Middle East Technical University Department of Computer Engineering

Formal Languages and Abstract Machines
Take Home Exam 2

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2099471

1 Context-Free Grammars (10 pts)

a) Give the rules of the Context-Free Grammars to recognize strings in the given languages where $\Sigma = \{a, b\}$ and S is the start symbol.

$L(G) = \{w \mid\ w \in \Sigma^*;\ |w| \geq 3;$ the first and the second from the last symbols of w are the same$\}$ (2/10 pts)

$S \rightarrow aAab \mid aAaa \mid bAbb \mid bAba$
$A \rightarrow aA \mid bA \mid a \mid b \mid e$

$L(G) = \{w \mid\ w \in \Sigma^*;$ the length of w is odd$\}$ (2/10 pts)

$S \rightarrow Aa \mid Ab$
$A \rightarrow Aaa \mid Aab \mid Aba \mid Abb \mid e$

$L(G) = \{w \mid\ w \in \Sigma^*;\ n(w, a) = 2 \cdot n(w, b)\}$ where $n(w, x)$ is the number of x symbols in w (3/10 pts)

$S \rightarrow BBb \mid BbB \mid bBB \mid e$
$B \rightarrow aS \mid Sa$

b) Find the set of strings recognized by the CFG rules given below: (3/10 pts)

$S \to X \mid Y$
$X \to aXb \mid A \mid B$
$A \to aA \mid a$
$B \to Bb \mid b$
$Y \to CbaC$
$C \to CC \mid a \mid b \mid \varepsilon$

$a^+b^+ \mid (a|b)^*\ ba\ (a|b)^*$

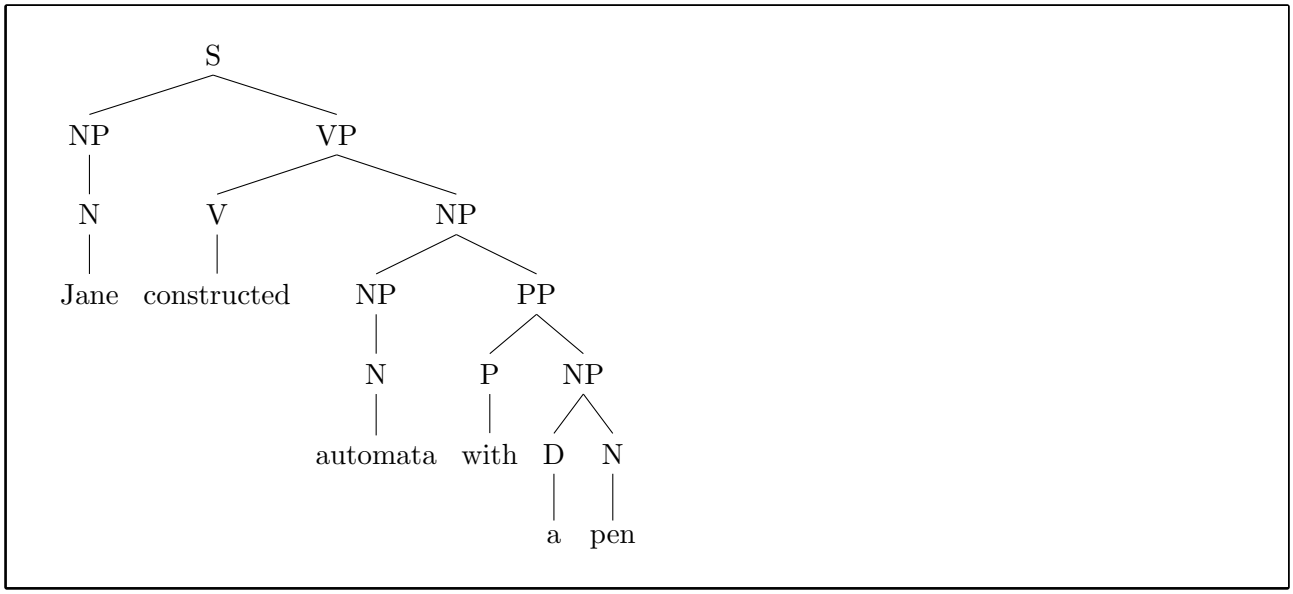
2 Parse Trees and Derivations (20 pts)

Given the CFG below, provide parse trees for given sentences in **a** and **b**.

