Formal Languages and Abstract Machines
Take Home Exam 2

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2099471

1 Context-Free Grammars (10 pts)

a) Give the rules of the Context-Free Grammars to recognize strings in the given languages where $\Sigma = \{a, b\}$ and S is the start symbol.

$L(G) = \{w \mid w \in \Sigma^*;\ |w| \geq 3;$ the first and the second from the last symbols of w are the same$\}$ (2/10 pts)

$S \rightarrow aAab \mid aAaa \mid bAbb \mid bAba$
$A \rightarrow aA \mid bA \mid a \mid b \mid e$

$L(G) = \{w \mid w \in \Sigma^*;$ the length of w is odd$\}$ (2/10 pts)

$S \rightarrow Aa \mid Ab$
$A \rightarrow Aaa \mid Aab \mid Aba \mid Abb \mid e$

$L(G) = \{w \mid w \in \Sigma^*;\ n(w, a) = 2 \cdot n(w, b)\}$ where $n(w, x)$ is the number of x symbols in w (3/10 pts)

$A \rightarrow BBa \mid BaB \mid aBB \mid e$
$B \rightarrow bS \mid Sb$

b) Find the set of strings recognized by the CFG rules given below: (3/10 pts)

$S \to X \mid Y$
$X \to aXb \mid A \mid B$
$A \to aA \mid a$
$B \to Bb \mid b$
$Y \to CbaC$
$C \to CC \mid a \mid b \mid \varepsilon$

$$\boxed{a^+b^+ \mid (a|b)^*\ ba\ (a|b)^*}$$

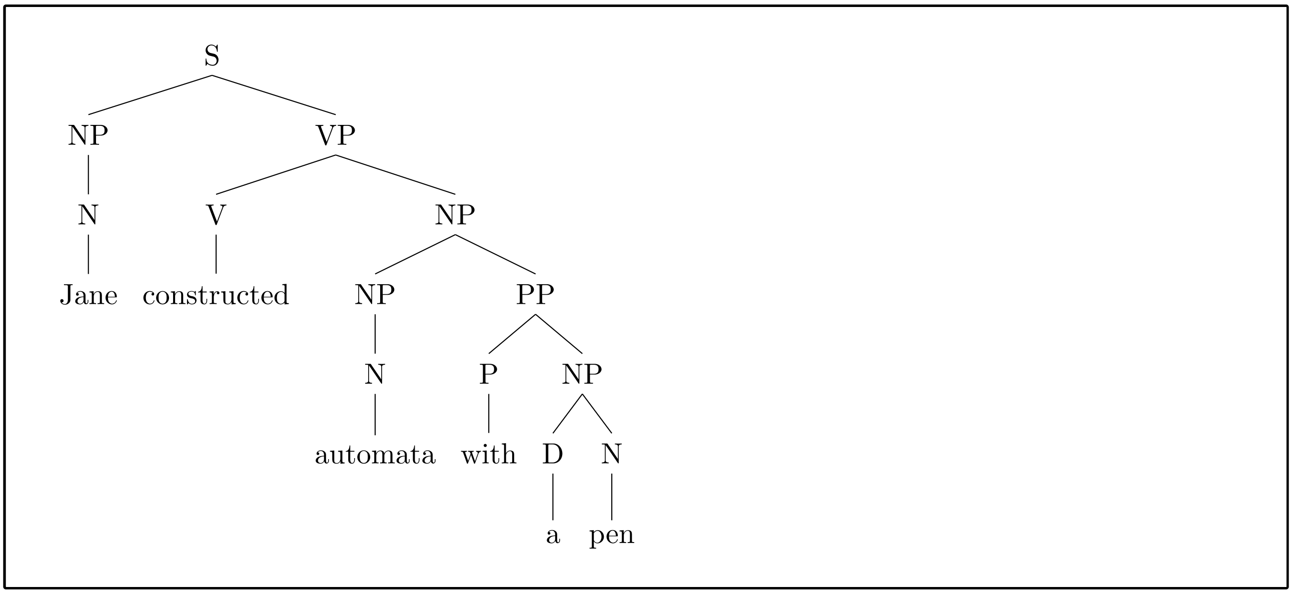
2 Parse Trees and Derivations (20 pts)

Given the CFG below, provide parse trees for given sentences in **a** and **b**.

