

## GATE 2015

## SECTION: GENERAL APTITUDE

- Q1. Choose the statement where underlined word is used correctly.
- (a) When the teacher eludes to different authors, he is being elusive
  - (b) When the thief keeps eluding the police he is being elusive
  - (c) Matters that are difficult to understand, identify or remember are allusive
  - (d) Mirages can be allusive but a better way to express them is illusory
- Q2. Fill in the blank with the correct idiom/phrase:
- The boy from the town was a \_\_\_\_\_ in sleepy village
- (a) dog out of herd
  - (b) sheep from the heap
  - (c) fish out of water
  - (d) bird from the flock
- Q3. Choose the appropriate word/phrase out of the four options given below, to complete the following sentence:
- Apparent lifelessness \_\_\_\_\_ dormat life.
- (a) harbours
  - (b) leads to
  - (c) supports
  - (d) affects
- Q4. Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?
- (a) 20
  - (b) 10
  - (c) 8
  - (d) 5
- Q5. Tanya is older than Eric. Cliff is older than Tanya. Eric is older than Cliff
- If the first two statement are true, then the third statement is:
- (a) True
  - (b) False
  - (c) Uncertain
  - (d) Data insufficient

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- Q6. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

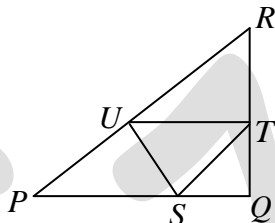
- I. No manager is a leader  
II. All leaders are executives

Conclusions:

- I No manager is an executive.  
II No executive is a manager.

- (a) Only conclusion I follows (b) Only conclusion II follows  
(c) Neither conclusion I nor II follows (d) Both conclusions I and II follow

- Q7. In the given figure angle  $Q$  is a right angle,  $PS:QS = 3:1$ ,  $RT:QT = 5.2$  and  $PU:UR = 1.1$ . If area of triangle  $QTS$  is  $20 \text{ cm}^2$ , then the area of triangle  $PQR$  in  $\text{cm}^2$  is \_\_\_\_\_.



- Q8. Select the appropriate option in place of underlined part of the sentence.  
Increased productivity necessary reflects greater efforts made by the employees.  
(a) Increase in productivity necessary  
(b) Increase productivity is necessary  
(c) Increase in productivity necessarily  
(d) No improvement required
- Q9. Right triangle  $PQR$  is to be constructed in the  $xy$ -plane so that the right angle is at  $P$  and line  $PR$  is parallel to the  $x$ -axis. The  $x$  and  $y$  coordinates of  $P, Q$  and  $R$  are to be integers that satisfy the inequalities:  $-4 \leq x \leq 5$  and  $6 \leq y \leq 16$ . How many different triangles could be constructed with these properties?  
(a) 110 (b) 1,100 (c) 9,900 (d) 10,000

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- Q10. A coin is tossed thrice. Let  $X$  be the event that head occurs in each of the first two tosses. Let  $Y$  be the event that a tail occurs on the third toss. Let  $Z$  be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?
- (a)  $X$  and  $Y$  are not independent                      (b)  $Y$  and  $Z$  are dependent  
(c)  $Y$  and  $Z$  are independent                      (d)  $X$  and  $Z$  are independent

## SECTION: PHYSICS

- Q1. A satellite is moving in a circular orbit around the Earth. If  $T, V$  and  $E$  are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?
- (a)  $V = -2T; E = -T$                       (b)  $V = -T; E = 0$   
(c)  $V = -\frac{T}{2}; E = \frac{T}{2}$                       (d)  $V = -\frac{3T}{2}; E = \frac{-T}{2}$
- Q2. An operator for a spin  $-\frac{1}{2}$  particle is given by  $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$ , where  $\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$ ,  $\vec{\sigma}$  denotes Pauli matrices and  $\lambda$  is a constant. The eigenvalues of  $\hat{A}$  are
- (a)  $\pm \frac{\lambda B}{\sqrt{2}}$                       (b)  $\pm \lambda B$                       (c)  $0, \lambda B$                       (d)  $0, -\lambda B$
- Q3. The Pauli matrices for three spin  $-\frac{1}{2}$  particles are  $\vec{\sigma}_1, \vec{\sigma}_2$  and  $\vec{\sigma}_3$ , respectively. The dimension of the Hilbert space required to define an operator  $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$  is \_\_\_\_\_
- Q4. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of  $30^\circ$  with respect to each other. The number of image charges is \_\_\_\_\_.
- Q5. In Bose-Einstein condensates, the particles
- (a) have strong interparticle attraction  
(b) condense in real space  
(c) have overlapping wavefunctions  
(d) have large and positive chemical potential