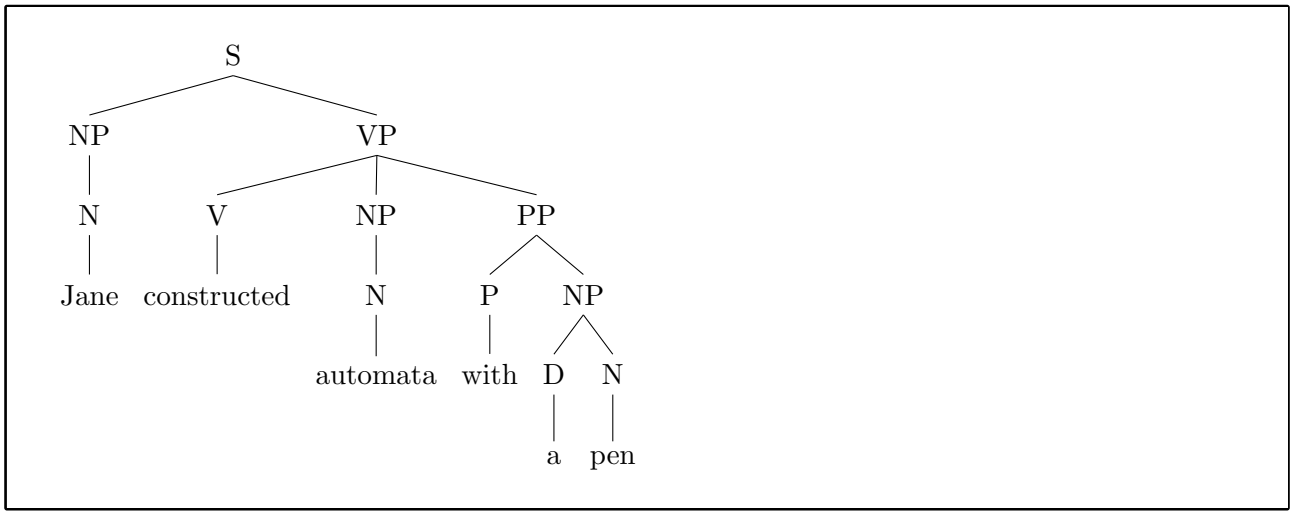
```
S   → NP VP
VP  → V NP | V NP PP
PP  → P NP
NP  → N | D N | NP PP
V   → wrote | built | constructed
D   → a | an | the | my
N   → John | Mary | Jane | man | book | automata | pen | class
P   → in | on | by | with
```

a) Jane constructed automata with a pen (4/20 pts)

First one:

