TDDE07 - Lab 3

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1 - Normal model, mixture of normal model with semi-conjugate prior

1a) - Normal model

The code for the Gibbs implementation can be seen in appendix 1.

By plotting the trajectories of the sampled Markov chains it can be seen that σ and μ converges around 39 and 32 respectively.

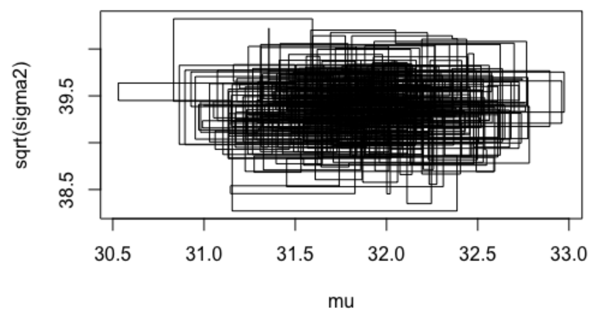
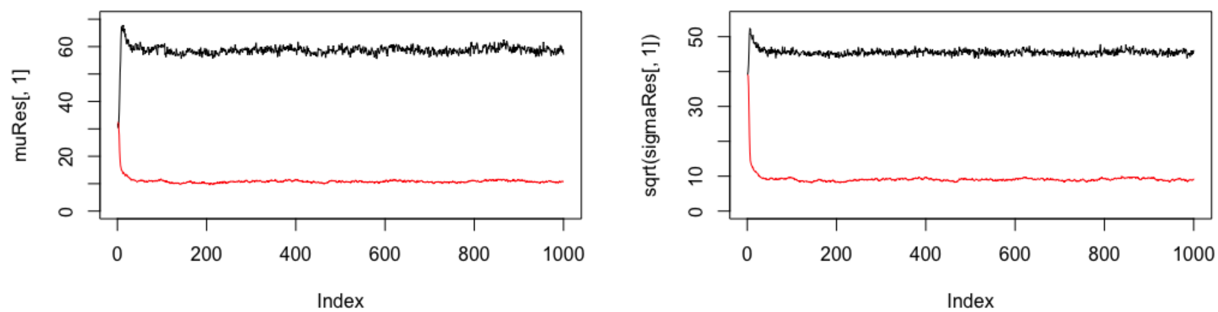


Figure 1: Analyzing the convergence

1b) - Mixture normal model

The results of the Gibbs sampling data augmentation algorithm given in NormalMixtureModel.R resulted can be seen below.



Both μ and σ converged after a few samples.

1c) - Graphical comparison

The figure below shows a comparison between the samplers. The blue line represents the normal density from excercise a, and the green line represents the mixture of normals density from excercise b. Both of the models have limitations when it comes to fit the data.

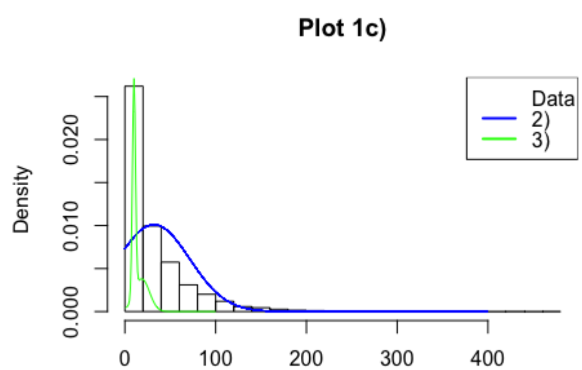
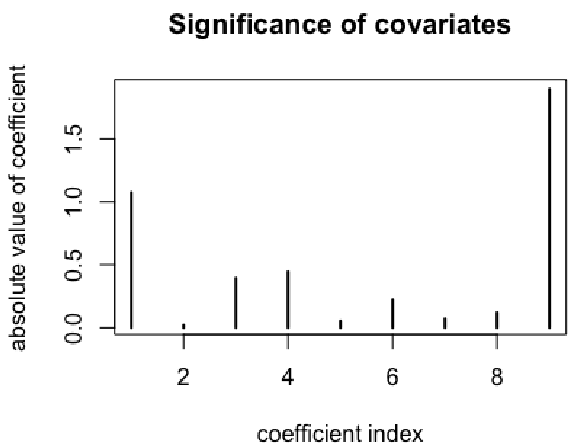


Figure 2: Graphical comparison

2 - Metropolis Random Walk for Poisson regression

2a)

The absolute values of the parameters obtained by a maximum likelihood estimation is shown below.



The figure shows that the ninth parameter, meaning minBidShare, is the most significant covariate, whereas PowerSeller is the least significant.

2b)

The logistic Poisson regression resulted in the following values for βs

:

	Const	PowerSeller	VerifyID	Sealed	Minblem	MajBlem	LargNeg	LogBook	MinBidShare
σ	0.4932	0.6767	1.4006	1.0888	1.0942	1.7190	1.1535	0.5236	1.0231545
$\hat{\beta}$	.33285	-0.28581	0.03928	-0.14391	-0.21800	-0.19890	-0.19947	-0.15492	-0.22178

Table 1: Posterior mode and stdev

2c)

The result below is from 50 000 sample draws from the posterior. In order to analyze the convergence, β_1 and β_2 are plotted in the same plot. As can be seen, most of the samples are drawn from a oval-shaped distribution centered around the modes of the betas. Thus, the algorithm has converged.

