F/km) [ c1 c0 ]:  
[ 11.9e-9 11.9e-9]

Line length (km):  
200

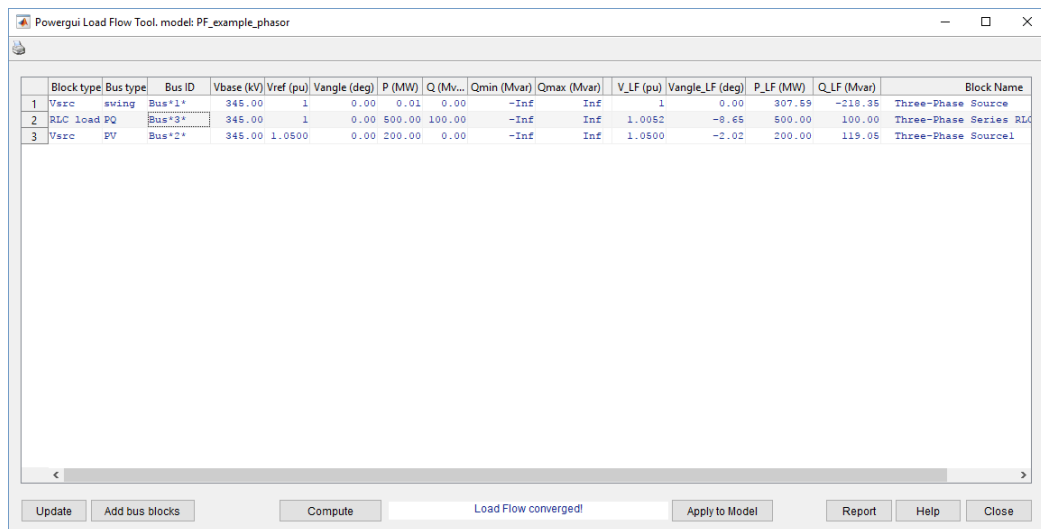
OK Cancel Help Apply

Figure 11 Transmission line parameters

## Load flow calculation

By clicking the powergui block and selecting the **Tools** menu, a Load flow calculation tool can be entered.

1. Check **Load flow settings** for 60 Hz,  $S_{\text{base}}=100$  MVA, voltage and power units as kV and MW.
2. Click the **Load Flow** button to bring up a window as in Figure 12.
3. Clicking the button **Add bus blocks**, results in the Load flow bus labels (red flags) being inserted next to each bus according to Figure 1.
4. Go back to the model page and edit the Load Flow Bus blocks.
  - a. In the blocks related to the slack-bus (Bus 1) and PV-bus (Bus-2) the target voltage and angle settings are given (only voltage for the PV-bus) See Figure 13.  $V_1 = 1.0 \angle 0^\circ \text{ pu}$ ,  $V_2 = 1.05 \text{ pu}$
  - b. Check the bus blocks numbering as defined earlier for this example: slack-bus (Bus 1), PV-bus (Bus-2) and PQ-bus (Bus-3).
5. In the powergui Load Flow Tool now press Compute to perform the load flow calculation.
6. Compare the results with earlier results from MATLAB fsolve load flow calculation in Table 1.
7. Click Apply to Model to save the load flow as initial conditions for the voltage sources of the system.
8. Close the Load flow tool.



Block type	Bus type	Bus ID	Vbase (kV)	Vref (pu)	Vangle (deg)	P (MW)	Q (Mvar)	Qmin (Mvar)	Qmax (Mvar)	V_LF (pu)	Vangle_LF (deg)	P_LF (MW)	Q_LF (Mvar)	Block Name
1	Vsrc swing	Bus*1*	345.00	1	0.00	0.01	0.00	-Inf	Inf	1	0.00	307.59	-219.35	Three-Phase Source
2	RLC load PQ	Bus*3*	345.00	1	0.00	500.00	100.00	-Inf	Inf	1.0052	-8.65	500.00	100.00	Three-Phase Series RLC
3	Vsrc PV	Bus*2*	345.00	1.0500	0.00	200.00	0.00	-Inf	Inf	1.0500	-2.02	200.00	119.05	Three-Phase Source

Update Add bus blocks Compute Load Flow converged! Apply to Model Report Help Close

Figure 12

Block Parameters: Bus\*3\*

Load Flow Bus (mask) (link)  
Identify and parameterize a load flow bus node.

