

# Reinforcement Learning

## Introduction to Multi-Agent Reinforcement Learning

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Based on slides by Stefano V. Albrecht, Filippos Christianos, Lukas Schäfer, and Leonard Hinckeldey



THE UNIVERSITY of EDINBURGH  
**informatics**

# Lecture Outline

- Multi-agent systems
- Game models
- Solution concepts
- Refinement concepts
- MARL learning framework
- Agent modeling
- Deep MARL
- Agent modeling with deep learning
- Outlook

## **Introduction: Multi-agent systems**

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## What is MARL?

**Multi-agent reinforcement learning (MARL) is about finding optimal decision policies for two or more artificial agents interacting in a shared environment.**

- Applying reinforcement learning (RL) algorithms to multi-agent systems
- Goal is to learn optimal policies for two or more agents

# MARL Applications



Computer games



Autonomous driving



Multi-robot warehouses



Automated trading

# Mutli-Agent Systems

A multi-agent system consists of:

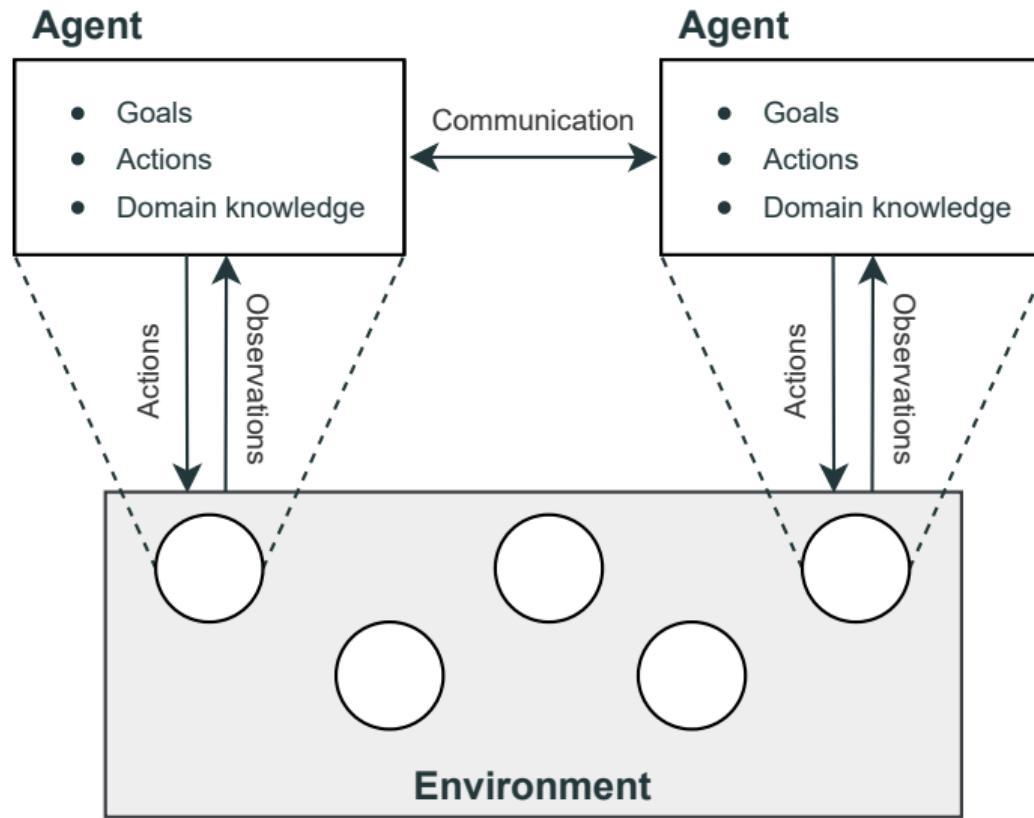
- **Environment:** The environment is a physical or virtual world whose state evolves and is influenced by the agents' actions within the environment.