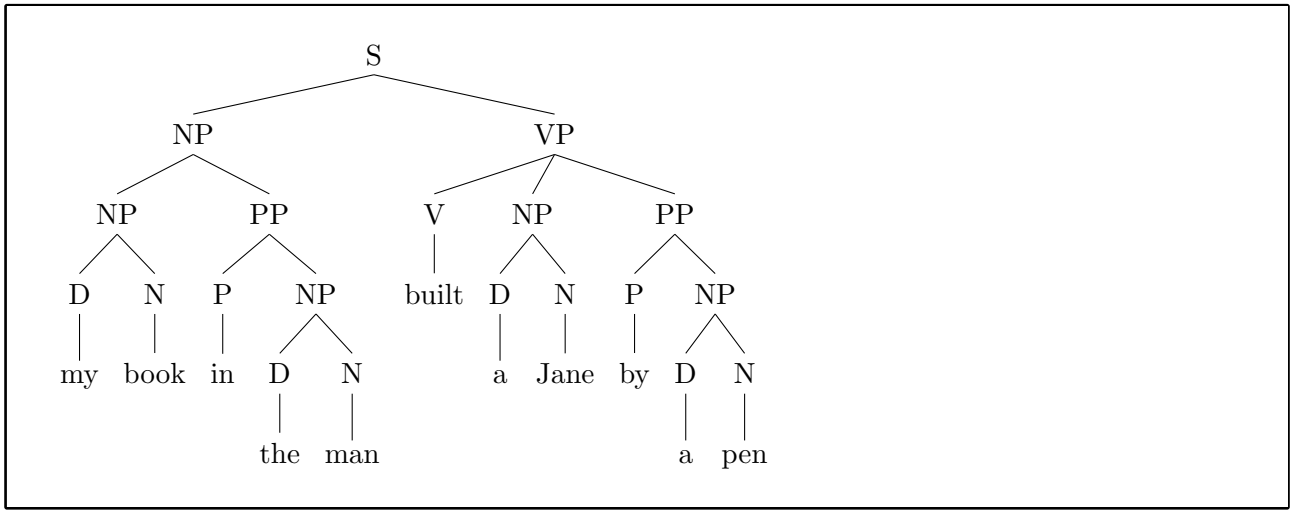


Second one:

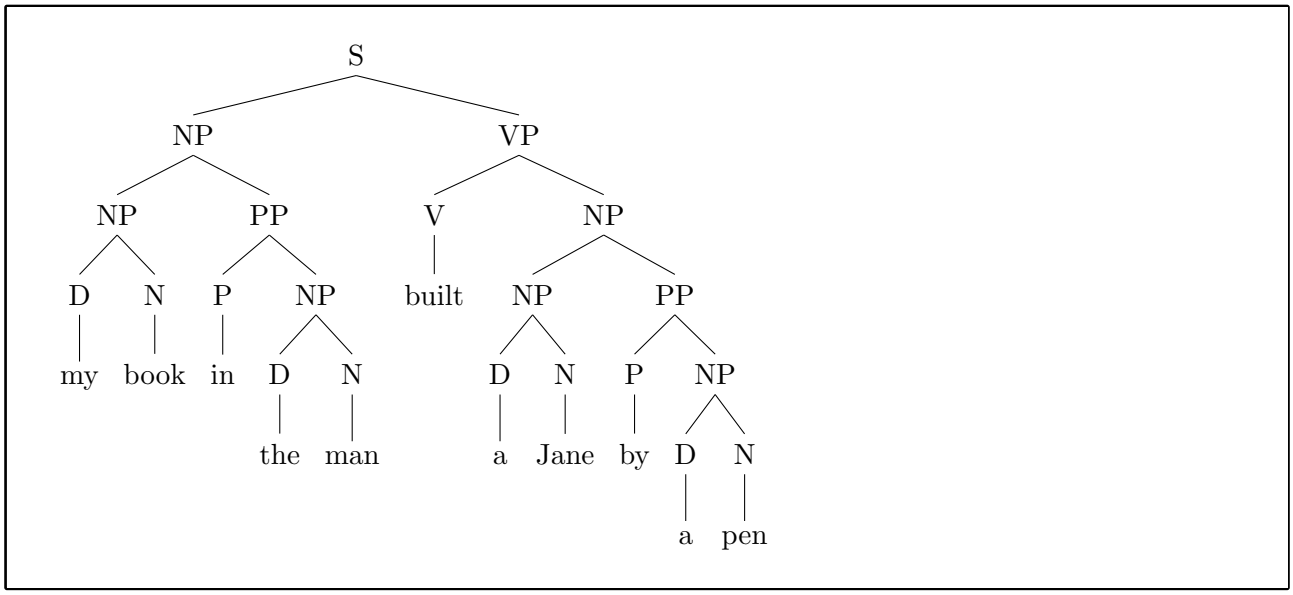


b) my book in the man built a Jane by a pen (4/20 pts)