Parameters Load Flow

Connectors: single

Bus identification Bus\*2\*

Base voltage (Vrms phase-phase)  
345000

Swing bus or PV bus voltage (pu)  
1.05

Swing bus voltage angle (degrees)  
0

OK Cancel Help Apply

Figure 13 Load flow bus-2 block settings

### Supplementary questions

1. Determine the surge impedance and surge impedance loading (SIL) for the transmission lines.  
 $Z_c = \sqrt{X \text{ per km} / B \text{ per km}} = 289 \text{ ohm}$   
 $SIL = V_{base}^2 / Z_c = 412 \text{ MW}$
2. Determine the active power loading of the individual lines as given by the load flow report.

sending bus – receiving bus	P into line [MW]		Q into line, sending end [Mvar]	Q into line, receiving end [Mvar]	Total Q into line [Mvar]
1-3	240	<SIL	-67	-4.7	-71.7
1-2	67.5	<SIL	-151	74.8	-76.2
2-3	266.8	<SIL	44.2	-95.3	-51.1
					-199

3. Determine which lines are loaded above and below SIL.
4. With the background of loading compared to SIL explain the levels of total Q of the lines.  
All lines have an excess of Q due to loading below SIL. Thereby the negative Q, corresponding to outflow of Q.

### Example 5.4, admittance matrix setup

Consider the 345 kV transmission from example 5.4 in [1] as shown in Figure 14. You shall setup the admittance matrix for this 3-bus system.

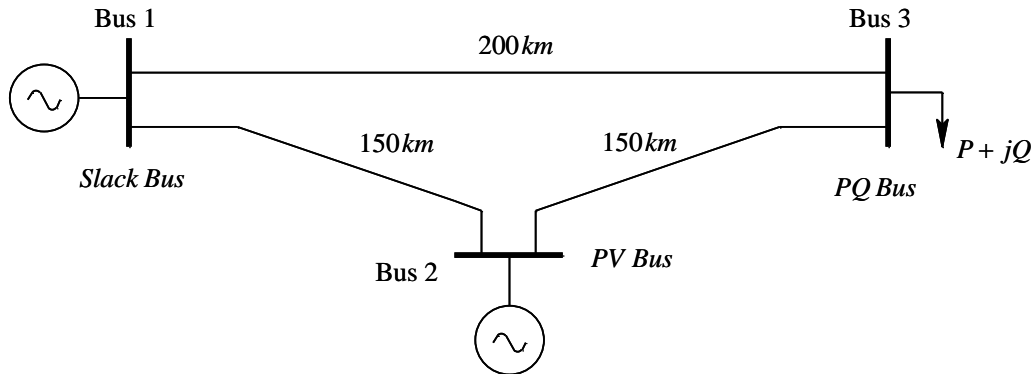


Figure 14 Transmission system 345 kV

Line data:

$$\begin{aligned} \text{series reactance} &= 0.376 \, \Omega / km, \\ \text{series resistance} &= 0.037 \, \Omega / km, \\ \text{shunt susceptance } B(=\omega C) &= 4.5 \, \mu S / km \end{aligned}$$

The actual impedance and susceptance of the line sections are defined related the actual line length:

$$\begin{aligned} Z_{12} &= (0.037 + j0.376) \cdot \text{Length}_{(1-2)} \, ohm \\ B_{12} &= 4.5 \cdot \text{Length}_{(1-2)} \, \mu ohm \end{aligned}$$

Convert the parameters to pu using

$$\begin{aligned} Z_{base} &= \frac{U_{ph}^2}{S_{ph}} = \frac{U_{LL}^2}{S_{3ph}} = \frac{U_{base}^2}{S_{base}} = \frac{345kV^2}{100MVA} = 1190 \, ohm \\ Y_{base} &= \frac{1}{Z_{base}} \end{aligned}$$

Setup the admittance matrix elements as defined below.