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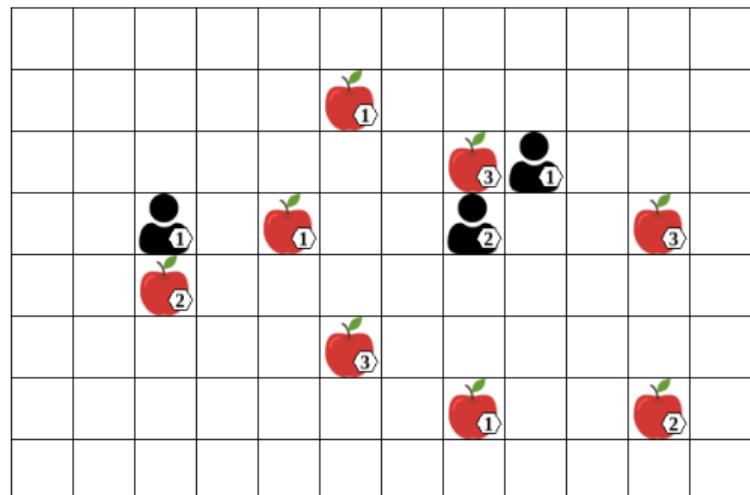
- **Environment:** The environment is a physical or virtual world whose state evolves and is influenced by the agents' actions within the environment.
- **Agents:** An agent is an entity which receives information about the state of the environment and can choose actions to influence the state.  
⇒ Agents are goal-directed, e.g. maximizing returns

# Multi-Agent Systems

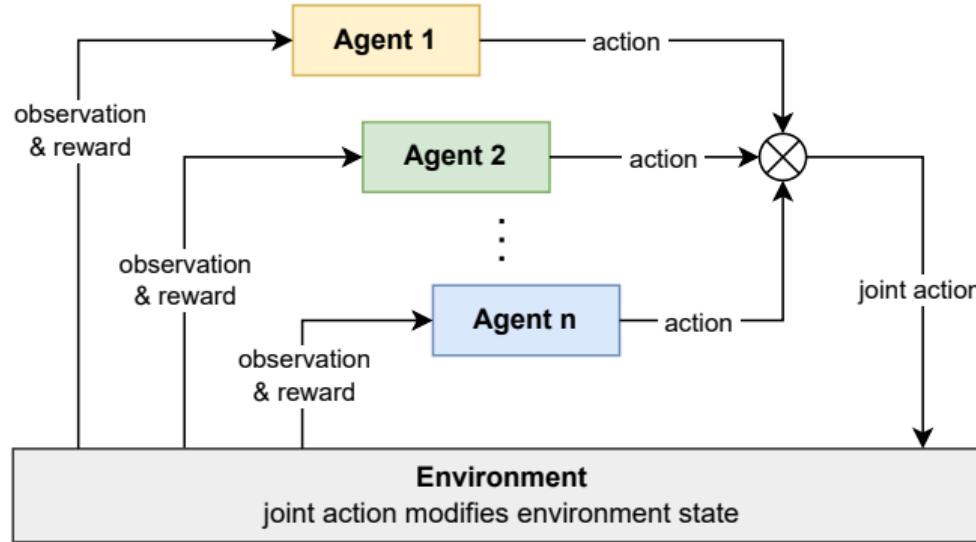


## Example: Level-Based Foraging

- Three agents (robots) with varying skill levels
- Goal: to collect all items (apples)
- Items can be collected if a group of one or more agents are located next to the item and the sum of agents' levels is greater than or equal to the item level
- Action space  
 $A = \{up, down, left, right, collect, noop\}$



# MARL for Solving Multi-Agent Systems



- **Goal:** learn optimal policies for a set of agents in a multi-agent system
- Each agent chooses an action based on its policy  $\Rightarrow$  joint action
- Joint action affects environment state; agents get rewards + new observations

## Why MARL?

Why should we use MARL to find optimal solutions to multi-agent systems rather than controlling multiple 'agents' using a single-agent RL (SARL) algorithm?

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## Decomposing a large problem

- In LBF example, controlling 3 robots each with 6 actions, the joint action space becomes  $6^3 = 216$ .  
⇒ Large action space for SARL!
- We can decompose this into three independent agents, each selecting from only 6 actions.  
⇒ Use MARL to train separate agent policies (more tractable)

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## Decentralized decision making

- There are many real-world scenarios where it is required for each agent to make decisions independently.
- E.g. autonomous driving is impractical for frequent long-distance data exchanges between a central agent and the vehicle.