First one:



Second one:

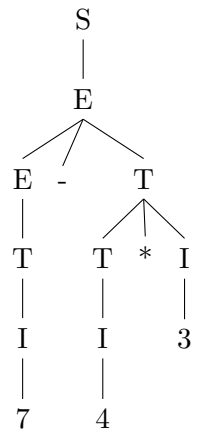


Given the CFG below, answer **c**, **d** and **e**

```
S  → E
E  → E + T | E - T | T
T  → T * I | T / I | I
I  → 0 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9
```

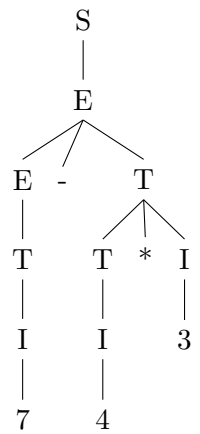
c) Provide the left-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \to E - T \to T - T \to I - T \to 7 - T \to\to 7 - T * I \to 7 - 4 * I \to 7 - 4 * 3$



d) Provide the right-most derivation of 7 - 4 * 3 step-by-step and plot the final parse tree matching that derivation (4/20 pts)

$S \to E \to E - T \to E - T * I \to E - T * 3 \to E - T * 3 \to E - I * 3 \to E - 4 * 3 \to T - 4 * 3 \to I - 4 * 3 \to 7 - 4 * 3$

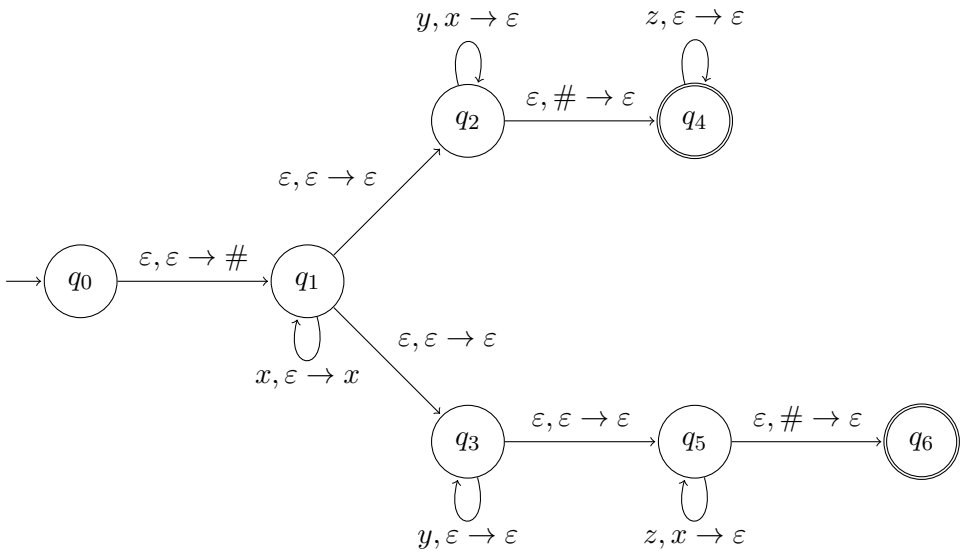


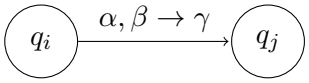
e) Are the derivations in **c** and **d** in the same similarity class? (4/20 pts)

Yes, because they can transform other one and one of them precedes other.

