Homework 5
Simultaneous system of two equations

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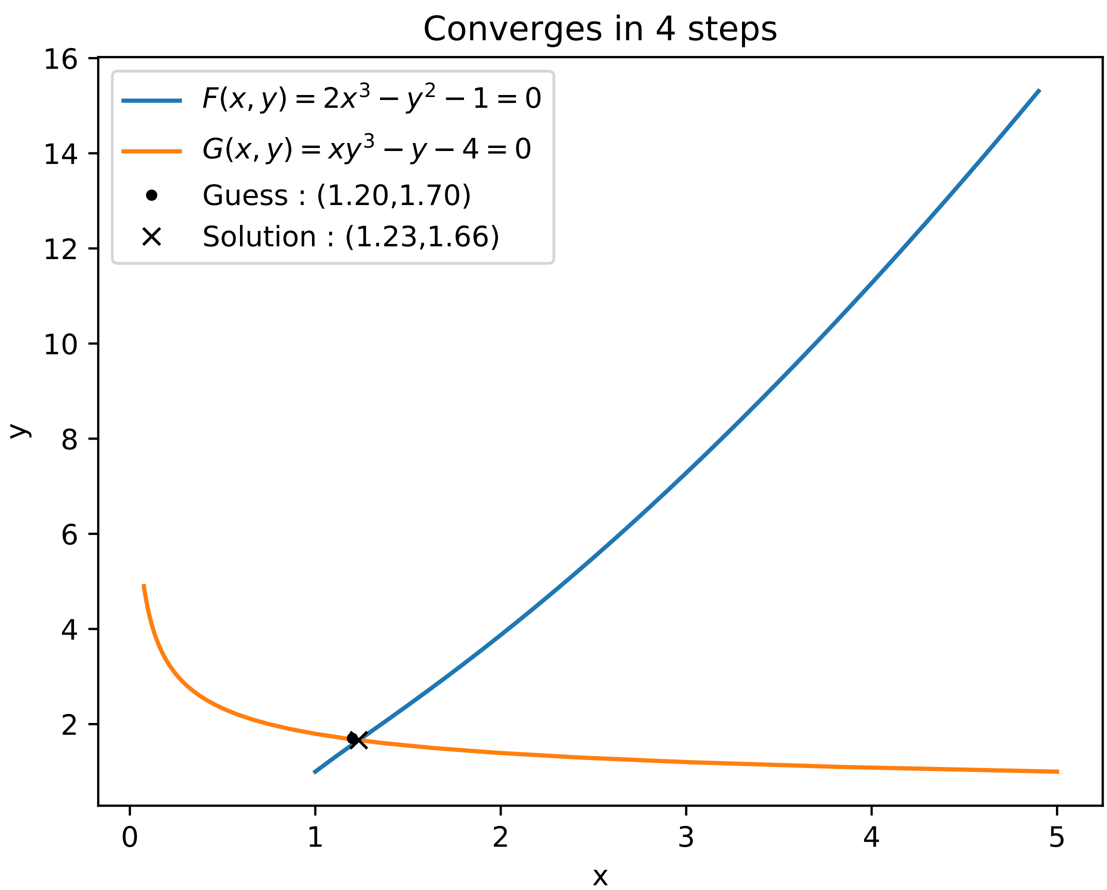
Python code and analysis

