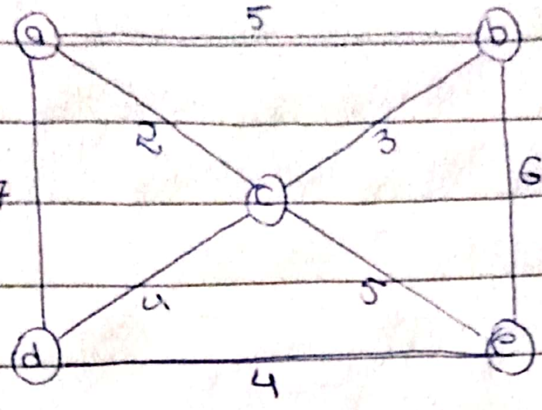
Exercise No:- 01

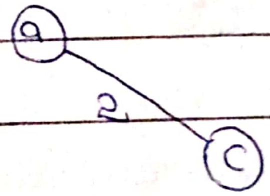
↳ Apply prims Algorithm the following graph:



Prims algorithm is a famous greedy algorithm used for finding Minimum Spaning Tree of given graph.

Step 01

i am selecting vertex a having edge with because of least weightage.



Step 02

Vertex c is further connecting with d,b,c. As with vertex b, it has least weightage and there is no edge that create a cycle, so i am selecting b.

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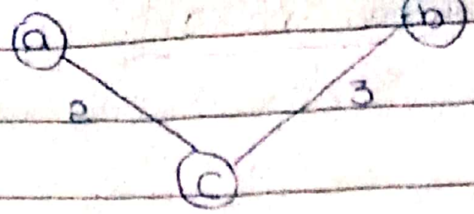
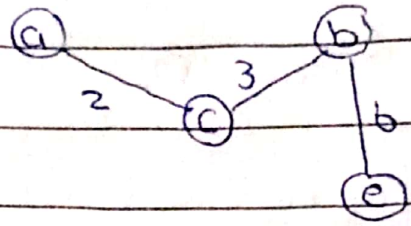


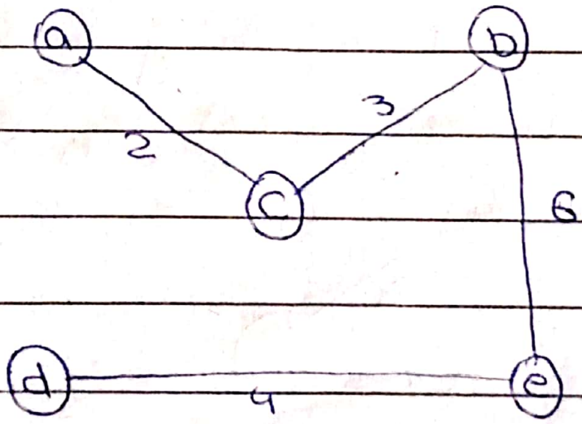
Step 03

Vertex b has edges with vertex a, c and e. As a and c are visited so i will select e.



Step 04

Vertex e has edges with b, c and d. As b and c are visited, so i will select d.



Since, all vertex are visited, so we will stop.

Now, cost of MST is

= sum of all edge weights

= 2+3+6+4 => 15 units.

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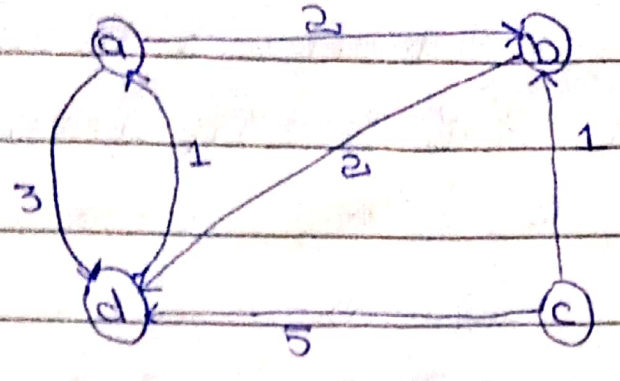
Exercise # 02

↳ For the given graph, show steps of the Floyd algorithm, including predecessor matrix:



Step 01

In the given graph, there are no self edges so we will write matrix for it.

$$D_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & \infty & 3 \\ b & \infty & 0 & \infty & 2 \\ c & \infty & 1 & 0 & 5 \\ d & 1 & \infty & \infty & 0 \end{array}$$

- It represents distance between every pair of vertices, in the form of given weights.
- For self loops, distance value = 0
- For vertices having no edge, distance value = ∞

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Step 02

$$D_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 2 & \infty & 3 \\ \infty & 0 & \infty & 2 \\ \infty & 1 & 0 & 5 \\ 1 & 3 & \infty & 0 \end{bmatrix} \end{array}$$

$$D_1'(2,3) = Min[\, D_0(2,3),\ D_0^0(2,1) + D_0(1,3) \,]$$
$$= \infty,\quad \infty + \infty$$
$$= \infty$$

$$D_1(2,4) = Min[\, D_0(2,4),\ D_0(2,1) + D_0(1,4) \,]$$
$$= 2,\quad \infty + 3$$
$$= 2,\quad \infty$$
$$= 2$$

$$D_1(3,2) = Min[\, D_0(3,2),\ D_0(3,1) + D_0(1,2) \,]$$
$$= 1,\quad \infty + 2$$
$$= 1$$

$$D_1(3,4) = Min[\, D_0(3,4),\ D_0(3,1) + D_0(1,4) \,]$$
$$= 5,\quad \infty + 3$$
$$= 5$$

$$D_1(4,2) = Min[\, D_0(4,2),\ D_0(4,1) + D_0(1,2) \,]$$
$$= \infty,\quad 1 + 2$$
$$= 3$$

$$D_1(4,3) = Min[\, D_0(4,3),\ D_0(4,1) + D_0(1,3) \,]$$
$$= \infty,\quad \infty + 1$$
$$= \infty$$

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Step 03

$$D_2 = \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 2 & \infty & 3 \\ \infty & 0 & \infty & 2 \\ 1 & 1 & 0 & 3 \\ 1 & 3 & \infty & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}$$

$D_2(1,3) = \text{Min}[D_1(1,3), D_1(1,2) + D_1(2,3)]$

$= \infty , 2 + \infty$

$= \infty$

$D_2(1,4) = \text{Min}[D_1(1,4), D_1(1,2) + D_1(2,4)]$

$= 3 , 2 + 2$

$= 3$

$D_2(3,1) = \text{Min}[D_1(3,1), D_1(3,2) + D_1(2,2)]$

$= \infty , 1 + 0$

$= 1$

$D_2(3,4) = \text{Min}[D_1(3,4), D_1(3,2) + D_1(2,4)]$

$= 5 , 1 + 2$

$= 3$

$D_2(4,1) = \text{Min}[D_1(4,1), D_1(1,2) + D_1(2,1)]$

$= 1 , 2 + \infty$

$= 1$

$D_2(4,3) = \text{Min}[D_2(4,3), D_2(4,2) + (2,3)]$

$= \infty , 3 + \infty$

$= \infty$

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Step 04

$$D_3 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & 0 & 3 \\ b & 1 & 0 & \infty & 2 \\ c & 1 & 1 & 0 & 3 \\ d & 1 & 3 & \infty & 0 \end{array}$$

$D_3(1,2) = \text{Min}[\,D_2(1,2),\ D_2(1,3) + D_2(3,2)\,]$

$= 2,\ 0+1$

$= 2$

$D_3(1,4) = \text{Min}[\,D_2(1,4),\ D_2(1,3) + D_2(3,4)\,]$

$= 3,\ 0+3$

Step 05

$$D_4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & \infty & 3 \\ b & \infty & 0 & \infty & 2 \\ c & 1 & 1 & 0 & 5 \\ d & 1 & \infty & \infty & 0 \end{array}$$

The predecessor matrix for all shortest path is as follow.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & \infty & 3 \\ b & \infty & 0 & \infty & 2 \\ c & \infty & 1 & 0 & 5 \\ d & 1 & \infty & \infty & 0 \end{array}$$

—x—x—

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