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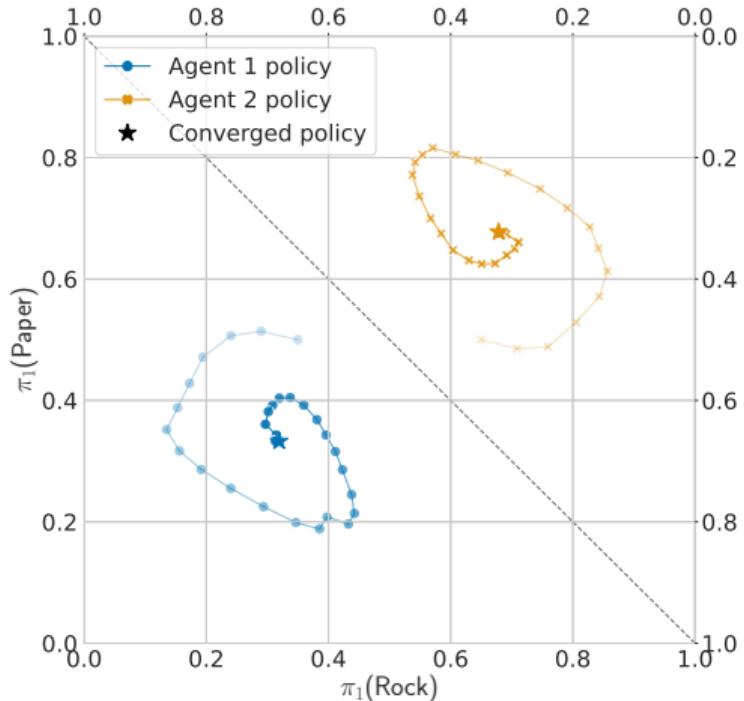
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- Optimality of policies and equilibrium selection
- Multi-agent credit assignment
- Scaling in number of agents

# Challenges of Multi-Agent Learning

## Non-stationary environment:

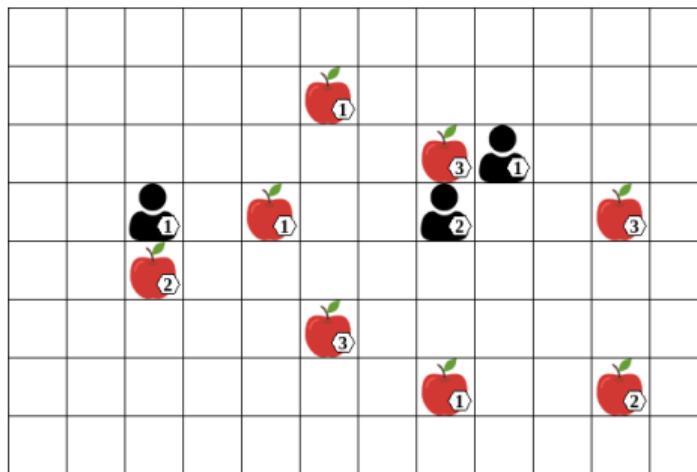
If multiple agents are learning, the environment becomes **non-stationary** from the perspective of individual agents

⇒ **Moving target**: each agent is optimizing against changing policies of other agents



