

Formal Languages and Abstract Machines

Take Home Exam 2

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2099471

1 Context-Free Grammars (10 pts)

a) Give the rules of the Context-Free Grammars to recognize strings in the given languages where $\Sigma = \{a, b\}$ and S is the start symbol.

$L(G) = \{w \mid w \in \Sigma^*; |w| \geq 3;$ (2/10 pts)
the first and the second from the last symbols of w are the same}

$$\begin{aligned} S &\rightarrow aAab \mid aAaa \mid bAbb \mid bAba \\ A &\rightarrow aA \mid bA \mid a \mid b \mid e \end{aligned}$$

$L(G) = \{w \mid w \in \Sigma^*; \text{the length of } w \text{ is odd}\}$ (2/10 pts)

$$\begin{aligned} S &\rightarrow Aa \mid Ab \\ A &\rightarrow Aaa \mid Aab \mid Aba \mid Abb \mid e \end{aligned}$$

$L(G) = \{w \mid w \in \Sigma^*; n(w, a) = 2 \cdot n(w, b)\}$ where $n(w, x)$ is the number of x symbols in w (3/10 pts)

$$\begin{aligned} S &\rightarrow BBb \mid BbB \mid bBB \mid e \\ B &\rightarrow aS \mid Sa \end{aligned}$$

b) Find the set of strings recognized by the CFG rules given below: (3/10 pts)

$S \rightarrow X \mid Y$
 $X \rightarrow aXb \mid A \mid B$
 $A \rightarrow aA \mid a$
 $B \rightarrow Bb \mid b$
 $Y \rightarrow CbaC$
 $C \rightarrow CC \mid a \mid b \mid \varepsilon$

$a^+b^+ \mid (a|b)^* ba (a|b)^*$

2 Parse Trees and Derivations

(20 pts)

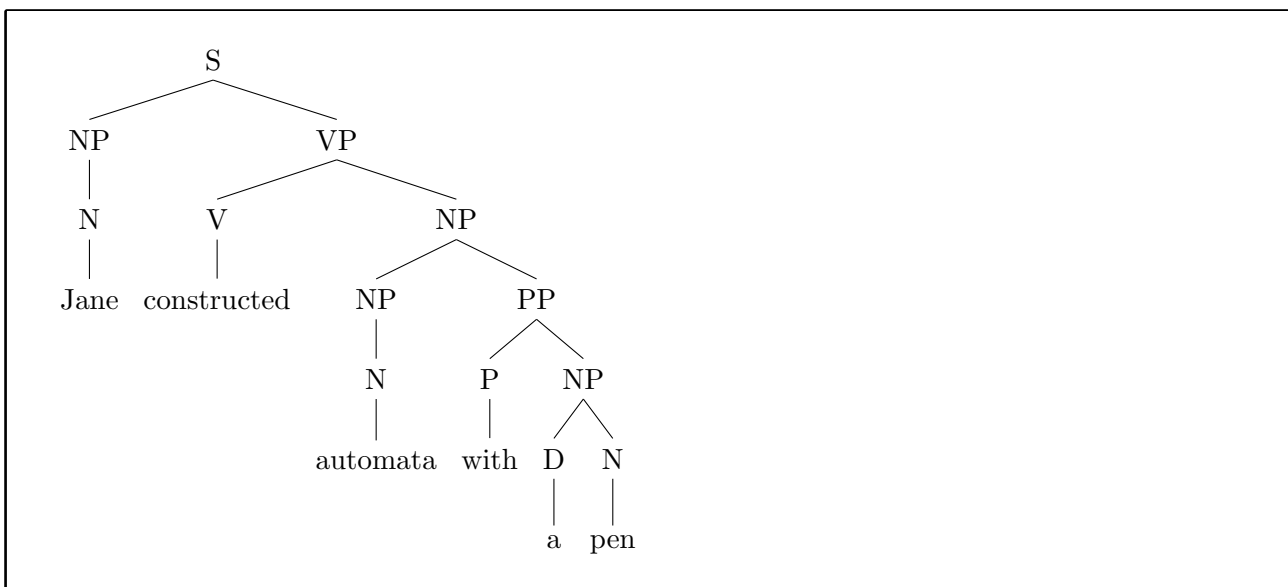
Given the CFG below, provide parse trees for given sentences in **a** and **b**.

```
S → NP VP
VP → V NP | V NP PP
PP → P NP
NP → N | D N | NP PP
V → wrote | built | constructed
D → a | an | the | my
N → John | Mary | Jane | man | book | automata | pen | class
P → in | on | by | with
```

