# Analysis of Crime Predictors

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## Introduction

We as human beings categorize actions as moral and immoral, and label serious moral transgressions such as murder, theft and fraud as crimes. To prevent crime, we impose punishments as deterrents, though other methods to reduce crime may exist. Moreover, crime rates vary by region, prompting the question: what makes a crime more likely to happen in a region?
Clay[1857], concentrating on crime in England, analysed the following characteristics: Beer-(ale)-house numbers, worship attendance and public school attendance. The argument he provides for the latter two, is both allow for the public access to Christian moral values. It is to note, the "public schools" mentioned by Clay concentrated mostly on teaching read and writing and were not free but accepted anyone who could afford it. In the case of beer-houses, he argues "the temptation to animal pleasure" corrupts a person. To help his case he tries to show the positive correlation of beer-houses, the negative correlations of public school and worship attendance to crime.
In this paper we want to investigate these effects of the aforementioned characteristics and try to predict crime rate from them using the same data set. However, before starting it is important to mention, as noticed by Clay[1857], the recorded crime rate may be inaccurate because of systematic reasons since each county handles crime and punishment differently on the executive and juridical levels. This will impact the accuracy of all possible analysis and models.

## Exploratory Data Analysis

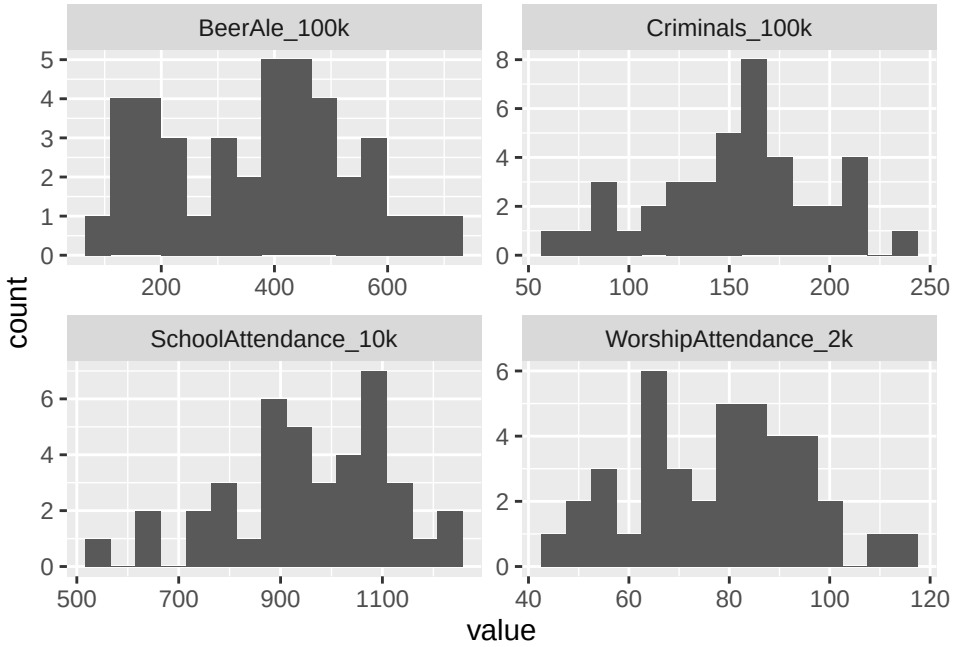
We begin our analysis with a look at the first few lines of our dataset.

| County | RegionName | RegionCode | Criminals_100k | BeerAle_100k | SchoolAttendance_10k | WorshipAttendance_2k |
|---|---|---|---|---|---|---|
| Middlesex | SouthEastern | 1 | 200 | 541 | 560 | 43 |
| Surrey | SouthEastern | 1 | 160 | 504 | 630 | 48 |
| Kent | SouthEastern | 1 | 160 | 552 | 790 | 68 |
| Sussex | SouthEastern | 1 | 147 | 295 | 820 | 67 |
| Hants | SouthEastern | 1 | 178 | 409 | 990 | 79 |
| Berks | SouthEastern | 1 | 205 | 568 | 930 | 69 |