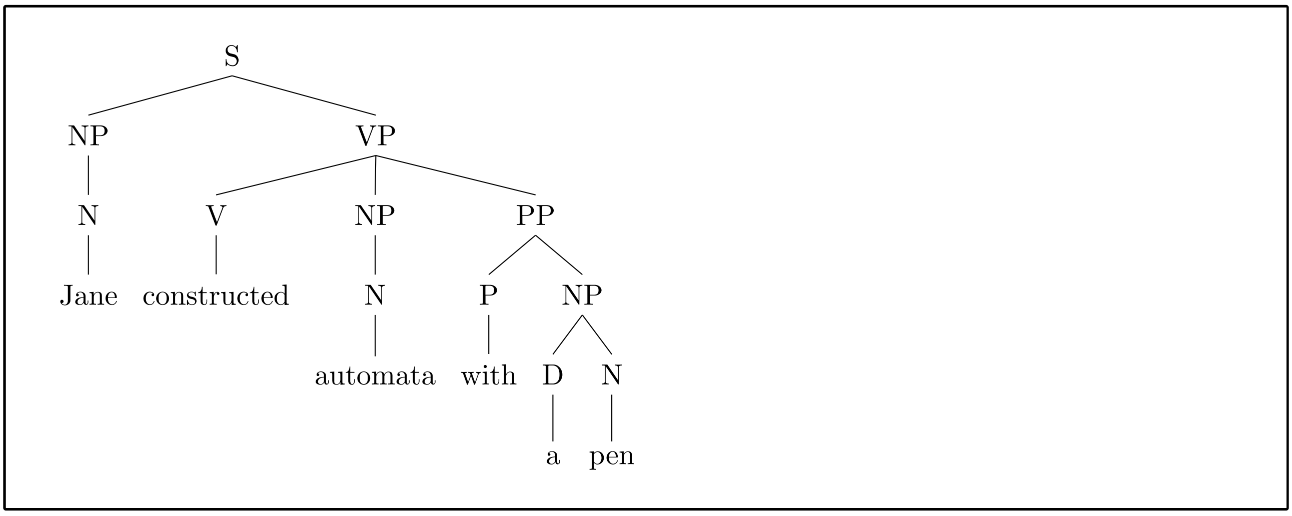
```
S   → NP VP
VP  → V NP | V NP PP
PP  → P NP
NP  → N | D N | NP PP
V   → wrote | built | constructed
D   → a | an | the | my
N   → John | Mary | Jane | man | book | automata | pen | class
P   → in | on | by | with
```

a) Jane constructed automata with a pen (4/20 pts)

First one:

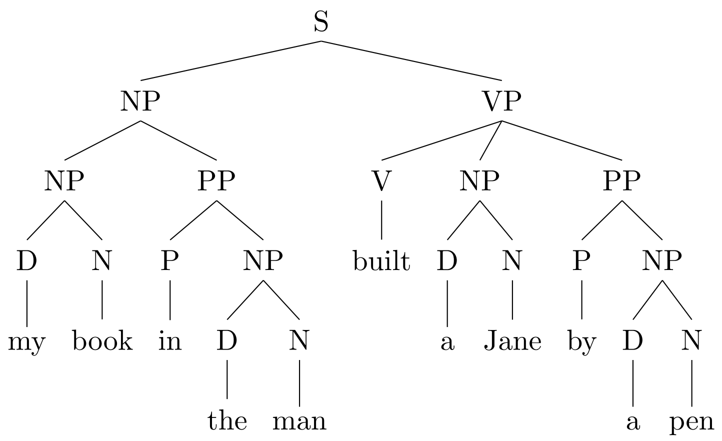


Second one:

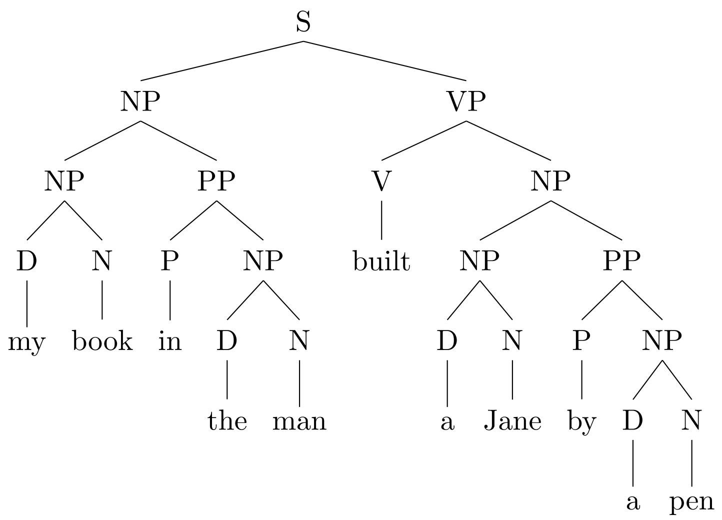


b) my book in the man built a Jane by a pen (4/20 pts)