# Multi-Agent Credit Assignment

**Multi-agent credit assignment:** which agent's actions contributed to received rewards?

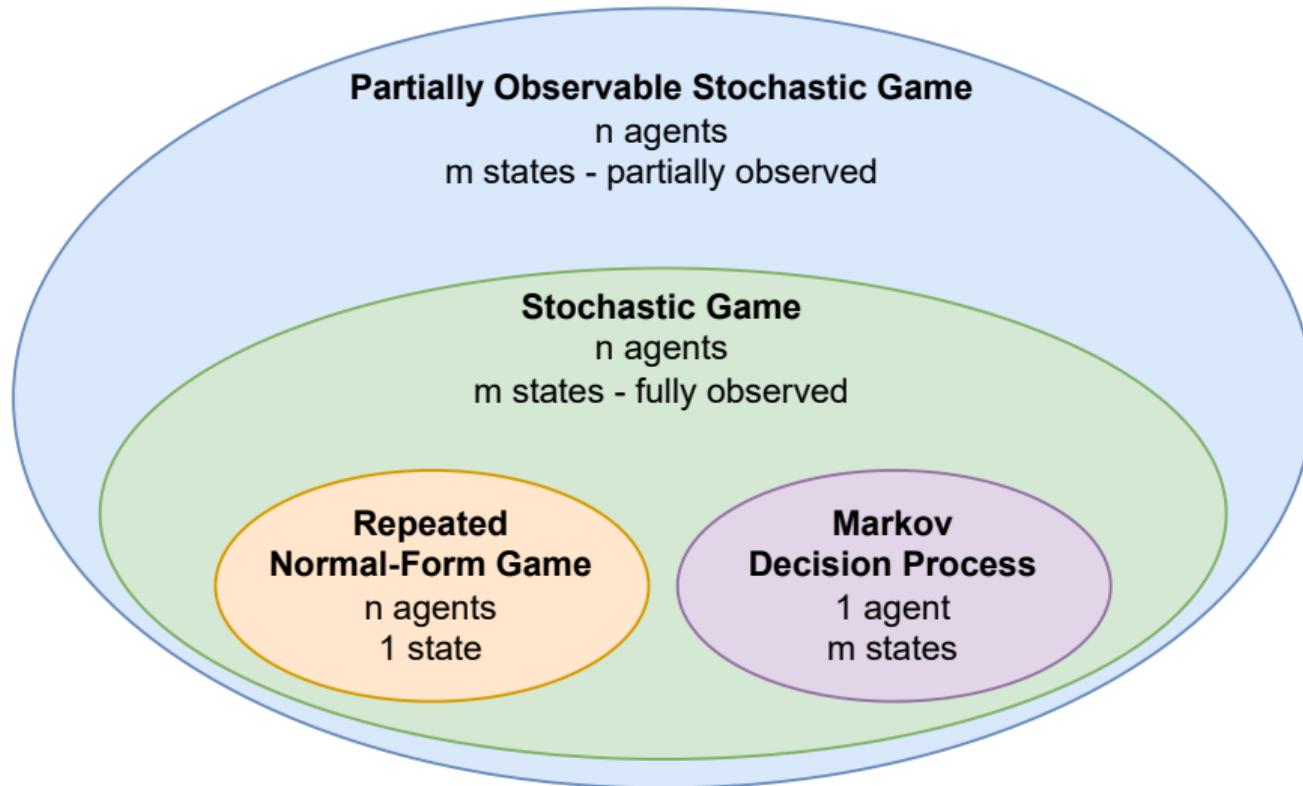


- At time step  $t$  all agents perform *collect*, each receiving reward +1
- Whose actions led to the reward?
- The agent on the left did not contribute
- Learning agents must disentangle the contributions of actions!

## Game models

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# Hierarchy of Games



## Normal-Form Games

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- $I$  is a finite set of agents  $I = \{1, \dots, n\}$
- For each agent  $i \in I$ :
  - $A_i$  is a finite set of actions
  - $\mathcal{R}_i$  is the reward function  $\mathcal{R}_i : A \rightarrow \mathbb{R}$  where  $A = A_1 \times \dots \times A_n$  (set of **joint** actions).

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2. The resulting actions from all agents form a **joint action**,  $a = (a_1, \dots, a_n)$
3. Each agent receives a reward based on its **individual** reward function and the **joint action**,  $r_i = \mathcal{R}_i(a)$

## Classes of Games

Games can be classified based on the relationship between the agents' reward functions.

- In **zero-sum games**, the sum of the agents' reward is always 0  
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- In **common-reward** games, all agents receive the same reward ( $R_i = R_j; \forall i, j \in I$ )
- In **general-sum** games, there are no restrictions on the relationship between reward functions.

## Matrix Games

Normal-form games with 2 agents are also called **matrix games** because they can be represented using reward matrices.