## Data summary:

| County | RegionName | RegionCode | Criminals_100k | BeerAle_100k | SchoolAttendance_10k | WorshipAttendance_2k |
|---|---|---|---|---|---|---|
| Length:40 | Length:40 | Min. :1.00 | Min. : 66.0 | Min. : 87.0 | Min. : 560.0 | Min. : 43.0 |
| Class :character | Class :character | 1st Qu.:1.00 | 1st Qu.:127.0 | 1st Qu.:209.0 | 1st Qu.: 880.0 | 1st Qu.: 65.0 |
| Mode :character | Mode :character | Median :3.00 | Median :157.5 | Median :407.0 | Median : 965.0 | Median : 79.5 |
| NA | NA | Mean :3.45 | Mean :152.9 | Mean :374.9 | Mean : 957.8 | Mean : 77.5 |
| NA | NA | 3rd Qu.:5.00 | 3rd Qu.:174.2 | 3rd Qu.:490.8 | 3rd Qu.:1082.5 | 3rd Qu.: 91.0 |
| NA | NA | Max. :8.00 | Max. :241.0 | Max. :708.0 | Max. :1250.0 | Max. :113.0 |

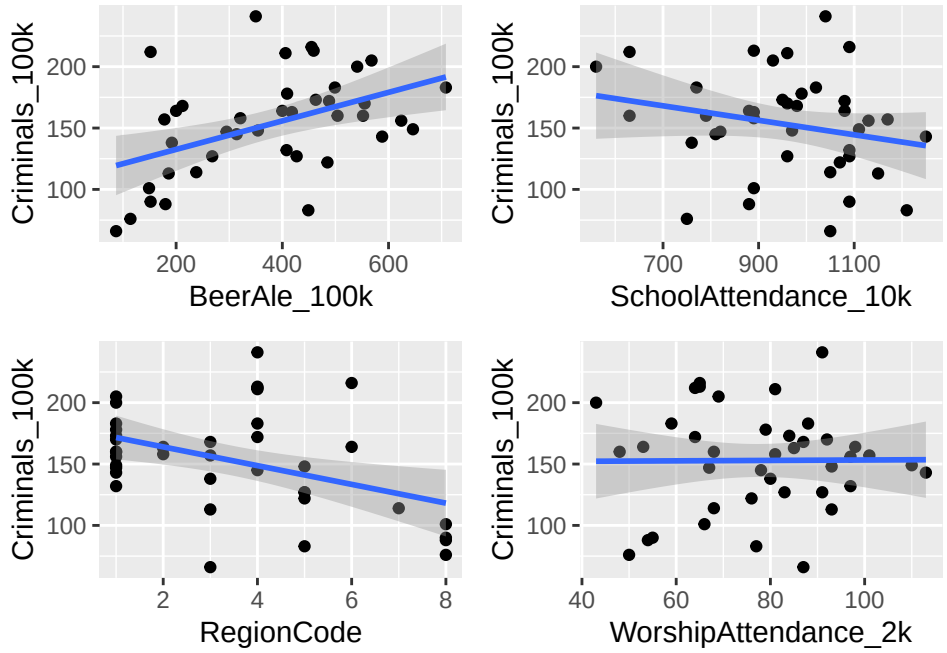
The dataset contains information about 40 different counties.County and RegionName are categorical variables, and each of the 8 regions is assigned a RegionCode, which is a number from 1 to 8. Criminals_100k is the number of criminals per 100,000 inhabitants for any particular county. Similarly, BeerAle_100k, SchoolAttendance_10k and WorshipAttendance_2k are social indicators measured numerically as a proportion of the population. In order to better visualise the numerical data, we plot histograms of each numerical variable, with the exception of the categorical RegionCode.




The BeerAle_100k variable appears to be bimodal. The histogram suggests that there are two common levels of alcohol consumption within the entire population, one around the 200, and another around 400 per 100,000 population. According to the data summary, Crimanals_100k has a mean of 152.9 and a median of 157.5, suggesting the symmetry that is also reflected in the histogram. SchoolAttendance_10k is slightly left skewed, while WorshipAttendance_2k has a varied but loosely symmetric distribution.