First one:



Second one:

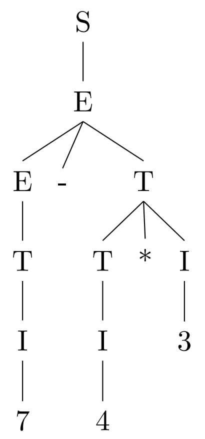


Given the CFG below, answer **c**, **d** and **e**

```
S  → E
E  → E + T | E - T | T
T  → T * I | T / I | I
I  → 0 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9
```

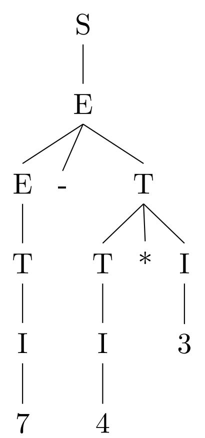
c) Provide the left-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \to E - T \to T - T \to I - T \to 7 - T \to\to 7 - T * I \to 7 - 4 * I \to 7 - 4 * 3$



d) Provide the right-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \to E \to E - T \to E - T * I \to E - T * 3 \to E - T * 3 \to E - I * 3 \to E - 4 * 3 \to T - 4 * 3 \to I - 4 * 3 \to 7 - 4 * 3$

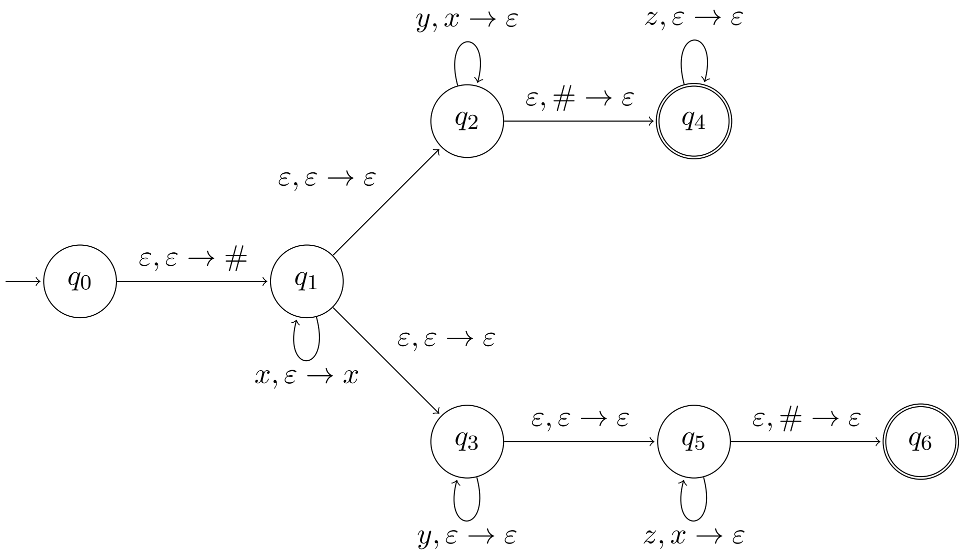


e) Are the derivations in **c** and **d** in the same similarity class? (4/20 pts)

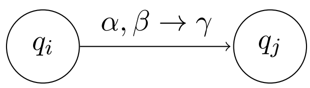
Yes, because they can transform other one and one of them precedes other.

3 Pushdown Automata (30 pts)

a) Find the language recognized by the PDA given below (5/30 pts)



where the transition $((q_i, \alpha, \beta), (q_j, \gamma))$ is represented as:



This language represents that:
$x^n y^n z^* \cup x^n y^* z^n$
So that, L = $\{x^n y^n z^k \cup x^n y^l z^n \mid k, l, n \geq 0\}$

b) Design a PDA to recognize language $L = \{x^n y^{m+n} x^m \mid\ n, m \geq 0;\ n, m \in \mathbb{N}\}$ (5/30 pts)

answer here ...

c) Design a PDA to recognize language $L = \{x^n y^m \mid n < m \leq 2n;\ n, m \in \mathbb{N}^+\}$ (10/30 pts)
Do not use multi-symbol push/pop operations in your transitions.
Simulate the PDA on strings xxy (with only one rejecting derivation) and $xxyyyy$ (accepting derivation) with transition tables.

answer here ...

d) Given two languages L' and L as $L' = \{w \mid \ w \in L;\ |w| = 4n + 2 \ for \ n \in \mathbb{N}\}$ (10/30 pts)
If L is a CFL, show that L' is also a CFL by constructing an automaton for L' in terms of another automaton that recognizes L.

answer here ...