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Rock-Paper-Scissors

	A	B
A	10	0
B	0	10

Coordination Game

	C	D
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Prisoner's Dilemma

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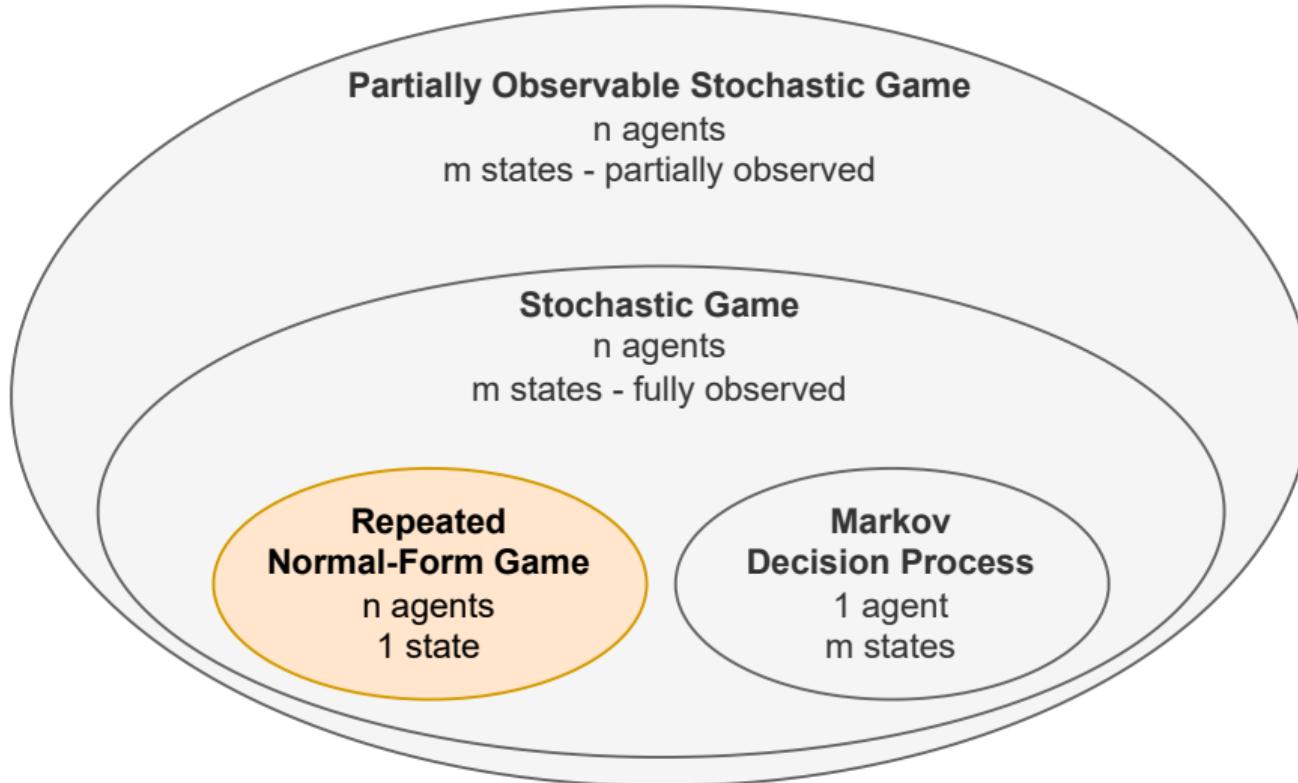
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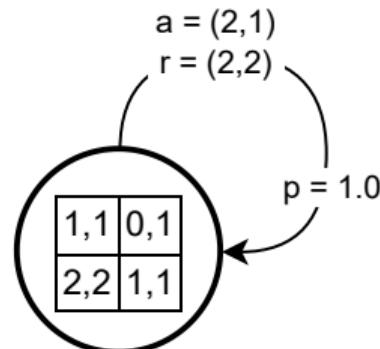
general-sum

# Repeated Normal-Form Games



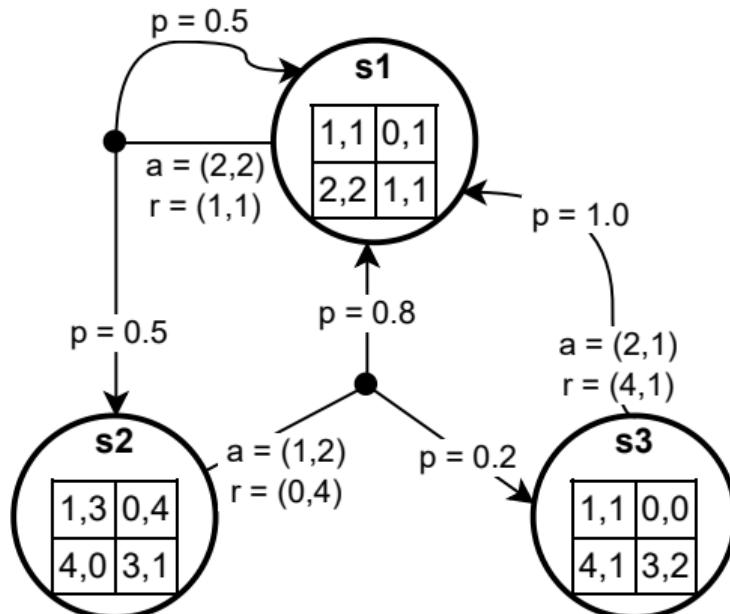
## Repeated Normal-Form Games

To extend normal-form games to **sequential** multi-agent interaction, we can repeat the same game over  $T$  timesteps.