The linear dependence between each pair of numeric variables is expressed in the following correlation matrix:


There appears to be negligible linear dependence between worship attendance and criminality. School attendance is slightly negatively correlated with the prevalence of crime. Criminal behaviour is positively correlated with BeerAle_100k with a correlation coefficient of 0.46, suggesting that counties with a more dominant culture of frequenting bars and pubs are also where more crime happens. In order to better visualise these dependencies, we regress Criminals_100k on each of these variables.



There appears to be a very clear linear dependence of criminal behaviour on bar attendance, with most datapoints falling within the 95% confidence interval of the regression line. There is high variation of criminality across school attendance and worship attendance.

## Model Fitting

We use least squres to minimize the sum of squared residuals in a polynomial regression. In order to predict Criminals_100k, we use a polynomial regression model which uses BeerAle_100k, SchoolAttendance_10k, and WorshipAttendance_2k as the features. We include the variable WorshipAttendance_2k in our analysis even if it has no correlation with the variable Criminals per 100k as no correlation doesn't mean no causation, it only suggests that there is no linear association.

Mathematically, the model can be represented as:

A polynomial regression model of degree n with three features (x1, x2, x3) can be represented as follows:

$$y = \beta_0 + \beta_{11}x_1 + \beta_{12}x_1^2 + ... + \beta_{1n}x_1^n + \beta_{21}x_2 + \beta_{22}x_2^2 + ... + \beta_{2n}x_2^n + \beta_{31}x_3 + \beta_{32}x_3^2 + ... + \beta_{3n}x_3^n + \epsilon$$

Where $\hat{y}$ is the predicted Criminals_100k population, and $x_1$, $x_2$, and $x_3$ represent BeerAle_100k, SchoolAttendance_10k and WorshipAttendance_2k respectively. $\beta_0$, $\beta_{ij}$ are the coefficients of the model where $i$ is the feature index and $j$ is the degree of the polynomial $\epsilon$ is the error term

We choose the model based on the exploratory data analysis and the correlation between the features and the target variable. On top of the linear relation, we also add polynomial terms to capture the non-linear relation between the features and the target variable. We cross validate to find the optimum polynomial degree among polynomials of degrees up to 6, using leave-one-out cross validation. The minimum cross validation errors corresponds to polynomials with degree 3 and 4.

