Computer exercise 4

Purpose of computer exercise: Implement a Monte Carlo simulation of a credit metrics type model that is used for estimating credit value-at-risk. For tractability we consider a two loans portfolio; one loan that today is BBB rated and one loan that today is A rated. The data shows the probability of transitioning from one rating to another in one year and associated losses (negative numbers are gains). Note: the part were we estimate credit spreads based on the ratings and then re-calculate the bond prices to estimate the losses are already done before the exercise starts.

1. For the BBB loan, answer the following question. How many upgrades and downgrades to each rating in one year do we expect, given the probability migration table and the fact that we are using $M = 100,000$ simulations (draws)? That is how many times out of 100,000 will the loan with an initial BBB rating get AAA, AA, ...D. **Answer in the script.**

2. In the sequel we will mainly focus on the portfolio of both loans. Start by calculating VaR for the portfolio using the normal distribution approximation. Again, assume that $\mu = 0$. For the portfolio we have $8 \cdot 8 = 64$ different possible outcomes (losses) and the probability of each outcome is not easily calculated unless we assume independence (a zero asset correlation). In this case the probability of each outcome is easily calculated. For example, the probability that the BBB loan will lose 8.99 *and* the A loan will lose 1.08 (corresponding to a portfolio loss of $8.99 + 1.08 = 10.07$) is equal to $0.0117 \cdot 0.0552 = 0.0646\%$. Continuing this way the probabilities for all possible losses may be calculated and hence the standard deviation of these 64 possible losses. The result is $\sigma_P = 3.35$ (you do not have to perform these calculations). As usual P denotes the portfolio. **Calculate VaR using the normal approximation for the portfolio at the $\alpha = 0.99, 0.999$ and 0.9999 confidence levels and answer in the script.**

3. a) Simulate 100,000 observations each of two normal distributed random variables: $x_1 \sim \Phi(0,1)$ and $x_2 \sim \Phi(0,1)$ where x_1 and x_2 are uncorrelated.

b) Calculate the quantiles based on the probability migration tables for both loans. These are the quantiles that will then be used in the simulation. This table shows some of the results for the BBB loan so you can check that you get it right:

$$
\begin{aligned}
z_D &= \Phi^{-1}(0.0018) = -2.91 \\
z_{CCC} &= \Phi^{-1}(0.0030) = -2.75 \\
&\vdots \\
z_{AAA} &= \Phi^{-1}(0.9998) = +3.54.
\end{aligned}
$$

4. Next, you should make the x_1 and x_2 numbers correlated by using the standard Cholesky decomposition:

$$\begin{aligned} q_1 &= x_1 \\ q_2 &= R \cdot x_1 + \sqrt{1-R^2} \cdot x_2. \end{aligned}$$

Use asset correlation $R = 0.35$. Check so that the empirical correlation between q_1 and q_2 is close to 0.35.

5. We want to check the simulated credit migrations (we below focus on the BBB loan but the procedure is the same for the A loan). Use the random numbers in the vector q1 to calculate the number of occurrences you get for each rating. Are the numbers similar to what you got in 1? (They should be.)

6. Simulate the loss distribution for each bond, use the numbers q1 and q2 that you have already constructed. For example for the BBB bond, if $q_1 = -2.20$, then the simulated loss is $L_1 = 8.99$. Then construct the portfolio losses by adding the loss for each bond for each Monte Carlo replications so you have a 100,000x1 vector of portfolio losses. Plot and look at the distribution of portfolio losses.

7. Calculate portfolio VaR $\alpha = 0.99, 0.999$ and 0.9999 based on the simulated portfolio losses using basic historical simulation. Write your answers manually in the script then simply change the asset correlation $R = 0$ and redo the calculation and once again write the answer in the script (you need to rerun all code from the point were you generate the random numbers).

8. Short questions, answer directly in the script. (a) Why is it that the normal approximation (question 2) seems to be very far away for the "correct" portfolio VaR for the two higher confidence levels but kind of reasonably OK for the lowest confidence level? Hint: Think about the shape of the loss distribution. (b) Does the asset correlation R seem to matter (for the VaR estimate)? Hint 1: Only for the very high confidence level (at least a little). Hint 2: We have only two loans in our portfolio. Use an "unreasonably" high asset correlation, for example $R = 0.95$. Then you will se a huge change in VaR for the highest confidence level (comparing with $R = 0$). Why?