**Make a MATLAB script file containing all calculations, rather than making calculations directly in the command window. This gives better flexibility to adjust and redo calculations when needed.**

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_n \end{bmatrix}$$

Elements in the diagonal are defined as:

$$Y_{kk} = Y_{kG} + \sum_{\substack{m \\ m \neq k}} \frac{1}{Z_{km}}$$

Where  $Y_{kG}$  is the total admittance to ground at bus (node) k. For each line, the total B is split equally between the ends of the line.

$$Y_{kG} = j \sum_{\substack{m \\ m \neq k}} \frac{B_{km}}{2}$$

For elements out of the diagonal, the negated admittance of the line between bus k and bus m is given by:

$$Y_{km} = -\frac{1}{Z_{km}}$$

y=

3.5994 -35.6400i	-2.0568 +20.9013i	-1.5426 +15.6760i
-2.0568 +20.9013i	4.1136 -40.9992i	-2.0568 +20.9013i
-1.5426 +15.6760i	-2.0568 +20.9013i	3.5994 -35.6400i

### Example 5.4, power flow solution

Using the admittance matrix defined in the previous task, you shall now solve the power flow for this three-bus system. The three busses of the system are defined as:

- Bus1, slack bus:  $V_1 = 1.0 \angle 0^\circ \text{ pu}$
- Bus2, PV-bus:  $V_2 = 1.05 \text{ pu}$ ,  $P_2^{sp} = 2.0 \text{ pu}$
- Bus3, PQ-bus:  $P_3^{sp} = -5.0 \text{ pu}$ ,  $Q_3^{sp} = -1.0 \text{ pu}$

The solution to the system implies knowing all three bus voltages to both magnitude and phase. Related to the conditions for the busses defined above we are missing three variables: Angle( $V_2$ ), magnitude and angle of  $V_3$ .

You will use MATLAB fsolve to solve the power flow of this system.

## fsolve

Solve system of nonlinear equations

Nonlinear system solver

Solves a problem specified by

$$F(x) = 0$$

for  $x$ , where  $F(x)$  is a function that returns a vector value.

$x$  is a vector or a matrix; see [Matrix Arguments](#).

Here  $F(x)$  are a set of equations that defined the solution. In our case we have three conditions which we want to arrive at as defined for the busses above:

$$\begin{aligned} P_2 &= P_2^{sp} = 2 \text{ pu} \\ P_3 &= P_3^{sp} = -5 \text{ pu} \\ Q_3 &= Q_3^{sp} = -1 \text{ pu} \end{aligned}$$

So, we have three equations to solve to get the three unknowns! The unknowns are defined as  $x(1)=\text{angle}(V_2)$ ,  $x(2)=\text{magnitude}(V_3)$ ,  $x(3)=\text{angle}(V_3)$ .

To use fsolve you will make the following function call in MATLAB to obtain the solution for the unknown  $x$ 's:

```
x=fsolve(@PFsolve,x0,[],y,vref,pqref)
```