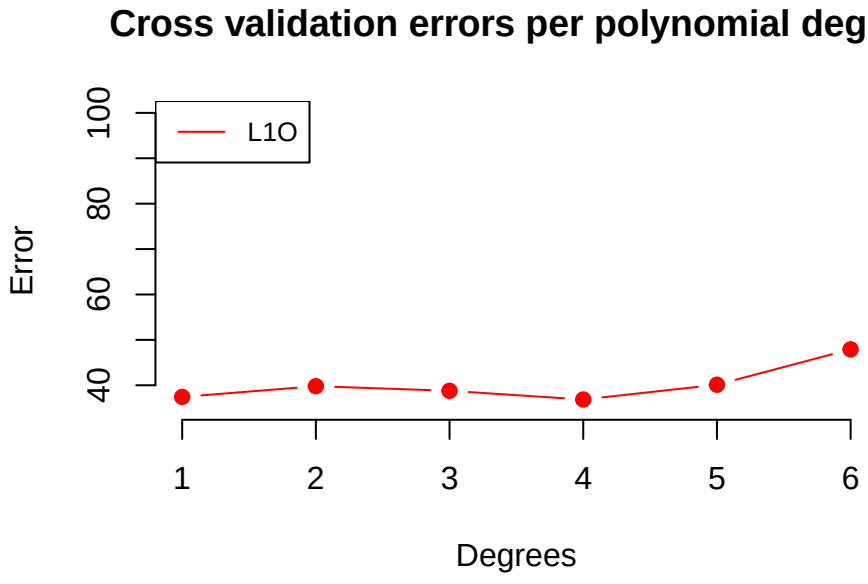


Table 5: Regression Model Summary

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 1257.042 | 623.021 | 2.018 | 0.052 |
| BeerAle_100k | 0.445 | 0.158 | 2.815 | 0.008 |
| SchoolAttendance_10k | -7.321 | 2.797 | -2.617 | 0.014 |
| WorshipAttendance_2k | 39.027 | 19.102 | 2.043 | 0.050 |
| I(BeerAle_100k^2) | 0.000 | 0.000 | -2.143 | 0.040 |
| I(SchoolAttendance_10k^2) | 0.008 | 0.003 | 2.593 | 0.014 |
| I(WorshipAttendance_2k^2) | -0.491 | 0.247 | -1.987 | 0.056 |
| I(SchoolAttendance_10k^3) | 0.000 | 0.000 | -2.593 | 0.014 |
| I(WorshipAttendance_2k^3) | 0.002 | 0.001 | 1.948 | 0.060 |

We further compare polynomial regression models of degrees 3 and 4 with the linear model using the Information Criteria.

Table 3: Information Criteria

| | DF | AIC | BIC |
|---|---|---|---|
| Linear model | 5 | 405.0249 | 413.4693 |
| Degree 3 polynomial | 11 | 402.5364 | 421.1140 |
| Degree 4 Polynomial | 14 | 401.7266 | 425.3710 |

The higher degree polynomial models fit better the data than the simpler linear model. Comparing the two models using the likelihood ratio test, we find that the restricted (degree 3) model cannot be rejected at the 5% significance level.

Table 4: Likelihood Ratio Test results

| Model | Df | LogLik | Chisq | Pr_Chisq |
|---|---|---|---|---|
| Degree 3 polynomial | 11 | -190.27 | | |
| Degree 4 polynomial | 14 | -186.86 | 6.81 | 7.82e-02 |

Then, we use backward selection based on the Akaike Information Criteria to select a subset of variables from the larger set of variables used in the polynomial model of degree 3. We compare all possible sub-models and pick the one that fits best the data.

We get the following model: $\hat{y} = \beta_0 + \beta_{11}x_1 + \beta_{12}x_1^2 + \beta_{21}x_2 + \beta_{22}x_2^2 + \beta_{23}x_2^3 + \beta_{31}x_3 + \beta_{32}x_3^2 + \beta_{33}x_3^3$

Where $\hat{y}$ is the predicted Criminals_100k population, and $x_1$, $x_2$, and $x_3$ represent BeerAle_100k, SchoolAttendance_10k and WorshipAttendance_2k respectively. The regression coefficients $\beta_0$ and $\beta_{ij}$ are estimated in Table 5. ¨

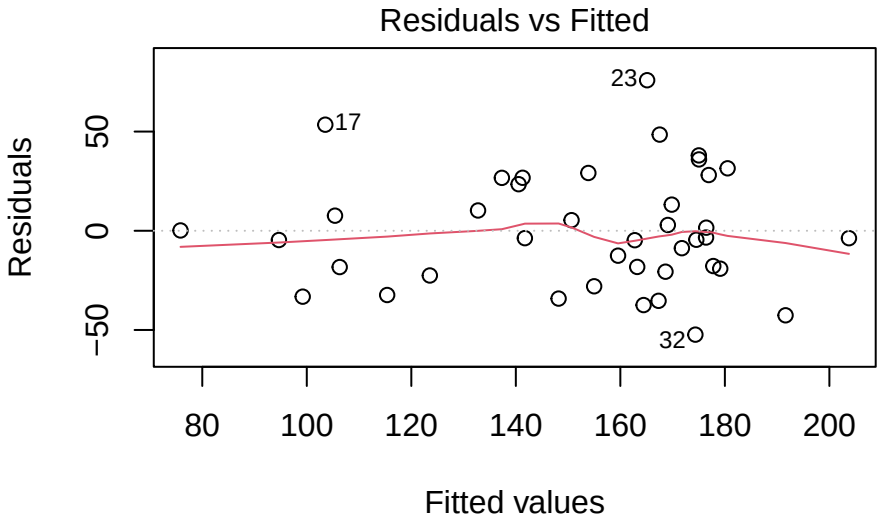
## Model assessment

In this section, we will verify the conditions for our regressions model to hold, namely that our residuals have zero mean, are uncorrelated, are homoscedastic and are normally distributed.