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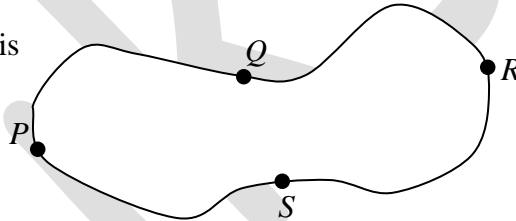
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Q6. Consider a complex function  $f(z) = \frac{1}{z\left(z + \frac{1}{z}\right)\cos(zx)}$ . Which one of the following

statements is correct?

- (a)  $f(z)$  has simple poles at  $z=0$  and  $z = -\frac{1}{2}$
- (b)  $f(z)$  has second order pole at  $z = -\frac{1}{2}$
- (c)  $f(z)$  has infinite number of second order poles
- (d)  $f(z)$  has all simple poles

Q7. Given that the magnetic flux through the closed loop  $PQRSP$  is  $\phi$ . If  $\int_P^R \vec{A} \cdot d\vec{l} = \phi_1$  along  $PQR$ , the value of  $\int_P^R \vec{A} \cdot d\vec{l}$  along  $PSR$  is

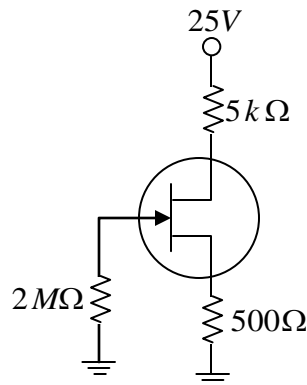


- (a)  $\phi - \phi_1$
- (b)  $\phi_1 - \phi$
- (c)  $-\phi_1$
- (d)  $\phi_1$

Q8. Which one of the following DOES NOT represent an exclusive OR operation for inputs  $A$  and  $B$ ?

- (a)  $(A + B)\overline{AB}$
- (b)  $\overline{AB} + \overline{BA}$
- (c)  $(A + B)(\overline{A} + \overline{B})$
- (d)  $(A + B)AB$

Q9. In the given circuit, the voltage across the source resistor is  $1V$ . The drain voltage (in  $V$ ) is \_\_\_\_\_



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- Q10. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because
- the chemical potential of the photons is zero
  - photons obey Pauli exclusion principle
  - photons are spin-1 particles
  - the entropy of the photons is very large
- Q11. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by,  $g(E) = CE^n$ , where  $C$  is constant. The value of  $n$  is \_\_\_\_\_
- Q12. Let  $\vec{L}$  and  $\vec{p}$  be the angular and linear momentum operators, respectively, for a particle. The commutator  $[L_x, p_y]$  gives
- $-i\hbar p_z$
  - 0
  - $i\hbar p_x$
  - $i\hbar p_z$
- Q13. Consider a system of  $N$  non-interacting spin  $-\frac{1}{2}$  particles, each having a magnetic moment  $\mu$ , is in a magnetic field  $\vec{B} = B\hat{z}$ . If  $E$  is the total energy of the system, the number of accessible microstates  $\Omega$  is given by
- $\Omega = \frac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)! \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$
  - $\Omega = \frac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$
  - $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)! \frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$
  - $\Omega = \frac{N!}{\left(N + \frac{E}{\mu B}\right)!}$
- Q14. The lattice parameters  $a, b, c$  of an orthorhombic crystal are related by  $a = 2b = 3c$ . In units of  $a$  the interplanar separation between the (110) planes is \_\_\_\_\_. (Upto three decimal places)

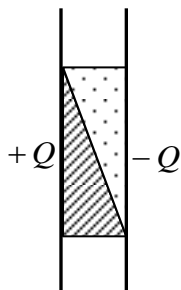
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- Q15. The space between two plates of a capacitor carrying charges  $+Q$  and  $-Q$  is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct?



