1. Specification

Given A of $n = 2^k$ (for some $k \in \mathbb{Z}^+$) non-zero real numbers, determine a maximal value M such that M is the sum of some subsequence of A.

Note: The assignment does not specify whether to consider 0, the sum of an empty subsequence, to be a valid value for M, which would be the case if A consists solely of negative values. The choice has been made to exclude this possibility.

2. Overview

Given the input $A = [a_0, a_1, ...a_{n-1}]$, do the following:

- Recursively find M for the two halves of A. For a 1-length split $A = [a]$, the only possibility is $M(A) = a$.
- Combine the splits (L, R) pairwise. M is the largest of the following:
  - $M(L)$, i.e. the maximum value of the left split.
  - $M(R)$, i.e. the maximum value of the right split.
  - $M(A_m)$ for any subsequence A_m that crosses the midpoint.

3. Implementation

For each recursive step, given A, determine a 4-tuple $T = (M, p, s, t)$ where $p = M(A_l)$ for any subsequence A_l that includes the first element (the prefix sum), s is defined analogously for the last element (the suffix sum), and t is the total sum of A.

For $A = [a]$, T is trivially (a, a, a, a). In other cases, we split A into (L, R) and determine the values of T as follows:

$$\begin{aligned} M(A) &= \max(M(L), M(R), s(L) + p(R)), \\ p(A) &= \max(p(L), t(L) + p(R)), \\ s(A) &= \max(s(R), t(R) + s(L)), \\ t(A) &= t(L) + t(R). \end{aligned}$$

The result is $M(T)$.

4. Complexity

Let $T(n)$ be the time complexity. For the base case, $T(n) = O(1)$ as $n = 1$. In the general case, we consider two portions, each half the size of the input, for a total cost of $2 \cdot T(\frac{n}{2})$.

This gives us the recurrence equation

$$T(n) = \begin{cases} O(1) & \text{if } n = 1, \\ 2 \cdot T(\frac{n}{2}) + O(1) & \text{otherwise.} \end{cases}$$

By the master theorem, this solves to

$$T(n) = \Theta\big(n^{\log_2 2}\big) = \Theta(n).$$