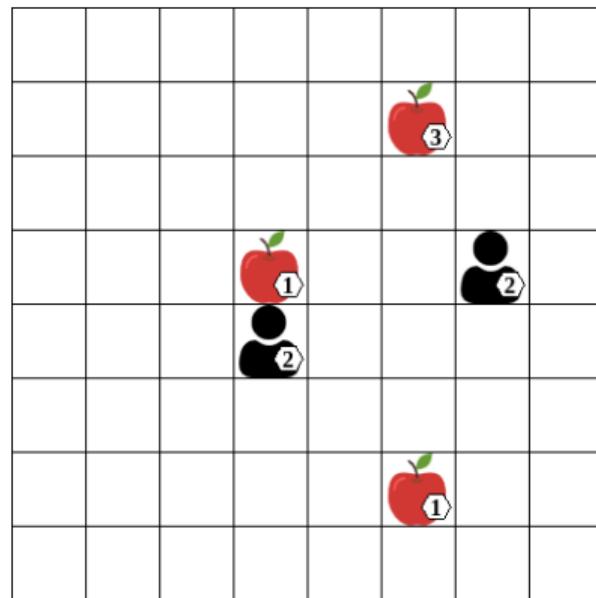
- At each time step  $t$  an agent  $i$  samples an action  $a_i^t$
- The policy is now conditioned on a **joint-action** history  $\pi_i(a_i^t | h^t)$  where  $h^t = (a^o, \dots, a^{t-1})$
- In special cases,  $h^t$  contains  $n$  last joint actions.  
E.g. in a tit-for-tat strategy (Axelrod and Hamilton 1981), the policy is conditioned on  $a^{t-1}$

# Stochastic Games



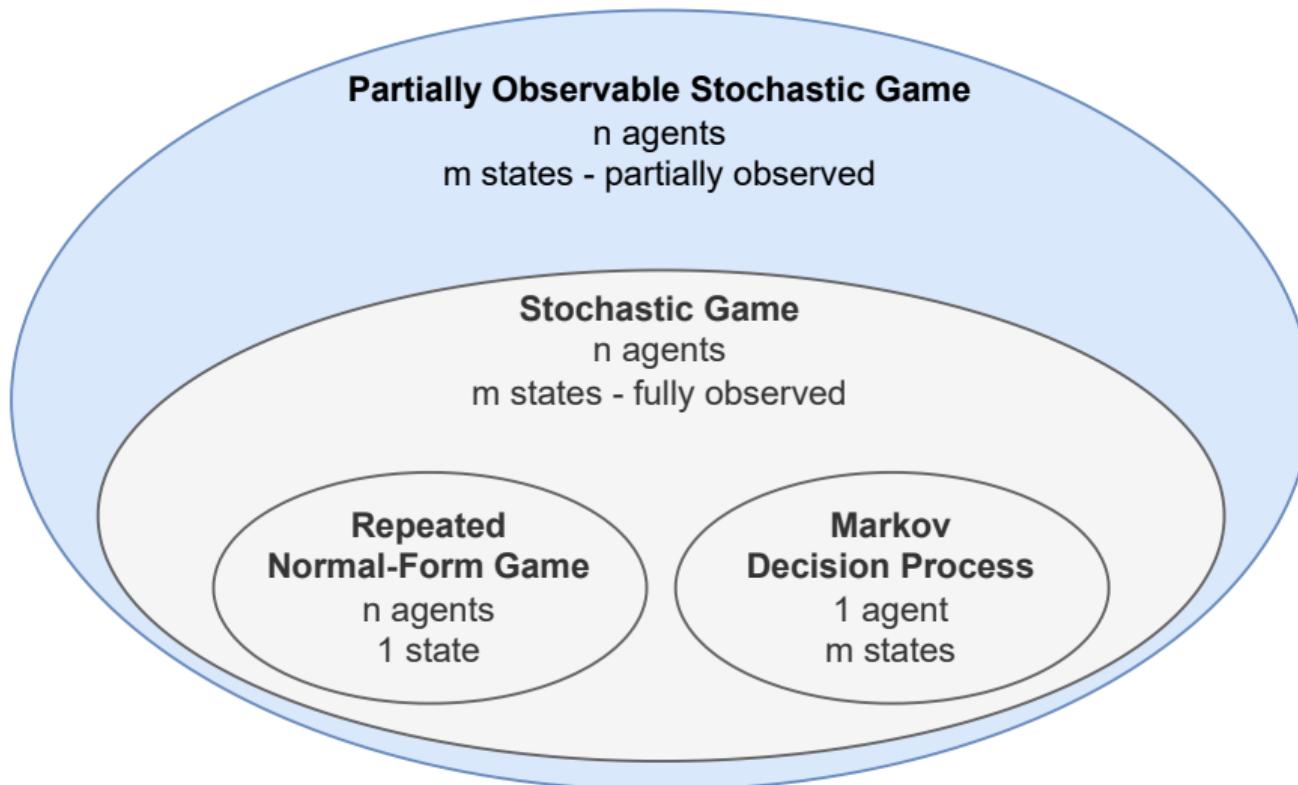
- Each **state** can be viewed as a **non-repeated normal-form game**
- Stochastic games can also be classified into: zero-sum, common-reward or general-sum
- The figure on the left shows a general-sum case