Zero mean error terms

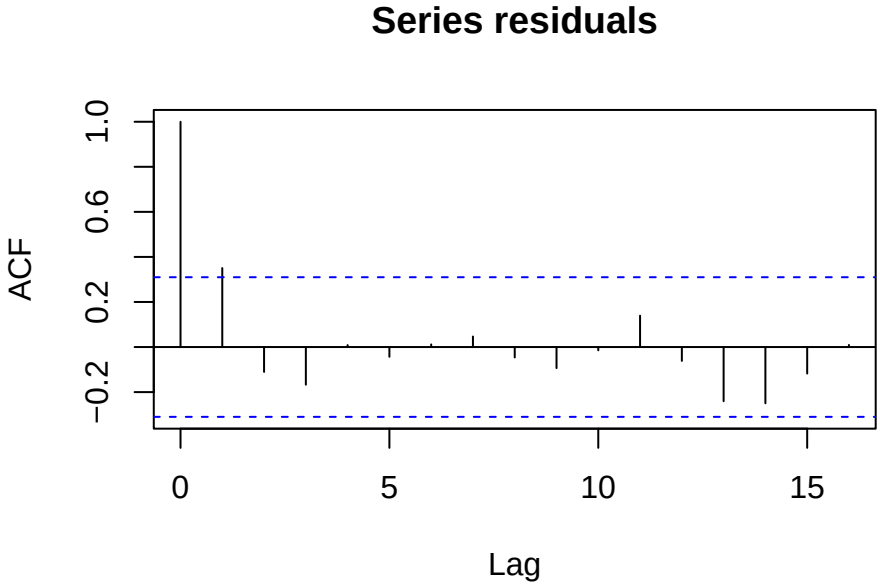
We compute our model residuals mean and get : $1.4210855 \times 10^{-15}$

Homoscedastic error term




We see from the plot above that our residuals are almost homoscedastic

Error terms are uncorrelated

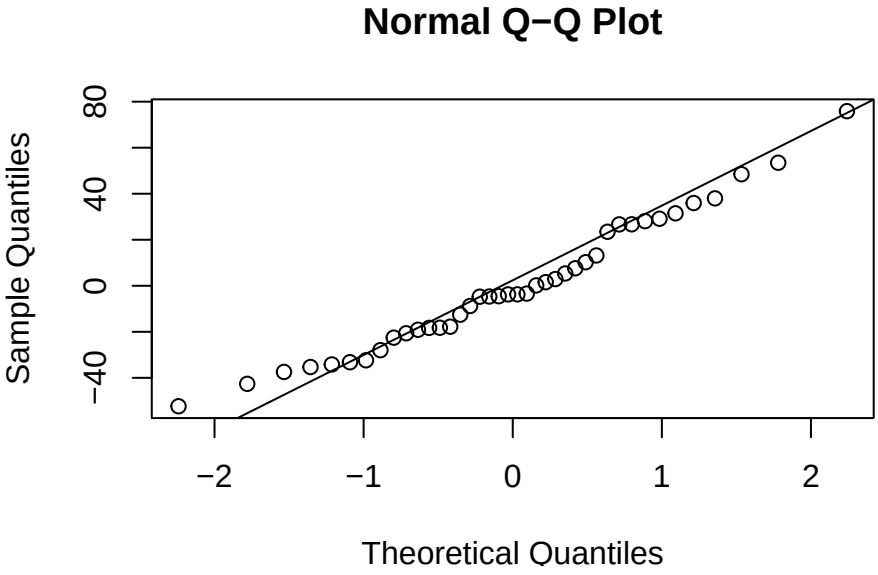



##

```
##  Durbin-Watson test
##
## data:  modelF2
## DW = 1.2826, p-value = 0.00126
## alternative hypothesis: true autocorrelation is greater than 0
```

Both the Autocorrelation function plot and the Durbin-Watson test (with autocorrelation and p-value ) suggest that we have a weak positive autocorrelation at lag 1.

Error terms are normally distributed




Given that our data set is small, we can assume from the Normal QQ plot that our residuals are approximately normally distributed