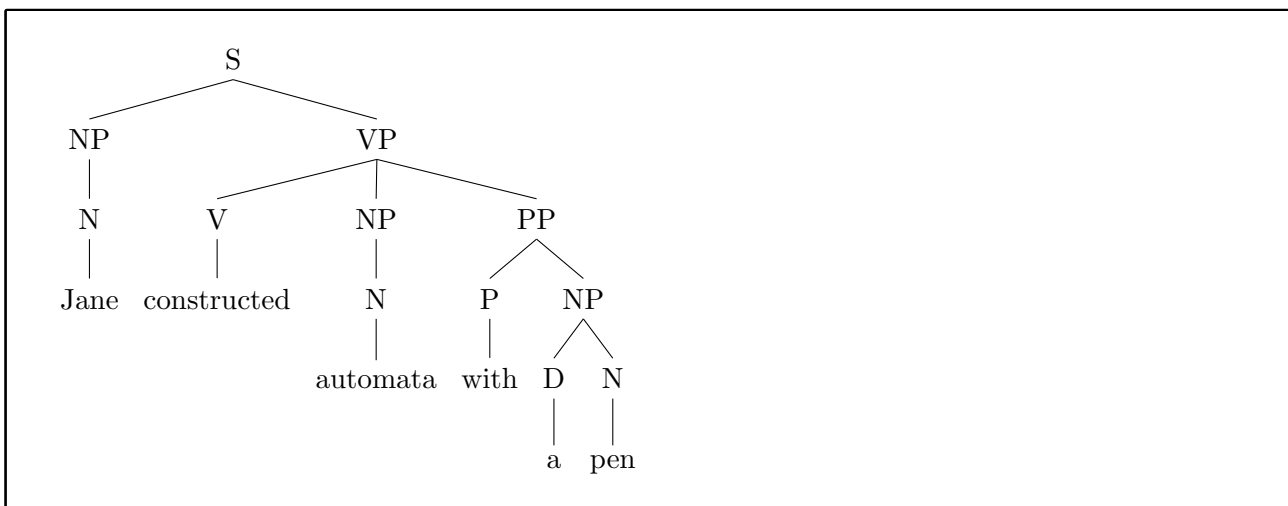
a) Jane constructed automata with a pen

(4/20 pts)

First one:



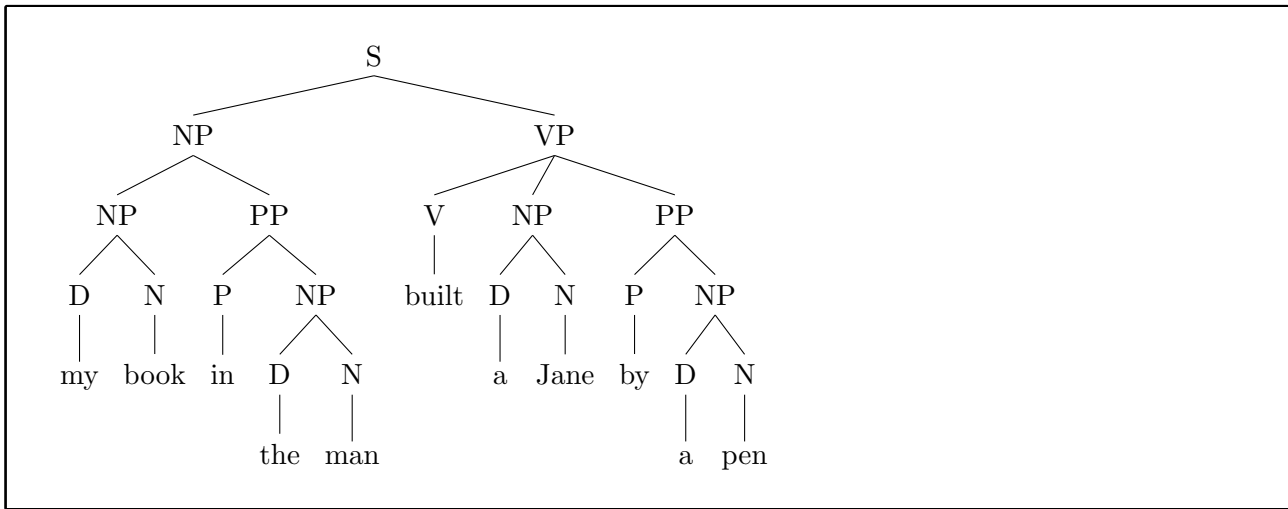
Second one:



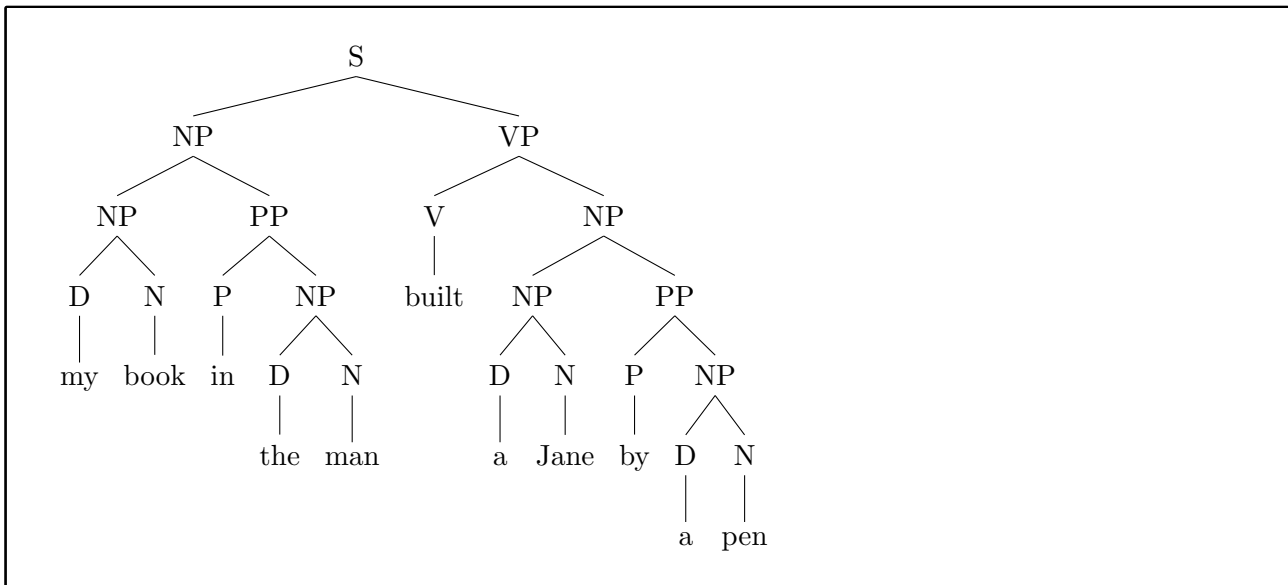
b) my book in the man built a Jane by a pen

(4/20 pts)

First one:



Second one:

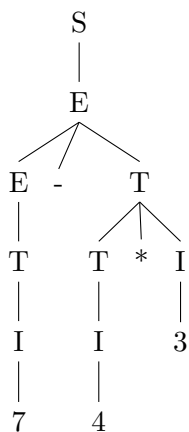


Given the CFG below, answer **c**, **d** and **e**

S	\rightarrow	E
E	\rightarrow	$E + T \mid E - T \mid T$
T	\rightarrow	$T * I \mid T / I \mid I$
I	\rightarrow	$0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 6 \mid 7 \mid 8 \mid 9$

