Python code and analysis

```
#!/usr/bin/python
 3 import numpy as np
 5 def trapezoidal(function, a, b, stepsize):
                Function that takes a function, lower limit a, upper limit b and stepsize and calculates the integral
                using trapezoidal method.
 8
                interval = np.arange(a,b+stepsize, stepsize) ## array of the interval points
                integral = 0 ## variable to store the sum
11
                for i in range (1, len(interval)-1, 2):
                          integral += (stepsize/2)*(function(interval[i-1]) + 2*function(interval[i]) + function(interval[i
                +1]))
14
                return integral
15
def simpsons (function, a, b, stepsize):
17
                Function that takes a function, lower limit a, upper limit b and stepsize and calculates the integral
18
                using simpsons method.
19
                interval = np.arange(a,b+stepsize,stepsize) ## array of interrval points
20
                integral = 0 ## variable to store the sum
21
                for i in range (1, len(interval) - 1, 2):
22
                          integral \; +\! = \; (\,stepsize \, / \, 3\,) * (\,function \, (\,interval \, [\, i \, -1]\,) \; + \; 4* \, function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\, i \, ]\,) \; + \; function \, (\,interval \, [\,
23
                +1]))
                return integral
24
27
def func_l(u):
                return 3*(1 - (u)**3)**(-1/3)
29
30
def func_r(s):
                return (3/2)*(1 - (s)**(3/2))**(-2/3)
32
33
a_{-1} = 0
b_l = (0.5) **(1/3)
no_of_bins = 10**3
stepsize_l = (b_l - a_l)/no_of_bins
a_r = 0
b_r = (0.5) **(2/3)
stepsize_r = (b_r - a_r)/no_of_bins
43 # The final answer in trapezoidal and simpsons methods respectively are
44 t_inte = trapezoidal(func_l, a_l, b_l, stepsize_l) + trapezoidal(func_r, a_r, b_r, stepsize_r)
45 s_inte = simpsons(func_l, a_l, b_l, stepsize_l) + simpsons(func_r, a_r, b_r, stepsize_r)
```

The outputs are shown below and the values that already converge are shaded

No. of bins	Step size	Step size	Trapezoidal	$\frac{Trapezoidal}{Theoretical}$	Simpsons	$\frac{Simpsons}{Theoretical}$
	(left)	(right)				
10	0.079370	0.062996	3.631300	1.001020	3.627696	1.000026
100	0.007937	0.006300	3.627636	1.000010	3.627599	1.000000
1000	0.000794	0.000630	3.627599	1.000000	3.627599	1.000000
10000	7.93700e-05	6.29960e-05	3.627599	1.000000	3.627599	1.000000

C++ code and analysis

```
1 #include <iostream>
2 #include <math.h>
3 using namespace std;
5 double f_l(double u){
          double e = -1.0/3.0;
          return 3.0*pow((1 - pow(u, 3.0)), e);
8 }
double f_r (double s) {
          double g = 3.0/2.0;
11
          double h = -2.0/3.0;
12
          return g*pow((1 - pow(s,g)),h);
13
15 }
17 int main(){
          double a_1 = 0.0;
18
          double b_{-1} = pow(0.5, 1.0/3.0);
19
          double number_of_steps = pow(10,4);
20
          double stepsize_l = (b_l - a_l)/number_of_steps;
21
          double trap_l = 0;
23
          double simp_l = 0;
          for (double i = a_l + stepsize_l; i < b_l; i += 2.0*stepsize_l)
25
                   trap\_l \; += \; (\; stepsize\_l \; / \; 2.0) * (\; f\_l \; (\; i-stepsize\_l \; ) \; + \; \; 2.0 * f\_l \; (\; i) \; + \; \; f\_l \; (\; i+stepsize\_l \; ) \; ) \; ;
26
                   simp_l += (stepsize_l/3.0)*(f_l(i-stepsize_l) + 4.0*f_l(i) + f_l(i+stepsize_l));
          }
28
29
          double a_r = 0.0;
31
          double b_r = pow(0.5, 2.0/3.0);
32
          double stepsize_r = (b_r - a_r)/number_of_steps;
33
34
          double trap_r = 0;
          double simp_r = 0;
36
          for (double i = a_r + stepsize_r; i < b_r; i += 2.0*stepsize_r){
37
                   trap_r += (stepsize_r/2.0)*(f_r(i-stepsize_r) + 2.0*f_r(i) + f_r(i+stepsize_r));
                   simp_r += (stepsize_r/3.0)*(f_r(i-stepsize_r) + 4.0*f_r(i) + f_r(i+stepsize_r));
39
          }
40
41
          double final_trap = trap_l + trap_r;
42
          double final_simp = simp_l + simp_r;
44
          45
          cout << "Simpsons method : " << final_simp << endl;</pre>
47
          return 0;
```

The results are:

No. of bins	Step size	Step size	Trapezoidal	$\frac{Trapezoidal}{Theoretical}$	Simpsons	$\frac{Simpsons}{Theoretical}$
	(left)	(right)				
10	0.079370	0.062996	3.6313	1.0010	3.6277	1.0000
100	0.007937	0.006300	3.62764	1.0000	3.6276	1.0000
1000	0.000794	0.000630	3.6276	1.0000	3.6276	1.0000
10000	7.93700e-05	6.29960e-05	3.6276	1.0000	3.6276	1.0000