- (a)  $\vec{E}$  and  $\vec{D}$  are continuous (b)  $\vec{E}$  is continuous and  $\vec{D}$  is discontinuous  
 (c)  $\vec{D}$  is continuous and  $\vec{E}$  is discontinuous (d)  $\vec{E}$  and  $\vec{D}$  are discontinuous
- Q16. The decay  $\mu^+ \rightarrow e^+ + \gamma$  is forbidden, because it violates  
 (a) momentum and lepton number conservations  
 (b) baryon and lepton number conservations  
 (c) angular momentum conservation  
 (d) lepton number conservation
- Q17. A beam of  $X$  - ray of intensity  $I_0$  is incident normally on a metal sheet of thickness  $2\text{ mm}$ . The intensity of the transmitted beam is  $0.025 I_0$ . The linear absorption coefficient of the metal sheet ( $\text{in m}^{-1}$ ) is \_\_\_\_\_ (upto one decimal place)
- Q18. The value of  $\int_0^3 t^2 \delta(3t-6) dt$  is \_\_\_\_\_ (upto one decimal place)
- Q19. The dispersion relation for phonons in a one dimensional monatomic Bravais lattice with lattice spacing  $a$  and consisting of ions of masses  $M$  is given by,  
 $\omega(k) = \sqrt{\frac{2c}{M}} [1 - \cos(ka)]$ , where  $\omega$  is the frequency of oscillation,  $k$  is the wavevector and  $C$  is the spring constant. For the long wavelength modes ( $\lambda \gg a$ ), the ratio of the phase velocity to the group velocity is \_\_\_\_\_

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Q20. The mean kinetic energy of a nucleon in a nucleus of atomic weight  $A$  varies as  $A^n$ , where  $n$  is \_\_\_\_\_ (upto two decimal places)

Q21. If  $f(x) = e^{-x^2}$  and  $g(x) = |x|e^{-x^2}$ , then

- (a)  $f$  and  $g$  are differentiable everywhere
- (b)  $f$  is differentiable everywhere but  $g$  is not
- (c)  $g$  is differentiable everywhere but  $f$  is not
- (d)  $g$  is discontinuous at  $x = 0$

Q22. Four forces are given below in Cartesian and spherical polar coordinates

$$(i) \vec{F}_1 = K \exp\left(\frac{-r^2}{R^2}\right) \hat{r}$$

$$(ii) \vec{F}_2 = K(x^3 \hat{y} - y^3 \hat{z})$$

$$(iii) \vec{F}_3 = K(x^3 \hat{x} + y^3 \hat{y})$$

$$(iv) \vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$$

where  $K$  is a constant Identify the correct option

- (a) (iii) and (iv) are conservative but (i) and (ii) are not
- (b) (i) and (ii) are conservative but (iii) and (iv) are not
- (c) (ii) and (iii) are conservative but (i) and (iv) are not
- (d) (i) and (iii) are conservative but (ii) and (iv) are not

Q23. Consider  $w = f(z) = u(x, y) + iv(x, y)$  to be an analytic function in a domain  $D$ . Which one of the following options is NOT correct?

- (a)  $u(x, y)$  satisfies Laplace equation in  $D$
- (b)  $v(x, y)$  satisfies Laplace equation in  $D$
- (c)  $\int_{z_1}^{z_2} f(z) dz$  is dependent on the choice of the contour between  $z_1$  and  $z_2$  in  $D$
- (d)  $f(z)$  can be Taylor expanded in  $D$

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Q24. In an inertial frame  $S$ , two events  $A$  and  $B$  take place at  $(ct_A = 0, \vec{r}_A = 0)$  and  $(ct_B = 0, \vec{r}_B = 2\hat{y})$ , respectively. The times at which these events take place in a frame  $S'$  moving with a velocity  $0.6c\hat{y}$  with respect to  $S$  are given by

(a)  $ct'_A = 0; ct'_B = -\frac{3}{2}$

(b)  $ct'_A = 0; ct'_B = 0$

(c)  $ct'_A = 0; ct'_B = \frac{3}{2}$

(d)  $ct'_A = 0; ct'_B = \frac{1}{2}$

Q25. In a Hall effect experiment, the hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