3 Pushdown Automata (30 pts)

a) Find the language recognized by the PDA given below (5/30 pts)

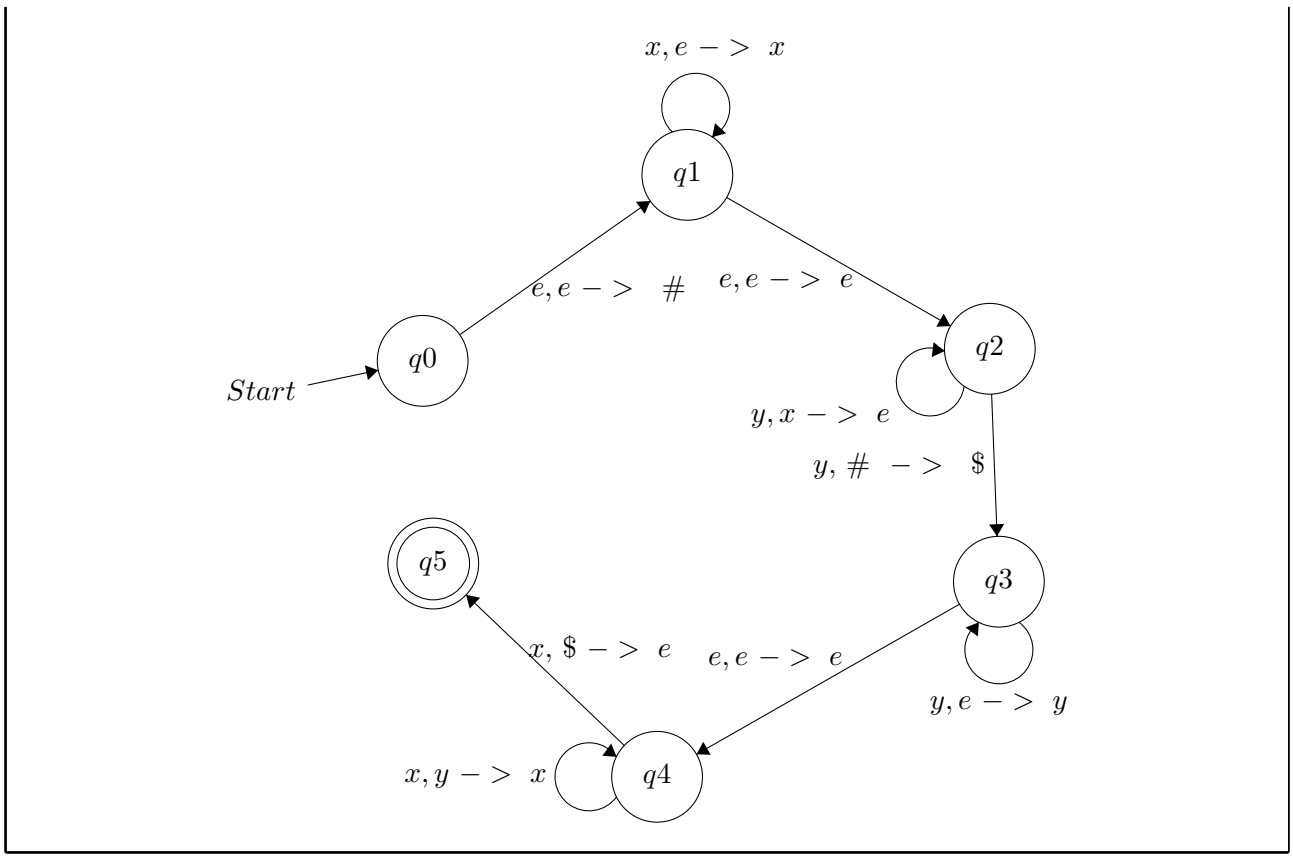


where the transition $((q_i, \alpha, \beta), (q_j, \gamma))$ is represented as: 

This language represents that:
$x^n y^n z^* \cup x^n y^* z^n$
So that, L = $\{x^n y^n z^k \cup x^n y^l z^n \mid k, l, n \geq 0\}$