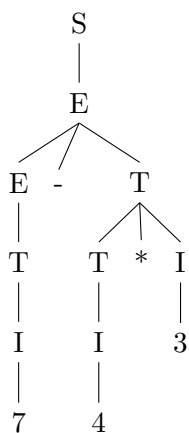
c) Provide the left-most derivation of $7 - 4 * 3$ step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \rightarrow E - T \rightarrow T - T \rightarrow I - T \rightarrow 7 - T \rightarrow 7 - T * I \rightarrow 7 - 4 * I \rightarrow 7 - 4 * 3$



d) Provide the right-most derivation of $7 - 4 * 3$ step-by-step and plot the final parse tree (4/20 pts)

$S \rightarrow E \rightarrow E - T \rightarrow E - T * I \rightarrow E - T * 3 \rightarrow E - T * 3 \rightarrow E - I * 3 \rightarrow E - 4 * 3 \rightarrow T - 4 * 3 \rightarrow I - 4 * 3 \rightarrow 7 - 4 * 3$



e) Are the derivations in **c** and **d** in the same similarity class?

(4/20 pts)

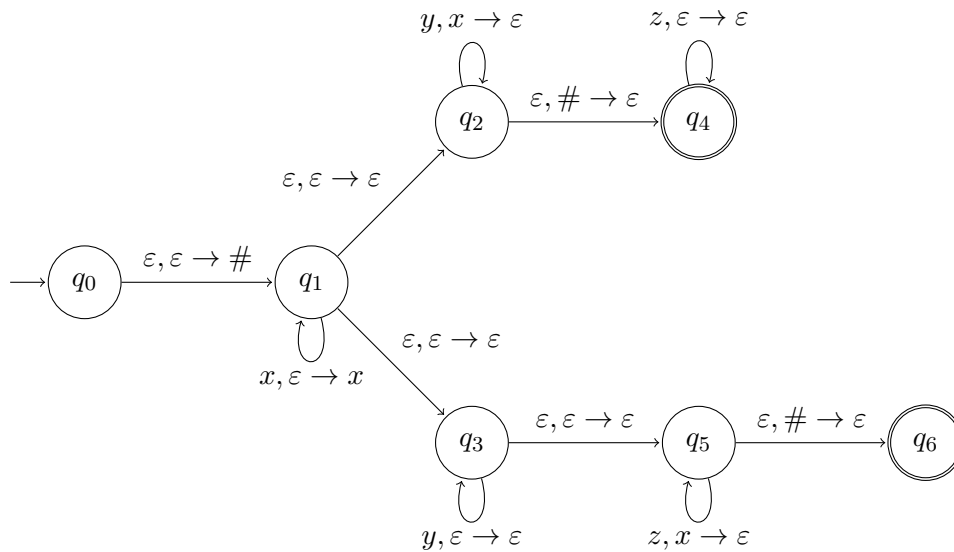
Yes, because they can transform other one and one of them precedes other.

3 Pushdown Automata

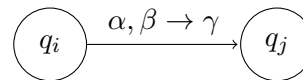
(30 pts)

a) Find the language recognized by the PDA given below

(5/30 pts)



where the transition $((q_i, \alpha, \beta), (q_j, \gamma))$ is represented as:



This language represents that:

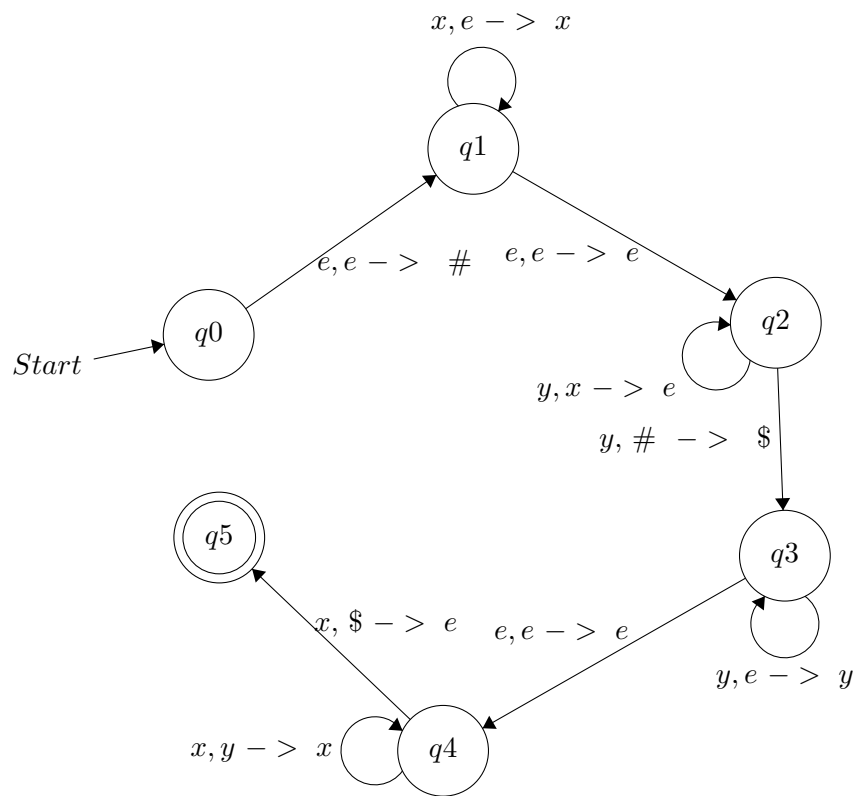
$$x^n y^n z^* \cup x^n y^* z^n$$

So that, $L = \{x^n y^n z^k \cup x^n y^l z^n \mid k, l, n \geq 0\}$

b) Design a PDA to recognize language $L = \{x^n y^{m+n} x^m \mid n, m \geq 0; n, m \in \mathbb{N}\}$

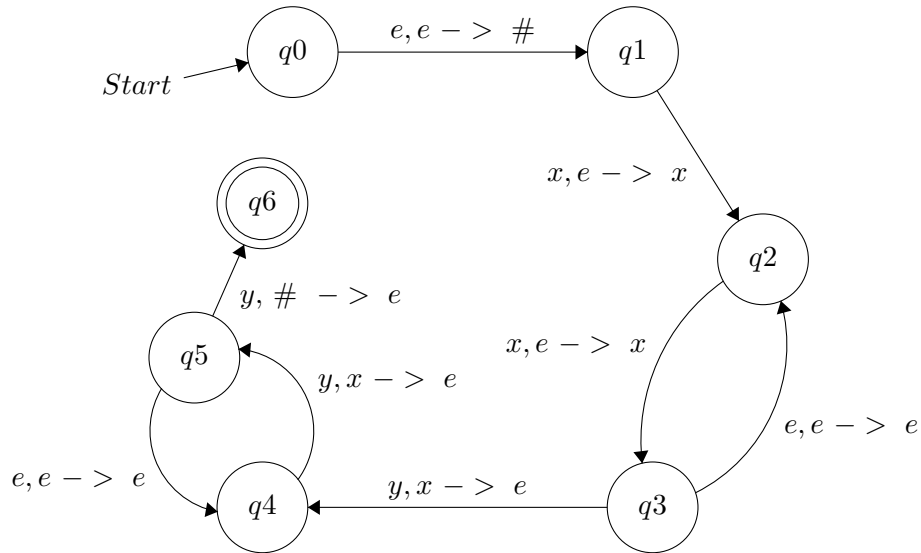
(5/30 pts)

Answer:



- c) Design a PDA to recognize language $L = \{x^n y^m \mid n < m \leq 2n; n, m \in \mathbb{N}^+\}$ (10/30 pts)
Do not use multi-symbol push/pop operations in your transitions.
Simulate the PDA on strings xy (with only one rejecting derivation) and $xyyyyy$ (accepting derivation) with transition tables.

Answer:



For the xy this automata will not work, because it will stuckwa.

For the $xyyyyy$ this automata will come its final state so that $xyyyyy$ will be accepted.

- d) Given two languages L' and L as $L' = \{w \mid w \in L; |w| = 4n + 2 \text{ for } n \in \mathbb{N}\}$ (10/30 pts)
If L is a CFL, show that L' is also a CFL by constructing an automaton for L' in terms of another automaton that recognizes L .

I worked hard but can not solve this problem...