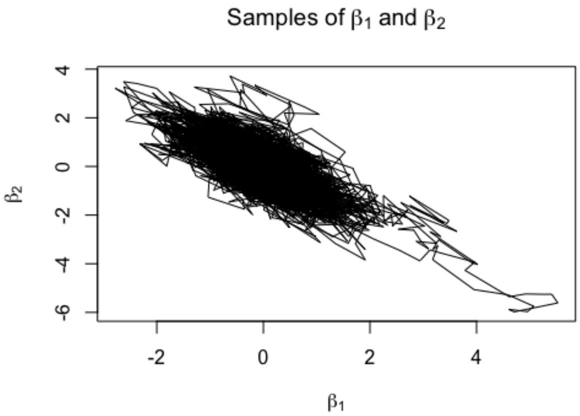


Figure 3: Significance of covariates

Code - 1

```
data = read.table("rainfall.txt", col.names = "x")

########### A ###########

# parameter init values
mu0 <- 30
tau0 <- 1
v0 <- 0
sigma0 <- 10

n <- nrow(data)
dataMean <- mean(data$x)
vn <- v0 + n

iter <- 1000

# draw mu
drawMu <- function(prevMu, prevSigma) {
  tauSq <- 1/( (n/prevSigma) + (1/tau0^2) )
  w <- (n/prevSigma)/((n/prevSigma) + (1/tau0^2))
  mu <- w*dataMean + (1-w)*mu0
  draw <- rnorm(1, mu, sqrt(tauSq))
  return (draw)
}
#inv chi square
invChiSquare <- function(v, s) {
  return(v*s / rchisq(1,v))
}

# draw sigma
drawSigma <- function(mu) {
  sum <- 0
  for (i in 1:n) {
    sum <- sum + (data[i,1] - mu)^2
  }
  s <- (v0*sigma0 + sum)/(n+v0)
  return(invChiSquare(vn, s))
}

mu <- c()
sigma2 <- c()

currMu <- 32
currSigma <- sigma0
for (i in 1:iter) {
  if(i %% 2 == 0) {
    currMu <- drawMu(currMu, currSigma)
  } else {
    currSigma <- drawSigma(currMu)
  }
  mu <- c(mu, currMu)
```

```r
  sigma2 <- c(sigma2, currSigma)
}

## plot trajactories of sampled mu and sigma
plot(mu, sqrt(sigma2),type='l')

######### C #########
xGrid <- seq(0,400,by=(100/n))
ndens <- dnorm(xGrid, mean(mu), mean(sqrt(sigma2)))

hist(data$x, 20, main = 'Plot 1c)', freq = FALSE, xlab='')
lines(xGrid, ndens, col='blue')
lines(mixDensMean, col = 'green')
legend("topright", box.lty = 1, legend = c("Data",'2)', '3)'),
       col = c("white",'blue', 'green'), lwd = 2)
```

Code - 2

```
library(mvtnorm)

data <- read.table("eBayNumberOfBidderData.txt", header = TRUE)


########## A ##########
fit <- glm(nBids ~ 0 + ., data, family = poisson)
coeff <- fit$coefficients
plot(abs(coeff), type='h',
     lwd=2,
     xlab = "coefficient index",
     main='Significance of covariates',
     ylab='absolute value of coefficient')

X <- as.matrix(data[,2:10])
## The most significant covariate is minBidShare.

########## B ##########
sigmaPrior <- 100 * solve(t(X)%*%X)

logPois <- function(beta, y, x, ...) {
  # log likelihood of poisson model

  n <- length(x)

  logLik <- 0
  for (i in 1:length(n)) {
    logLik <- logLik + y[i] * t(beta)%*%x[i,] - exp( t(beta)%*%x[i,] - log(factorial(y[i])))
  }

  # log of prior
  logPrior <- dmvnorm(beta, mean = rep(0, 9), sigma = sigmaPrior, log=TRUE)

  # add
  return(logLik + logPrior)
}

OptimResults<-optim(coeff,logPois,gr=NULL, y = data$nBids,x = X,method=c("BFGS"),control=list(fnscale=-


postCov <- -solve(OptimResults$hessian)
st_div <- sqrt(diag(postCov))
betaMode <- OptimResults$par

########## C ##########

gaussianSample <- function(theta, sigma, c) {
  val <- rmvnorm(1, theta, c*sigma)
  return (val)
}
```

```r
RWMSampler <- function(c, it, initBeta, fn, ...) {
  accRate <- 0
  sample <- c()
  prev <- gaussianSample(initBeta, postCov, c)
  for (i in 1:it) {
    candidate <- gaussianSample(prev, postCov, c)
    alpha <- min(1,exp(fn(prev, ...) - fn(candidate, ...)))
    u <- runif(1, 0, 1)
    if (alpha <= u) {
      # accept candidate
      prev <- candidate
      accRate <- accRate + 1
      # as matrix
      sample <- rbind(sample, prev)
    }
  }
  return (sample)
}

sample = RWMSampler(1,50000,betaMode, logPois, data$nBids, X)
plot(sample[,1], sample[,2],
     type='l',
     xlab = expression(beta[1]),
     ylab = expression(beta[2]),
     main = expression("Samples of" ~ beta[1] ~ "and" ~ beta[2]))
```