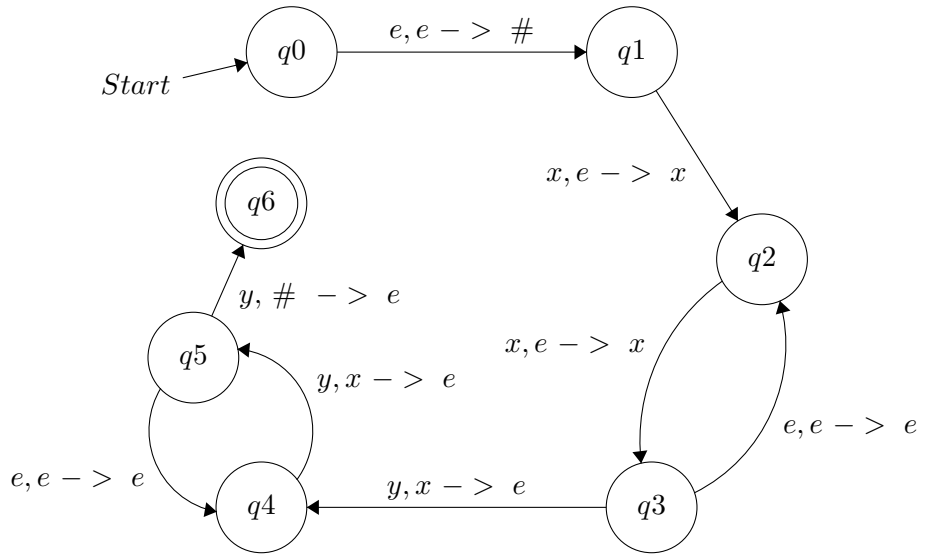
b) Design a PDA to recognize language $L = \{x^n y^{m+n} x^m \mid \ n, m \geq 0;\ n, m \in \mathbb{N}\}$ (5/30 pts)

Answer:


x, e -> x
q1
e, e -> #
e, e -> e
q0
q2
Start
y, x -> e
y, # -> $
q5
q3
x, $ -> e
e, e -> e
y, e -> y
x, y -> x
q4

c) Design a PDA to recognize language $L = \{x^n y^m \mid n < m \leq 2n;\ n, m \in \mathbb{N}^+\}$ (10/30 pts)
Do not use multi-symbol push/pop operations in your transitions.
Simulate the PDA on strings *xxy* (with only one rejecting derivation) and *xxyyyy* (accepting derivation) with transition tables.

Answer:



For the xxy this automata will not work, because it will stuckwa.
For the xxyyyy this automata will come its final state so that xxyyyy will be accepted.

d) Given two languages L' and L as $L' = \{w \mid w \in L;\ |w| = 4n + 2\ for\ n \in \mathbb{N}\}$ (10/30 pts)
If L is a CFL, show that L' is also a CFL by constructing an automaton for L' in terms of another automaton that recognizes L.

I worked hard but can not solve this problem...

4 Closure Properties (20 pts)

Let L_1 and L_2 be context-free languages which are not regular, and let L_3 be a regular language. Determine whether the following languages are necessarily CFLs or not. If they need to be context-free, explain your reasoning. If not, give one example where the language is a CFL and a counter example where the language is not a CFL.

a) $L_4 = L_1 \cap (L_2 \setminus L_3)$ (10/20 pts)

No.
$L_2 - L_3 = L_2 \cap L_3'$ since $(L_3)'$ is regular because regular languages are closed under complement and intersection of them is context free.
$L_1 \cap L_3$ not certainly context free.
$L_1 = q^k z^k x^l$ and $L_2 - L_3 = z^k x^k$

b) $L_5 = (L_1 \cap L_3)^*$ (10/20 pts)

Yes.
All regular languages are subset of context-free so that $L_1 \cap L_3$ is also context-free.
Context-free languages are closed under Kleene Star so that L_5 is CFL.