(a)  $n \approx p$

(b)  $n > p$

(c)  $\mu_e > \mu_h$

(d)  $m_e^* > m_h^*$

Q26. Consider a system of eight non-interacting, identical quantum particles of spin  $-\frac{3}{2}$  in a one dimensional box of length  $L$ . The minimum excitation energy of the system, in units of  $\frac{\pi^2 \hbar^2}{2mL^2}$  is \_\_\_\_\_

Q27. The average energy  $U$  of a one dimensional quantum oscillator of frequency  $\omega$  and in contact with a heat bath at temperature  $T$  is given by

(a)  $U = \frac{1}{2} \hbar \omega \coth\left(\frac{1}{2} \beta \hbar \omega\right)$

(b)  $U = \frac{1}{2} \hbar \omega \sinh\left(\frac{1}{2} \beta \hbar \omega\right)$

(c)  $U = \frac{1}{2} \hbar \omega \tanh\left(\frac{1}{2} \beta \hbar \omega\right)$

(d)  $U = \frac{1}{2} \hbar \omega \cosh\left(\frac{1}{2} \beta \hbar \omega\right)$

Q28. In a rigid rotator of mass  $M$ , if the energy of the first excited state is  $(1 \text{ meV})$ , then the fourth excited state energy (in  $\text{meV}$ ) is \_\_\_\_\_.

Q29. The Lagrangian for a particle of mass  $m$  at a position  $\vec{r}$  moving with a velocity  $\vec{v}$  is given by  $L = \frac{m}{2} \vec{v}^2 + C \vec{r} \cdot \vec{v} - V(r)$ , where  $V(r)$  is a potential and  $C$  is a constant. If  $\vec{p}_c$  is the canonical momentum, then its Hamiltonian is given by

(a)  $\frac{1}{2m} (\vec{p}_c + C\vec{r})^2 + V(r)$

(b)  $\frac{1}{2m} (\vec{p}_c - C\vec{r})^2 + V(r)$

(c)  $\frac{p_c^2}{2m} + V(r)$

(d)  $\frac{1}{2m} p_c^2 + C^2 r^2 + V(r)$

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Q30. The entropy of a gas containing  $N$  particles enclosed in a volume  $V$  is given by

$$S = Nk_B \ln \left( \frac{aVE^{3/2}}{N^{5/2}} \right), \text{ where } E \text{ is the total energy, } a \text{ is a constant and } k_B \text{ is the}$$

Boltzmann constant. The chemical potential  $\mu$  of the system at a temperature  $T$  is given by

$$(a) \mu = -k_B T \left[ \ln \left( \frac{aVE^{3/2}}{N^{5/2}} \right) - \frac{5}{2} \right] \quad (b) \mu = -k_B T \left[ \ln \left( \frac{aVE^{3/2}}{N^{5/2}} \right) - \frac{3}{2} \right]$$

$$(c) \mu = -k_B T \left[ \ln \left( \frac{aVE^{3/2}}{N^{3/2}} \right) - \frac{5}{2} \right] \quad (d) \mu = -k_B T \left[ \ln \left( \frac{aVE^{3/2}}{N^{3/2}} \right) - \frac{3}{2} \right]$$

Q31. The Heaviside function is defined as  $H(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$  and its Fourier transform is

given by  $-2i/\omega$ . The Fourier transform of  $\frac{1}{2}[H(t+1/2) - H(t-1/2)]$  is

$$(a) \frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} \quad (b) \frac{\cos\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}}$$

$$(c) \sin\left(\frac{\omega}{2}\right) \quad (d) 0$$

Q32. The Hamiltonian for a system of two particles of masses  $m_1$  and  $m_2$  at  $\vec{r}_1$  and  $\vec{r}_2$  having

velocities  $\vec{v}_1$  and  $\vec{v}_2$  is given by  $H = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2} \hat{z} \cdot (\vec{r}_1 \times \vec{r}_2)$ , where

$C$  is constant. Which one of the following statements is correct?