4 Closure Properties (20 pts)

Let L_1 and L_2 be context-free languages which are not regular, and let L_3 be a regular language. Determine whether the following languages are necessarily CFLs or not. If they need to be context-free, explain your reasoning. If not, give one example where the language is a CFL and a counter example where the language is not a CFL.

a) $L_4 = L_1 \cap (L_2 \setminus L_3)$ (10/20 pts)

No.
$L_2 - L_3 = L_2 \cap L_3'$ since $(L_3)'$ is regular because regular languages are closed under complement and intersection of them is context free.
$L_1 \cap L_3$ not certainly context free.
$L_1 = q^k z^k x^l$ and $L_2 - L_3 = z^k x^k$

b) $L_5 = (L_1 \cap L_3)^*$ (10/20 pts)

Yes.
All regular languages are subset of context-free so that $L_1 \cap L_3$ is also context-free.
Context-free languages are closed under Kleene Star so that L_5 is CFL.

5 Pumping Theorem (20 pts)

a) Show that $L = \{a^n m^n t^i \mid n \leq i \leq 2n\}$ is not a Context Free Language (10/20 pts)
using Pumping Theorem for CFLs.

$uvxyz = aammtt$ and $vxy = aam \rightarrow$
$uv^2xy^2z = aaammmtt \rightarrow$
$n > i$
so that this language is not Context-Free.

b) Show that $L = \{a^n b^{2n} a^n \mid n \in \mathbb{N}+\}$ is not a Context Free Language (10/20 pts)
using Pumping Theorem for CFLs.

$uvxyz = abba$ and $vxy = abb \rightarrow$
$uv^2xy^2z = aabbba \rightarrow$
$n = 2$, $2n = 3$, $n = 1$
so that this language is not Context-Free.

6 CNF and CYK (not graded)

a) Convert the given context-free grammar to Chomsky Normal Form.

$S \to XSX \mid xY$
$X \to Y \mid S$
$Y \to z \mid \varepsilon$

answer here ...

b) Use the grammar below to parse the given sentence using Cocke–Younger–Kasami algorithm. Plot the parse trees.

S → NP VP
S → X1 VP
X1 → Aux NP
S → book | include | prefer
S → Verb NP
S → X2 PP
S → Verb PP
S → VP PP
NP → I | she | me | Houston
NP → Det Nom
Nom → book | flight | meal | money
Nom → Nom Noun
Nom → Nom PP

VP → book | include | prefer
VP → Verb NP
VP → X2 PP
X2 → Verb NP
VP → Verb PP
VP → VP PP
PP → Prep NP
Det → that | this | the | a
Noun → book | flight | meal | money
Verb → book | include | prefer
Aux → does
Prep → from | to | on | near | through

book the flight through Houston

Empty parse table:

1:5 → 1:1 2:5 1:5 → 1:2 3:5 1:5 → 1:3 4:5 1:5 → 1:4 5:5				
1:4 → 1:1 2:4 1:4 → 1:2 3:4 1:4 → 1:3 4:4	2:5 → 2:2 3:5 2:5 → 2:3 4:5 2:5 → 2:4 5:5			
1:3 → 1:1 2:3 1:3 → 1:2 3:3	2:4 → 2:2 3:4 2:4 → 2:3 4:4	3:5 → 3:3 4:5 3:5 → 3:4 5:5		
1:2 → 1:1 2:2	2:3 → 2:2 3:3	3:4 → 3:3 4:4	4:5 → 4:4 5:5	
1:1	2:2	3:3	4:4	5:5
book	the	flight	through	Houston

rest of the answer here ...

7 Deterministic Pushdown Automata (not graded)

Provide a DPDA to recognize the given languages, the DPDA must read its entire input and finish with an empty stack.

a) $a^*bc \cup a^nb^nc$

answer here ...

b) $(aa)^*c \cup a^n b^n c$

answer here ...