4 Closure Properties

(20 pts)

Let L_1 and L_2 be context-free languages which are not regular, and let L_3 be a regular language. Determine whether the following languages are necessarily CFLs or not. If they need to be context-free, explain your reasoning. If not, give one example where the language is a CFL and a counter example where the language is not a CFL.

a) $L_4 = L_1 \cap (L_2 \setminus L_3)$

(10/20 pts)

No.

$L_2 - L_3 = L_2 \cap L_3'$ since L_3' is regular because regular languages are closed under complement and intersection of them is context free.

$L_1 \cap L_3$ not certainly context free.

$L_1 = q^k z^k x^l$ and $L_2 - L_3 = z^k x^k$

b) $L_5 = (L_1 \cap L_3)^*$

(10/20 pts)

Yes.

All regular languages are subset of context-free so that $L_1 \cap L_3$ is also context-free.

Context-free languages are closed under Kleene Star so that L_5 is CFL.

5 Pumping Theorem

(20 pts)

- a) Show that $L = \{a^n m^n t^i \mid n \leq i \leq 2n\}$ is not a Context Free Language (10/20 pts)
using Pumping Theorem for CFLs.

$uvxyz = ammtt$ and $vxy = am \rightarrow$
 $uv^2xy^2z = aaammmtt \rightarrow$
 $n > i$
so that this language is not Context-Free.

- b) Show that $L = \{a^n b^{2n} a^n \mid n \in \mathbb{N}^+\}$ is not a Context Free Language (10/20 pts)
using Pumping Theorem for CFLs.

$uvxyz = abba$ and $vxy = abb \rightarrow$
 $uv^2xy^2z = aabbba \rightarrow$
 $n = 2, 2n = 3, n = 1$
so that this language is not Context-Free.

6 CNF and CYK

(not graded)

a) Convert the given context-free grammar to Chomsky Normal Form.

$$S \rightarrow XSX \mid xY$$

$$X \rightarrow Y \mid S$$

$$Y \rightarrow z \mid \varepsilon$$

answer here ...

b) Use the grammar below to parse the given sentence using Cocke–Younger–Kasami algorithm. Plot the parse trees.

S → NP VP	VP → book include prefer
S → X1 VP	VP → Verb NP
X1 → Aux NP	VP → X2 PP
S → book include prefer	X2 → Verb NP
S → Verb NP	VP → Verb PP
S → X2 PP	VP → VP PP
S → Verb PP	PP → Prep NP
S → VP PP	Det → that this the a
NP → I she me Houston	Noun → book flight meal money
NP → Det Nom	Verb → book include prefer
Nom → book flight meal money	Aux → does
Nom → Nom Noun	Prep → from to on near through
Nom → Nom PP	

book the flight through Houston

Empty parse table:

<div> <div>1:5 → 1:1 2:5 1:5 → 1:2 3:5 1:5 → 1:3 4:5 1:5 → 1:4 5:5</div> </div>				
<div> <div>1:4 → 1:1 2:4 1:4 → 1:2 3:4 1:4 → 1:3 4:4</div> </div>		<div> <div>2:5 → 2:2 3:5 2:5 → 2:3 4:5 2:5 → 2:4 5:5</div> </div>		
<div> <div>1:3 → 1:1 2:3 1:3 → 1:2 3:3</div> </div>		<div> <div>2:4 → 2:2 3:4 2:4 → 2:3 4:4</div> </div>	<div> <div>3:5 → 3:3 4:5 3:5 → 3:4 5:5</div> </div>	
<div>1:2 → 1:1 2:2</div>		<div>2:3 → 2:2 3:3</div>	<div>3:4 → 3:3 4:4</div>	<div>4:5 → 4:4 5:5</div>
1:1	2:2	3:3	4:4	5:5
book	the	flight	through	Houston

rest of the answer here ...

7 Deterministic Pushdown Automata

(not graded)

Provide a DPDA to recognize the given languages, the DPDA must read its entire input and finish with an empty stack.

a) $a^*bc \cup a^n b^n c$

answer here ...

b) $(aa)^*c \cup a^nb^nc$

answer here ...