- (a) The total energy and total momentum are conserved
- (b) Only the total energy is conserved
- (c) The total energy and the  $z$  - component of the total angular momentum are conserved
- (d) The total energy and total angular momentum are conserved

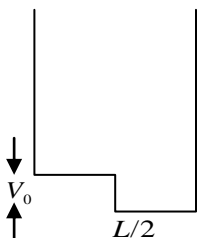
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Q33. A particle is confined in a box of length  $L$  as shown below.



If the potential  $V_0$  is treated as a perturbation, including the first order correction, the ground state energy is

(a)  $E = \frac{\hbar^2 \pi^2}{2mL^2} + V_0$

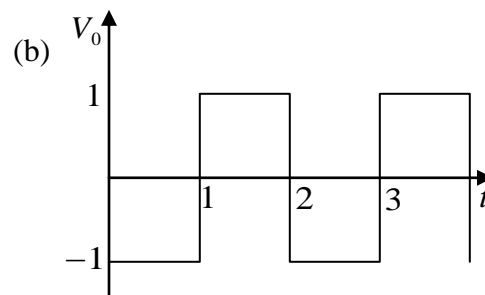
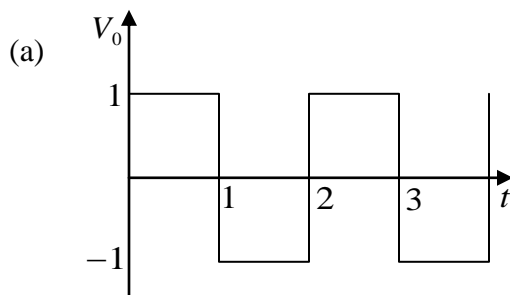
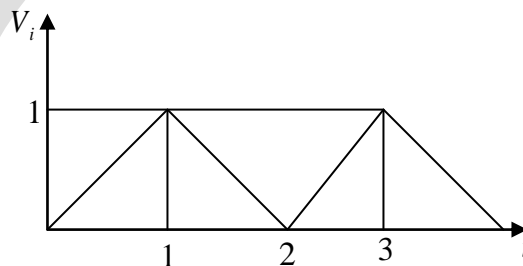
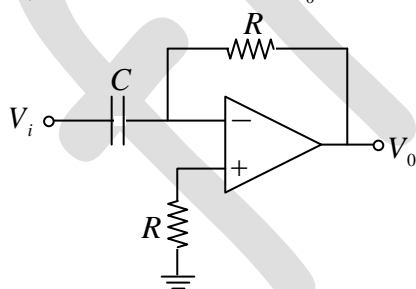
(b)  $E = \frac{\hbar^2 \pi^2}{2mL^2} - \frac{V_0}{2}$

(c)  $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{4}$

(d)  $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{2}$

Q34. The band gap of an intrinsic semiconductor is  $E_g = 0.72 \text{ eV}$  and  $m_n^* = 6m_g^*$ . At  $300 \text{ K}$ , the Fermi level with respect to the edge of the valence band (in  $\text{eV}$ ) is at \_\_\_\_\_ (upto three decimal places)  $k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$

Q35. Consider the circuit shown in the figure, where  $RC = 1$ . For an input signal  $V_i$  shown below, choose the correct  $V_0$  from the options:

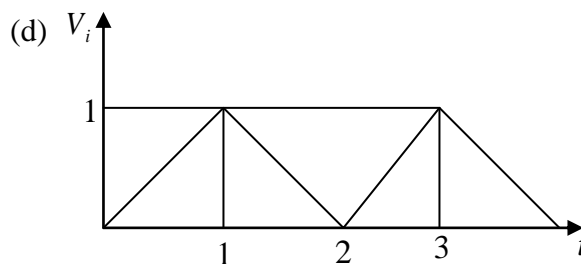
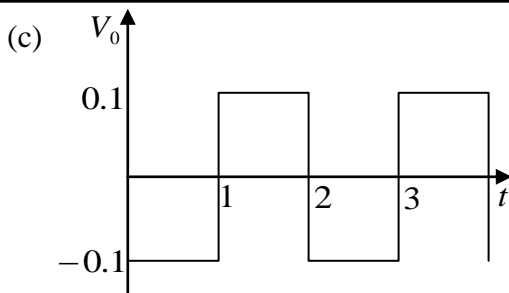