- Here, PFsolve is a function given for use together with fsolve for this example. See function listing below.
- $x_0$  is an initial guess for the solution. Use 0 for initial angles and 1 for voltage, such as  $x_0=[0 \ 1 \ 0]$ .
- $y$  is the admittance matrix.
- $V_{ref}$  defines the known voltages and is a vector of three elements. Set unknown element to zero.
- $P_{qref}$  is the reference for P and Q, also a vector of three elements. Set unknown element to zero and give elements where both P and Q are defined as  $P+jQ$ .

```
function [ dpq,pq,vk,ik ] = PFsolve( x,y,vref,pqref )
%PFsolve Summary of this function goes here
% x: variables for solution
% y: admittance matrix
% vref: voltage references for slack-bus or PV-busses.
%      Complex value if angle is defined for slack-bus.
% pqref: p-references for PV-busses and p+jq for PQ-
busses.
```

```
% Node voltages
vk(1,1)=vref(1);
```

```
vk(2,1)=vref(2)*exp(1j*x(1));
vk(3,1)=x(2)*exp(1j*x(3));

% Injected currents into nodes
ik=y*vk;

% Injected power into nodes
pq=vk.*conj(ik);

% Equations to solve for zero through variables in x
dpq(1)=real(pqref(2)-pq(2));
dpq(2)=real(pqref(3)-pq(3));
dpq(3)=imag(pqref(3)-pq(3));
end
```

After getting a solution (x) from fsolve, all the values for P, Q, voltage and current for the three busses, can be obtained through the function call of PFsolve directly, with the solution x as the first parameter:

```
[ dpq,pq,vk,ik ] = PFsolve( x,y,vref,pqref )
```

- dpq is the deviation from the ideal solution
- pq is the P+jQ injection in each bus
- vk is the bus voltage
- ik is the injected current in each bus

1. Fill the table with the load flow results obtained:

Bus no	Voltage magnitude [pu]	Voltage angle [deg]	Active power [pu]	Reactive power [pu]
1	1.0000	0	3.0772	-2.1748
2	1.0500	-2.0432	2.0000	1.1902
3	1.0049	-8.7117	-5.0000	- 1.0000

*Table 1 Load flow results*

2. How much are the total active power losses in the system given in MW?

The total sum of active power infeed of all busses equals the losses:

$(3.0772+2-5) = 0.0772 \text{ pu} = 0.0772 * S_{\text{base}} = 7.7 \text{ MW}.$

3. How much are the total reactive power losses in the system given in Mvar?

The total sum of reactive power infeed of all busses equals the total reactive power consumption of the lines:

$(-2.1748+1.1902-1) = -1.98 \text{ pu} = -1.98 * S_{\text{base}} = -198 \text{ Mvar}.$

## 10.20

In the power flow of example 5.4, what will the voltage be at bus-3 if the power demand (P and Q) at bus-3 is increased by 100%?

## Dynamic simulation

By changing the model used for the PQ-load from the Three-phase source to a **Three-phase Dynamic Load**, stepping of the load during a time simulation can be done. In Figure 15 below the Simscape setup using a dynamic load is shown.

**Save your previous model to a new name** for preparation of this modified setup. Retain the previous model for load flow calculations without the dynamic simulation.

Parameters of the dynamic load shall be setup,

1. defining the base voltage and frequency,
2. as well as the initial P and Q.
3. Initial voltage and angle is not required to fill, since using the Load flow tool will do this.
4. Furthermore, external control of PQ shall be selected.