5 Pumping Theorem (20 pts)

a) Show that $L = \{a^n m^n t^i \mid n \leq i \leq 2n\}$ is not a Context Free Language using Pumping Theorem for CFLs. (10/20 pts)

$uvxyz = aammtt$ and $vxy = aam \rightarrow$
$uv^2xy^2z = aaammmtt \rightarrow$
$n > i$
so that this language is not Context-Free.

b) Show that $L = \{a^n b^{2n} a^n \mid n \in \mathbb{N}+\}$ is not a Context Free Language using Pumping Theorem for CFLs. (10/20 pts)

$uvxyz = abba$ and $vxy = abb \rightarrow$
$uv^2xy^2z = aabbba \rightarrow$
$n = 2$, $2n = 3$, $n = 1$
so that this language is not Context-Free.

6 CNF and CYK (not graded)

a) Convert the given context-free grammar to Chomsky Normal Form.

$S \to XSX \mid xY$
$X \to Y \mid S$
$Y \to z \mid \varepsilon$

answer here ...

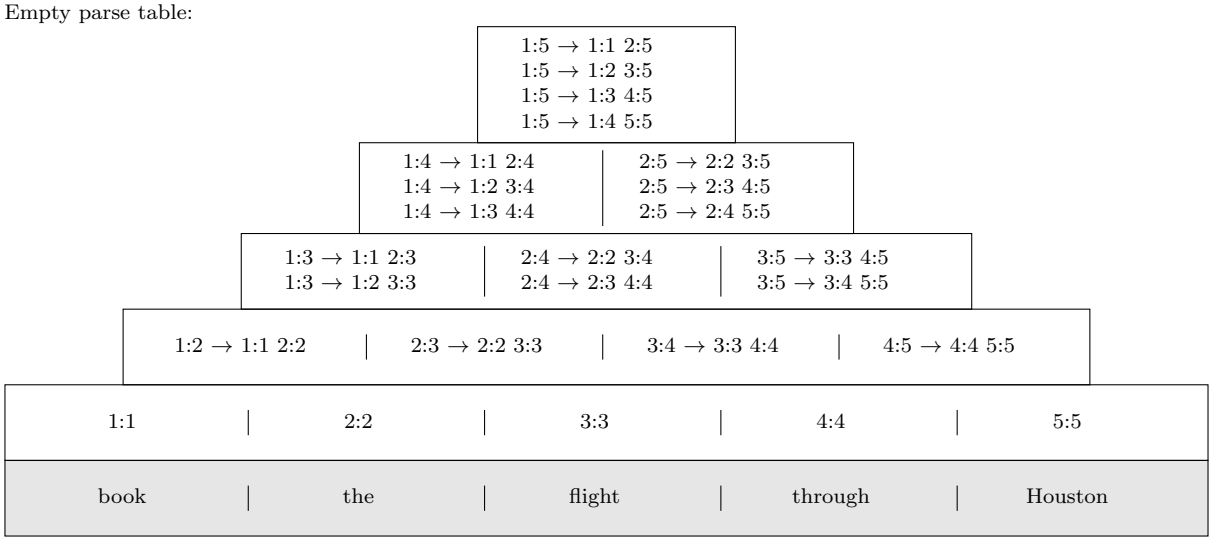
b) Use the grammar below to parse the given sentence using Cocke–Younger–Kasami algorithm. Plot the parse trees.

S → NP VP
S → X1 VP
X1 → Aux NP
S → book | include | prefer
S → Verb NP
S → X2 PP
S → Verb PP
S → VP PP
NP → I | she | me | Houston
NP → Det Nom
Nom → book | flight | meal | money
Nom → Nom Noun
Nom → Nom PP

VP → book | include | prefer
VP → Verb NP
VP → X2 PP
X2 → Verb NP
VP → Verb PP
VP → VP PP
PP → Prep NP
Det → that | this | the | a
Noun → book | flight | meal | money
Verb → book | include | prefer
Aux → does
Prep → from | to | on | near | through

book the flight through Houston

Empty parse table:



rest of the answer here ...

7 Deterministic Pushdown Automata (not graded)

Provide a DPDA to recognize the given languages, the DPDA must read its entire input and finish with an empty stack.

a) $a^*bc \cup a^n b^n c$

answer here ...

b) $(aa)^*c \cup a^n b^n c$

answer here ...