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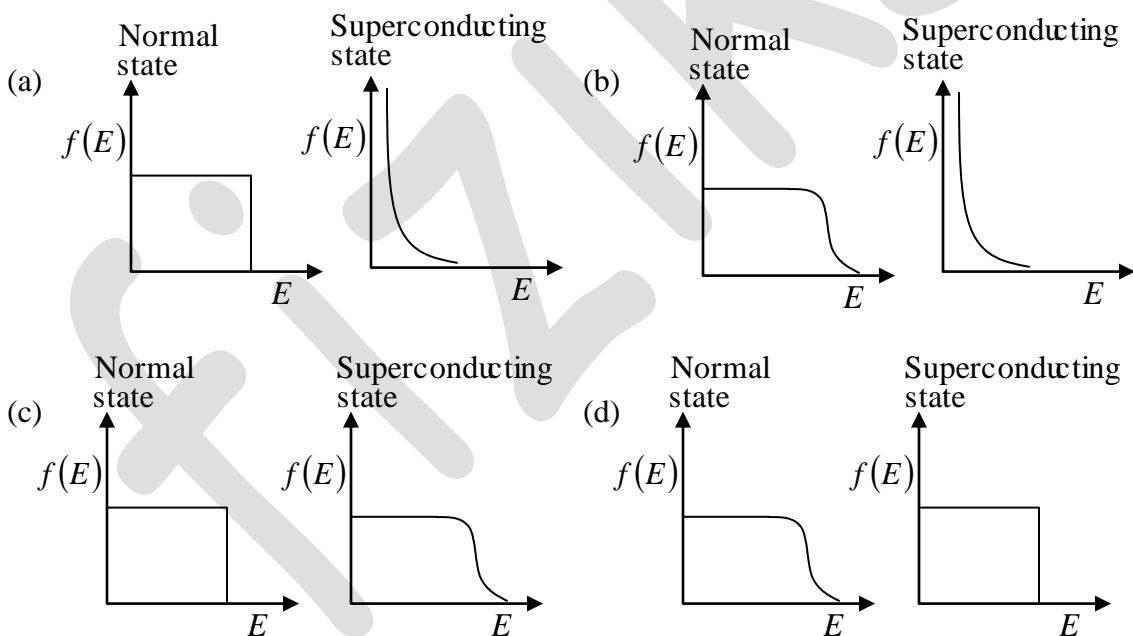
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Q36. The atomic masses of  $^{152}_{63}\text{Eu}$ ,  $^{152}_{62}\text{Sm}$ ,  $^1_1\text{H}$  and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the  $Q$ -value of the reaction  $^{152}_{63}\text{Eu} + n \rightarrow ^{152}_{62}\text{Sm} + p$  is \_\_\_\_\_  $\times 10^{-3}$  amu (upto three decimal places)

Q37. Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states?



Q38. The binding energy per molecule of  $\text{NaCl}$  (lattice parameter is  $0.563\text{nm}$ ) is  $7.956\text{ eV}$ .

The repulsive term of the potential is of the form  $\frac{K}{r^9}$ , where  $K$  is a constant. The value of the Madelung constant is \_\_\_\_\_ (upto three decimal places)  
(Electron charge  $e = -1.6 \times 10^{-19}\text{ C}$ ;  $\epsilon_0 = 8.854 \times 10^{-12}\text{ C}^2\text{N}^{-1}\text{m}^{-2}$ )