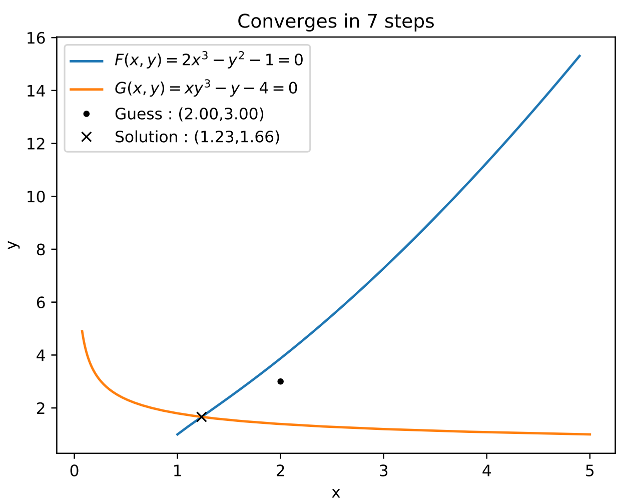
```python
import numpy as np

def F(x,y):
    return 2*(x**3.0) - (y**2.0) - 1

def G(x,y):
    return x*(y**3.0) - y - 4

def partialx(function,x0,y0,stepsize):
    """
    Returns partial first derivative of the function "function" at the point x0,y0 with respect to x by considering points, one and two steps on either side of x0.
    The Accuracy is of order (stepsize)^4.
    """
    return (function(x0 - 2*stepsize, y0) - 8*function(x0 - stepsize, y0) + 8*function(x0 + stepsize, y0) - function(x0 + 2*stepsize, y0))/(12*stepsize)

def partialy(function,x0,y0,stepsize):
    """
    Returns partial first derivative of the function "function" at the point x0,y0 with respect to y by considering points, one and two steps on either side of y0.
    The Accuracy is of order (stepsize)^4.
    """
    return (function(x0, y0 - 2*stepsize) - 8*function(x0, y0 - stepsize) + 8*function(x0, y0 + stepsize) - function(x0, y0 + 2*stepsize))/(12*stepsize)

def det(a,b,c,d):
    return a*d - b*c

def Jacobian(F,G,x,y,stepsize):
    """
    Returns the jacobian of F and G at the point x and y
    """

    a = partialx(F,x,y,stepsize)
    b = partialy(F,x,y,stepsize)
    c = partialx(G,x,y,stepsize)
    d = partialy(G,x,y,stepsize)

    return det(a,b,c,d)


def NewtonSolver(F,G,x0,y0,stepsize):
    """
    Solves the system of equations F(x,y) = 0 and G(x,y) = 0 using the Newton Raphson method
    """
     J = Jacobian(F,G,x0,y0,stepsize)
     s = 0
     if (J == 0):
         return "Warning! : Jacobian is zero! Please enter a better guess!"
     else:
         s += 1
         h = (-1.0/J)*det(F(x0,y0),partialy(F,x0,y0,stepsize),G(x0,y0),partialy(G,x0,y0,stepsize))
         k = (-1.0/J)*det(partialx(F,x0,y0,stepsize),F(x0,y0),partialx(G,x0,y0,stepsize),G(x0,y0))
         x1 = x0 + h
         y1 = y0 + k
         while((abs(x1-x0) > 10**(-8)) and (abs(y1-y0) > 10**(-8))):
             x0 = x1
             y0 = y1
```

```
            J = Jacobian(F,G,x0,y0,stepsize)
            if (J == 0):
                return "Warning! : Jacobian is zero! Please enter a better guess!"
            else:
                h = (-1.0/J)*det(F(x0,y0),partialy(F,x0,y0,stepsize),G(x0,y0),partialy(G,x0,y0,stepsize))
                k = (-1.0/J)*det(partialx(F,x0,y0,stepsize),F(x0,y0),partialx(G,x0,y0,stepsize),G(x0,y0))
                x1 = x0 + h
                y1 = y0 + k
                s += 1
                print(h,k)
                print("step : ",s)
    return x1, y1, s

x_guess = 0
y_guess = 1.0

x1, y1, s = NewtonSolver(F,G,x_guess,y_guess,10**(-8))
```

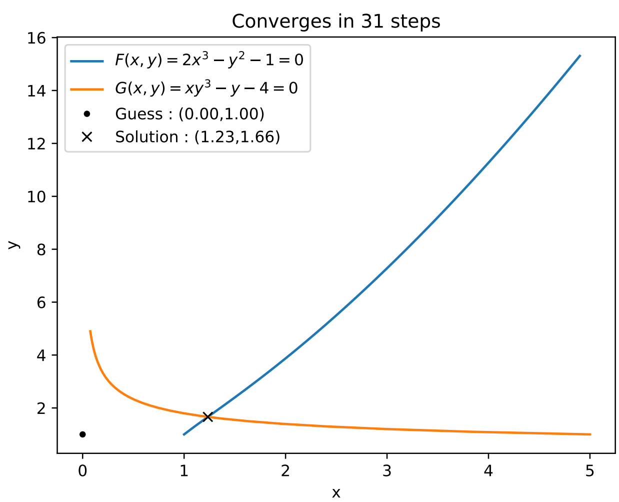
Plots

The title of the plots signifies how many iterations it took to converge to the solution. The chosen initial conditions are tabulated in the legend.





(a)



(b)