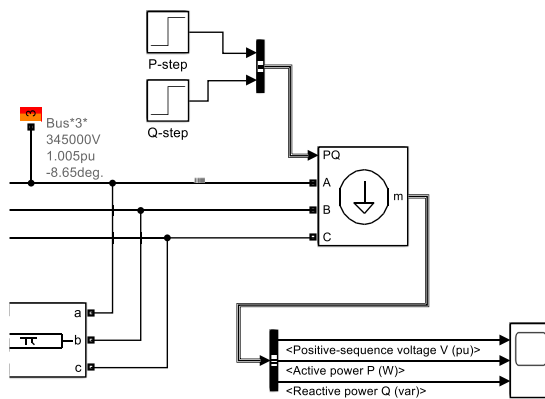


Figure 15 PQ-load replaced by Dynamic load



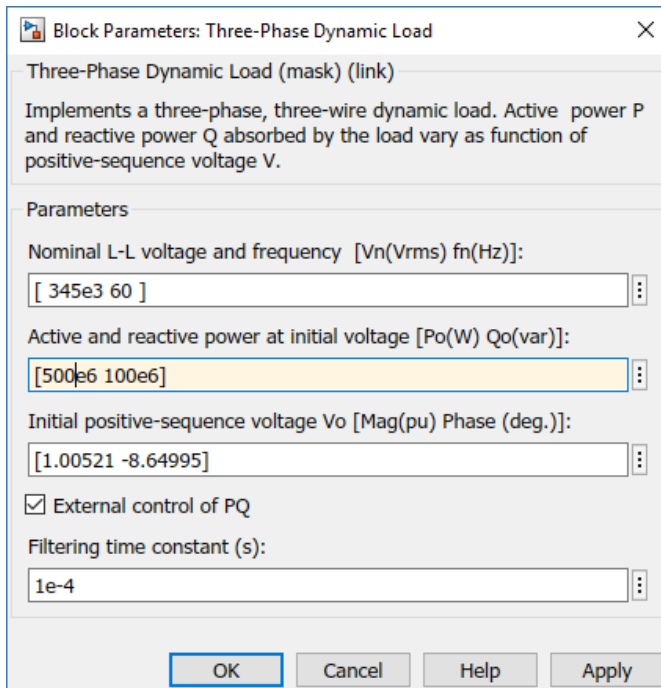





Figure 16 Dynamic load settings

To complete the setup, the external PQ input and the measurement of resulting voltage and PQ-flow, shall be defined. The complete Simulink library is reached

through the button .

1. Add two **Step** blocks (found in Simulink/Sources, or by typing Step after pointing at the model area).
  - a. Setup the Step blocks for a step at 1s, where P changes from 500 to 1000 MW, and Q from 100 to 200 Mvar.
2. Add a **Bus Creator** (in Simulink/Commonly used blocks) to make a 2-dimensional bus for interfacing the dynamic load PQ input.
3. Add a **Bus Selector** to separate the three signals obtained at the m-output.
  - a. Double click the Bus Selector to perform setup. You can see the three available signals to the left. By marking them and clicking select they will move to the right-hand side. Delete any initial undefined signals on the right side.
4. Add a **Scope** (in Simulink/Commonly used blocks) for plotting.
  - a. Double click to setup. Selecting **File/Number of input ports/More...** brings up the following window.
  - b. Set number of input ports to 3
  - c. Maximize axes: Auto

- d. Axes scaling: Auto
  - e. Click layout to define the layout of the 3 sub-plots.
5. Under the menu Model Configuration Parameters  and the tab **Solver**,
  - b. set solver options to variable-step and **ode23tb(stiff/TR-BDF2)**.
  - a. Simulation stop time: 2 s.
6. Open the powergui Load Flow Tool and press Compute to perform the load flow calculation related to the initial conditions with PQ-load of 500 MW and 100 Mvar.
  - a. Compare the results with earlier Simscape load flow.
  - b. Click Apply to Model to save the load flow as initial conditions for the new Dynamic Load.
  - c. Close the Load flow tool.
7. Start simulation through  button.
8. On the Scope graphs you may now determine the final voltage level after the PQ-load increase.