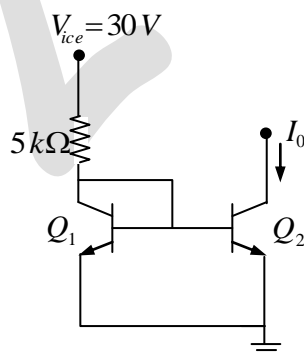
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- Q39. A particle of mass  $0.01 \text{ kg}$  falls freely in the earth's gravitational field with an initial velocity  $(0) = 10 \text{ ms}^{-1}$ . If the air exerts a frictional force of the form,  $f = -kv$ , then for  $k = 0.05 \text{ Nm}^{-1}\text{s}$ , the velocity (in  $\text{ms}^{-1}$ ) at time  $t = 0.2 \text{ s}$  is \_\_\_\_\_ (upto two decimal places). (use  $g = 10 \text{ ms}^{-2}$  and  $e = 2.72$ )
- Q40. Let the Hamiltonian for two spin- $1/2$  particles of equal masses  $m$ , momenta  $\vec{p}_1$  and  $\vec{p}_2$  and positions  $\vec{r}_1$  and  $\vec{r}_2$  be  $H = \frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 + \frac{1}{2} m \omega^2 (r_1^2 + r_2^2) + k \vec{\sigma}_1 \cdot \vec{\sigma}_2$ , where  $\vec{\sigma}_1$  and  $\vec{\sigma}_2$  denote the corresponding Pauli matrices,  $\hbar\omega = 0.1 \text{ eV}$  and  $k = 0.2 \text{ eV}$ . If the ground state has net spin zero, then the energy (in  $\text{eV}$ ) is \_\_\_\_\_
- Q41. A monochromatic plane wave (wavelength =  $600 \text{ nm}$ )  $E_0 \exp[i(kz - \omega t)]$  is incident normally on a diffraction grating giving rise to a plane wave  $E_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$  in the first order of diffraction. Here  $E_1 < E_0$  and  $\vec{k}_1 = |\vec{k}_1| \left[ \frac{1}{2} \hat{x} + \frac{\sqrt{3}}{2} \hat{z} \right]$ . The period (in  $\mu\text{m}$ ) of the diffraction grating is \_\_\_\_\_ (upto one decimal place)
- Q42. In the simple current source shown in the figure,  $Q_1$  and  $Q_2$  are identical transistors with current gain  $\beta = 100$  and  $V_{BE} = 0.7 \text{ V}$



The current  $I_0$  (in  $\text{mA}$ ) is \_\_\_\_\_ (upto two decimal places)

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Q43. A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance  $r$  from the axis of the solenoid is proportional to

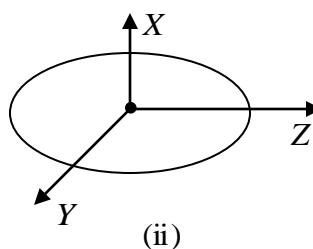
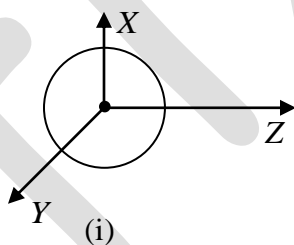
- (a)  $r^2$  inside the solenoid and  $\frac{1}{r}$  outside      (b)  $r$  inside the solenoid and  $\frac{1}{r^2}$  outside  
(c)  $r^2$  inside the solenoid and  $\frac{1}{r^2}$  outside      (d)  $r$  inside the solenoid and  $\frac{1}{r}$  outside

Q44. Suppose a linear harmonic oscillator of frequency  $\omega$  and mass  $m$  is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[ |\psi_0\rangle + e^{i\frac{\pi}{2}} |\psi_1\rangle \right] \text{ at } t=0 \text{ where } |\psi_0\rangle \text{ and } |\psi_1\rangle \text{ are the ground and the first}$$

excited states, respectively. The value of  $\langle\psi|x|\psi\rangle$  in the units of  $\sqrt{\frac{\hbar}{m\omega}}$  at  $t=0$  is \_\_\_\_\_

Q45. A charge  $-q$  is distributed uniformly over a sphere, with a positive charge  $q$  at its center in (i). Also in (ii), a charge  $-q$  is distributed uniformly over an ellipsoid with a positive charge  $q$  at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?



- (a) The dipole moment is zero in both (i) and (ii)  
(b) The dipole moment is non-zero in (i) but zero in (ii)  
(c) The dipole moment is zero in (i) but non-zero in (ii)  
(d) The dipole moment is non-zero in both (i) and (ii)

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Q46. A function  $y(z)$  satisfies the ordinary differential equation  $y'' + \frac{1}{z} y' - \frac{m^2}{z^2} y = 0$ , where

$m = 0, 1, 2, 3, \dots$ . Consider the four statements P, Q, R, S as given below.

P:  $z^m$  and  $z^{-m}$  are linearly independent solutions for all values of  $m$

Q:  $z^m$  and  $z^{-m}$  are linearly independent solutions for all values of  $m > 0$

R:  $\ln z$  and 1 are linearly independent solutions for  $m = 0$

S:  $z^m$  and  $\ln z$  are linearly independent solutions for all values of  $m$

The correct option for the combination of valid statements is

(a) P, R and S only (b) P and R only (c) Q and R only (d) R and S only

Q47. Match the phrases in Group I and Group II and identify the correct option.

