

Homework 4

Harmonic and Anharmonic Oscillator

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Python code and analysis

```
1 import numpy as np
2 import cmath as c
3
4 def aofx(x,k,alpha):
5     """
6     Returns the function that equals  $d^2x/dt^2$ 
7     """
8     return -k*(x**alpha)
9
10 def euler_cromer(x0, t0, t, delta_t, k, alpha):
11     """
12     Returns an array of x values from time  $t = t_0$  to  $t = t$ 
13     """
14     t_array = np.arange(t0, t + delta_t, delta_t)
15     x_array = np.zeros(len(t_array))
16     v_array = np.zeros(len(t_array))
17     x_array[0] = x0
18
19     for i in range(1, len(t_array)):
20         v_array[i] = v_array[i-1] + delta_t*aofx(x_array[i-1],k,alpha)
21         x_array[i] = x_array[i-1] + delta_t*v_array[i]
22
23     return t_array, x_array, v_array
24
25 def FourierTransform(t_array, f_array, w_ini, w_fin):
26     """
27     Function to calculate fourier transform (to calculate time period of oscillation)
28     """
29     w_array = np.linspace(w_ini, w_fin, 1000)
30     f_of_w = np.zeros(len(w_array))
31     for wi in range(len(w_array)):
32         integral = 0
33         for ti in range(len(t_array)):
34             fl = f_array[ti]*(c.exp(-1j*w_array[wi]*t_array[ti]))
35             integral = integral + fl/np.sqrt(2.0*np.pi)
36         f_of_w[wi] = integral
37
38     return w_array, f_of_w
39
40 # Initial conditions
41
42 x01 = 0.2 # amplitude
43 t0 = 0.0 # start time
44 delta_t = 0.1 # time step
45 t = 10.0 # end time
46
47 t_array1, x_array1, v_array1 = euler_cromer(x01, t0, t, delta_t, k, alpha)
48 w_array1, f_of_w1 = FourierTransform(t_array1, x_array1, wi, wf)
49 w_sel = w_array1[np.where(f_of_w1 == max(f_of_w1))[0][1]]
50 T_period = 2.0*np.pi/w_sel ### Time period of oscillation
```

The time period was obtained by getting a fourier transform of the obtained motion. The obtained values are shown below :

	Harmonic Oscillator			Anharmonic Oscillator		
Amplitude (m)	0.2	0.5	1.0	0.2	0.5	1.0
Time Period (s)	6.22	6.22	6.22	37.00	14.82	7.40

Plotting the motion for both harmonic and anharmonic oscillator