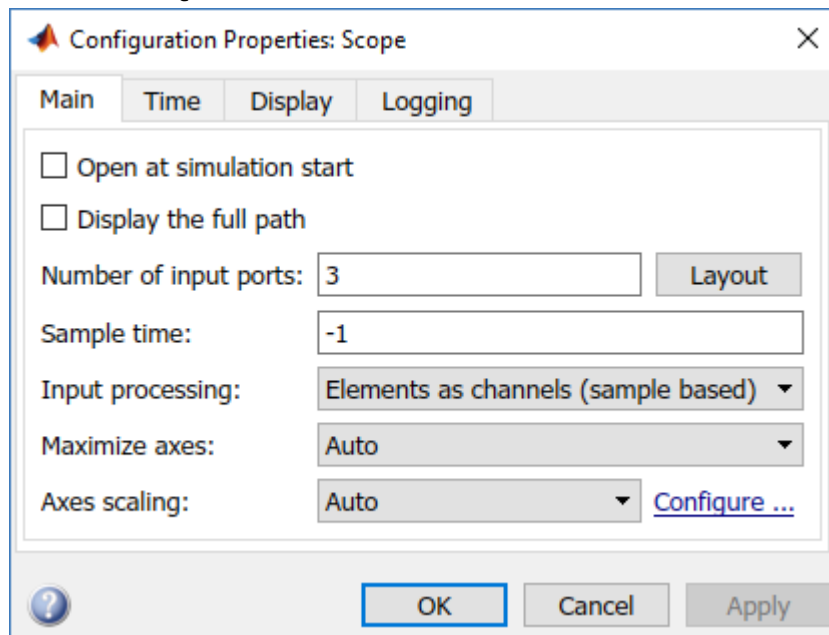
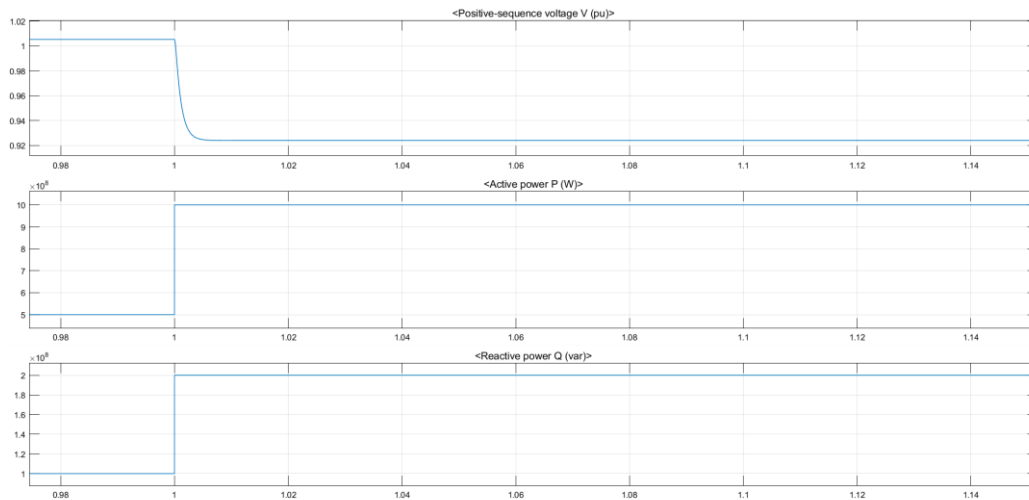


Figure 17 Scope sub-plot layout and settings



## 10.21

For the problem 10.20, with bus-3 loading increased by 100%, what reactive power compensation needs to be provided at bus-3 to bring its voltage to 1 pu? Use Simscape model without dynamic simulation, changing bus-3 **Three phase source** from PQ-bus to a PV-bus with 1000 MW load and 1 pu voltage setting to find the net reactive power for this voltage level. Determine reactive power compensation at bus-3 that would be required in combination with the original inductive Q-load of 200 Mvar.

	V(pu)	Angle (deg)	P(MW)	Q(Mvar)
Bus*1*	1	0.00	832.9	-165.7
Bus*2*	1.05	-7.67	200	184.5
Bus*3*	1	-19.88	-1000	42.9
Total			32.9	61.8

The net reactive power infeed at bus 3 to maintain 1 pu voltage is 42.9 Mvar. With the reactive power load being 200 Mvar inductive, this results in a need for capacitive compensation of 242.9 Mvar.