Group I

Group II

(P) Electron spin resonance (ESR)

(i) radio frequency

(Q) Nuclear magnetic resonance (NMR)

(ii) visible range frequency

(R) Transition between vibrational states of a molecule

(iii) microwave frequency

(S) Electronic transition

(iv) far-infrared range

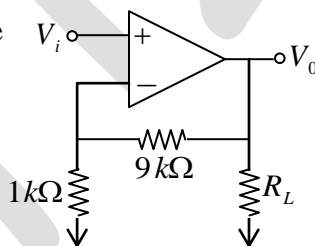
(a) (P-i), (Q-ii), (R-iii), (S-iv)

(b) (P-ii), (Q-i), (R-iv), (S-iii)

(c) (P-iii), (Q-iv), (R-i), (S-ii)

(d) (P-iii), (Q-i), (R-iv), (S-ii)

Q48. In the given circuit, if the open loop gain  $A = 10^5$  the feedback configurations and the closed loop gain  $A_f$  are



(a) series-shunt,  $A_f = 9$

(b) series-series,  $A_f = 10$

(c) series-shunt,  $A_f = 10$

(d) shunt-shunt,  $A_f = 10$

Q49. The excitation wavelength of laser in a Raman effect experiment is  $546\text{nm}$ . If the Stokes' line is observed at  $552\text{nm}$ , then the wavenumber of the anti-Stokes' line (in  $\text{cm}^{-1}$ ) is \_\_\_\_\_

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Q50. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If  $\vec{F}_c$  and  $\vec{F}_{Co}$  denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

- (a)  $\vec{F}_c$  is radially outward and  $\vec{F}_{Co} = \vec{F}_c$   
 (b)  $\vec{F}_c$  is radially inward and  $\vec{F}_{Co} = -2\vec{F}_c$   
 (c)  $\vec{F}_c$  is radially outward and  $\vec{F}_{Co} = -2\vec{F}_c$   
 (d)  $\vec{F}_c$  is radially outward and  $\vec{F}_{Co} = 2\vec{F}_c$

Q51. A particle with rest mass  $M$  is at rest and decays into two particles of equal rest masses  $\frac{3}{10}M$  which move along the  $z$  axis. Their velocities are given by

- (a)  $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$  (b)  $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$   
 (c)  $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$  (d)  $\vec{v}_1 = (0.6c)\hat{z}; \vec{v}_2 = (-0.8c)\hat{z}$

Q52. Given that the Fermi energy of gold is  $5.54 \text{ eV}$ , the number density of electrons is \_\_\_\_\_  $\times 10^{28} \text{ m}^{-3}$  (upto one decimal place)

(Mass of electron =  $9.11 \times 10^{-31} \text{ kg}$ ;  $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ ;  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ )

Q53. The number of permitted transitions from  $^2P_{3/2} \rightarrow ^2S_{1/2}$  in the presence of a weak magnetic field is \_\_\_\_\_

Q54. A plane wave  $(\hat{x} + i\hat{y})E_0 \exp[i(kz - \omega t)]$  after passing through an optical element emerges as  $(\hat{x} - i\hat{y})E_0 \exp[i(kz - \omega t)]$ , where  $k$  and  $\omega$  are the wavevector and the angular frequency, respectively. The optical element is a

- (a) quarter wave plate (b) half wave plate  
 (c) polarizer (d) Faraday rotator

Q55. In the nuclear shell model, the potential is modeled as  $V(r) = \frac{1}{2}m\omega^2 r^2 - \lambda \vec{L} \cdot \vec{S}$ ,  $\lambda > 0$ .

The correct spin-parity and isospin assignments for the ground state of  $^{13}\text{C}$  is

- (a)  $\frac{1^-}{2}; \frac{-1}{2}$  (b)  $\frac{1^+}{2}; \frac{-1}{2}$  (c)  $\frac{3^+}{2}; \frac{1}{2}$  (d)  $\frac{3^-}{2}; \frac{-1}{2}$

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