### Supplementary questions

- Determine the surge impedance and surge impedance loading (SIL) for the transmission lines.  
 $Z_c = \sqrt{X \text{ per km} / B \text{ per km}} = 289 \text{ ohm}$   
 $SIL = V_{base}^2 / Z_c = 412 \text{ MW}$
- Determine the active power loading of the individual lines as given by the load flow report.

sending bus – receiving bus	P into line [MW]		Q into line, sending end [Mvar]	Q into line, receiving end [Mvar]	Total Q into line [Mvar]
1-3	547	>SIL	-11.6	92.8	81.2
1-2	286	<SIL	-154	114.3	-39.7
2-3	481.5	>SIL	70.3	-49.8	20.5
					62

- Determine which lines are loaded above and below SIL.
- With the background of loading compared to SIL explain the levels of total Q of the lines.

Only the line 1-2 has an excess of Q due to loading below SIL. The other lines are loaded above SIL and therefore a positive demand of Q.

- Compare the line loading and Q consumption with the supplementary questions to the original load flow calculation.

Generally all the lines are here loaded closer to SIL resulting in a smaller total Q flow.

## 10.22 Series compensation

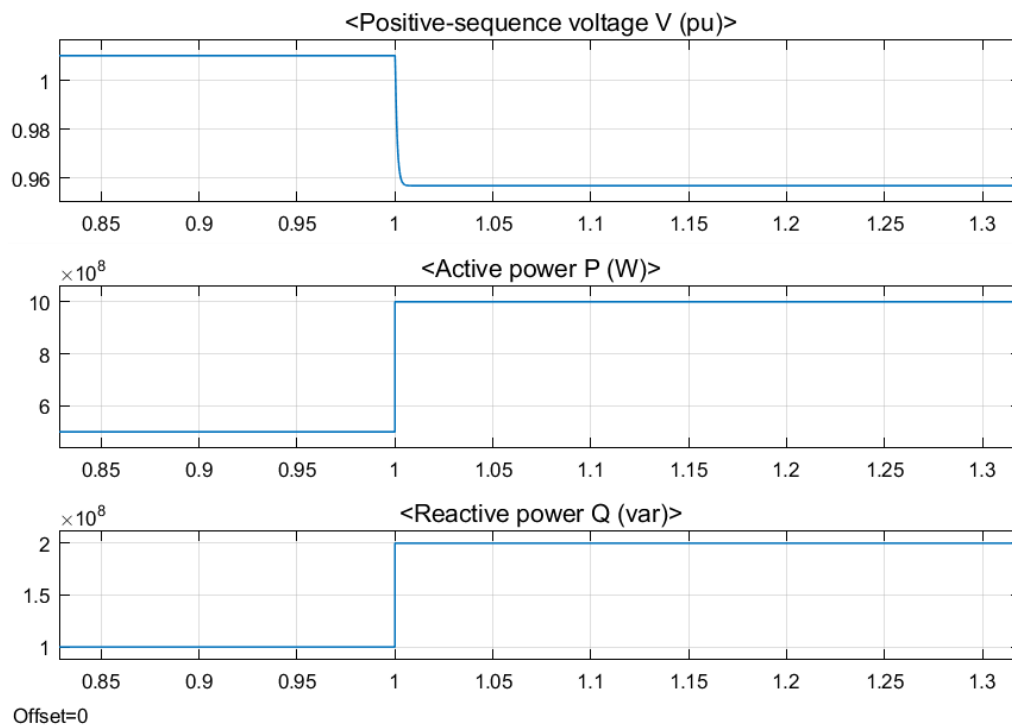
200km line  $Z=0.376 \cdot 200=37.6$  ohm

$Z+X_c=0.376 \cdot 100$

$X_c=-18.8$  ohm

For a 60 Hz system

$C=-1/(wX_c)=70.5 \mu\text{F}$



## Literature:

- [1] [Electric Power Systems a first course, Ned Mohan, Wiley 2012](#)