## Example: Level-Based Foraging



- $s \in S$ : vector of x-y positions for agents/items, binary collection flags, levels for agents/items
- $a_i \in A_i$ : move in four directions, collect item, or no operation (noop)
- $\mathcal{T}$ : joint actions update state, e.g., two agents collecting an item switch its flag
- $\mathcal{R}$ :
  - common-reward: +1 reward for any item collected by any agent
  - general-sum: +1 reward only for agents directly involved in item collection

# Partially Observable Stochastic Games (POSG)



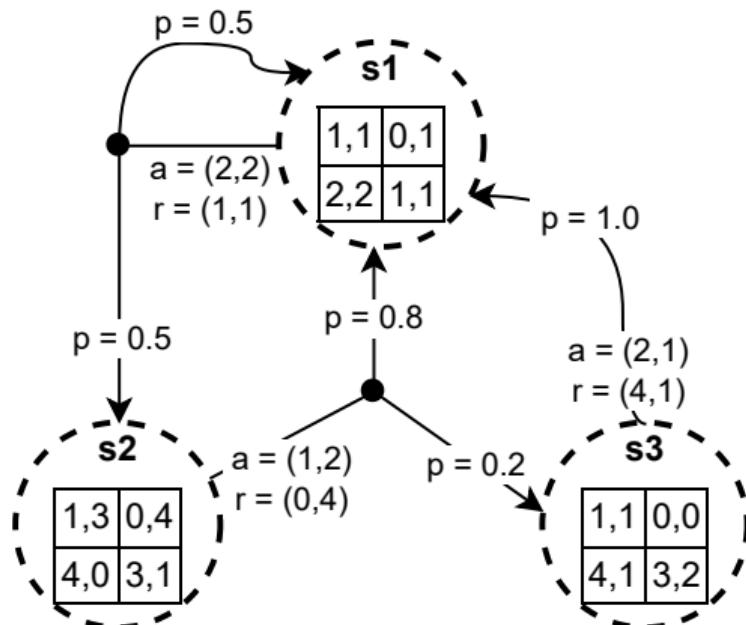
## Partially Observable Stochastic Games (POSG)

At the top of the game model hierarchy, the most **general** model is the POSG

- POSGs represent complex decision processes with **incomplete information**
- Unlike in stochastic games, agents receive **observations** providing **incomplete information** about the state and agents' actions
- POSGs apply to scenarios where agents have limited sensing capabilities
  - ⇒ e.g. autonomous driving
  - ⇒ e.g. strategic games (e.g. card games) with private, hidden information

## POSG Definition

POSG is defined in the same way stochastic games are, with two additions. Thus it is defined as a 8 tuple  $(I, S, \{A_i\}_{i \in I}, \{\mathcal{R}_i\}_{i \in I}, \mathcal{T}, \mu, \{\mathcal{O}_i\}_{i \in I}, \{\mathcal{O}_i\}_{i \in I})$



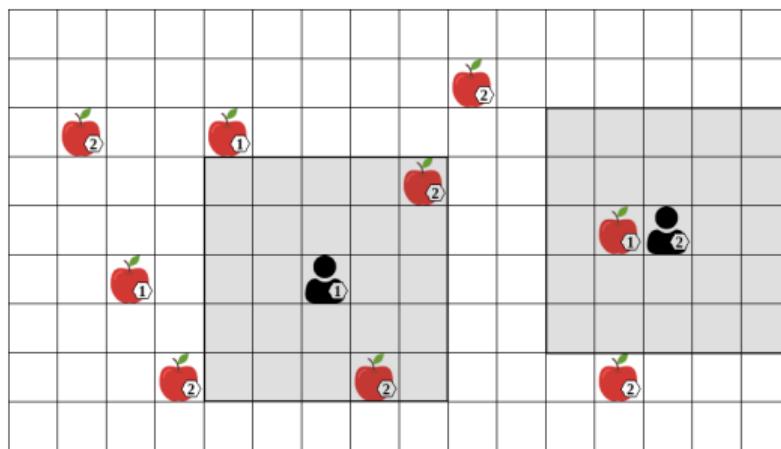
For each agent  $i$  we additionally define:

- a finite set of observation  $\mathcal{O}_i$
- an observation function  $\{\mathcal{O}_i\}_{i \in I}$  such that  $\mathcal{O}_i : A \times S \times \mathcal{O}_i \rightarrow [0, 1]$  and  $\forall a \in A, s \in S : \sum_{o_i \in \mathcal{O}_i} \mathcal{O}_i(a, s, o_i) = 1$

# The Observation Function

POSG can represent diverse observability conditions. For example:

- modeling noise by adding uncertainty in the possible observation
- to limit the visibility region of agents (see LBF example)

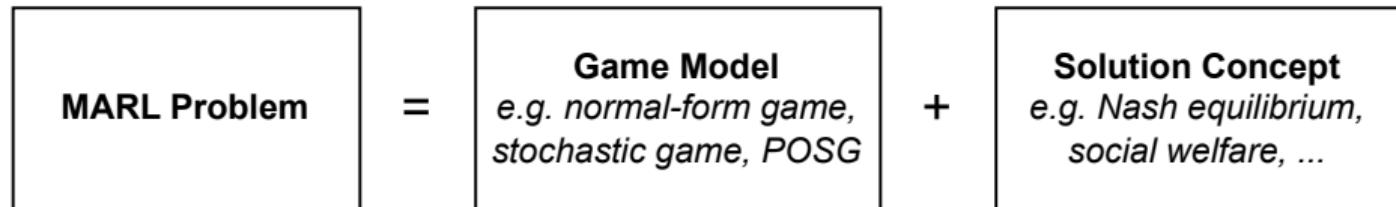


- Here, the agent can only see some parts of the state and joint action
- $o_i^t = (\bar{s}^t, \bar{a}^t)$  where  $\bar{s}^t \subset s^t, \bar{a}^t \subset a^t$

## **Solution concepts for games**

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# Solution Concepts for Games



What does it mean to act **optimally** in a multi-agent system?

- Maximizing returns of one agent is no longer enough to determine a solution
- We must consider the **joint policy** of all agents
- This is less straightforward, and there are many different solution concepts

## Nash Equilibrium

**Nash equilibrium** solution concept applies the idea of a **mutual best response** to general-sum games with two or more agents.

- No agent  $i$  can improve its expected returns by changing its policy  $\pi_i$ ; assuming other agents policies remain fixed:

$$\forall i, \pi'_i : U_i(\pi'_i, \pi_{-i}) \leq U_i(\pi)$$

- Each agent's policy is a **best response** to all other agent's policies
- John Nash (1950) proved the existence of such a solution for **general-sum non-repeated normal-form games**

# Nash Equilibrium in Matrix Games

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Rock Paper Scissors

*Can you identify the Nash equilibria?*

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NE at D, D (-3, -3)

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Coordination Game  
Two NE's at A, A (10) and  
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Rock Paper Scissors  
NE is to choose actions  
uniformly at random with  
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- **Non-uniqueness:**
  - There can be multiple (even infinitely many) equilibria, each with different expected returns
- **Incompleteness:**
  - Equilibria for sequential games don't specify actions for **off-equilibrium paths**, i.e. paths not specified by equilibrium policy  $\Pr(\hat{h}|\pi) = 0$
  - If there is a temporary disturbance that leads to an off-equilibrium path, the equilibrium policy  $\pi$  does not specify actions to return to a **on-equilibrium** path

## Refinement concepts

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## Pareto Optimality

To address some of these limitations, we can add additional solution requirements such as **Pareto optimality**.

A joint policy  $\pi$  is **Pareto-optimal** if it is not **Pareto-dominated** by any other joint policy. A joint policy  $\pi$  is Pareto-dominated by another policy  $\pi'$  if

$$\forall i : U_i(\pi') \geq U_i(\pi) \quad \text{and} \quad \exists i : U_i(\pi') > U_i(\pi).$$

### Intuition

A joint policy is **Pareto-optimal** if there is no other joint policy that improves the expected return for at least one agent without reducing the expected return for any other agent.

## Social Welfare and Fairness

To further constrain the space of desirable solutions, we can consider social welfare and fairness concepts.

### Welfare optimality:

$$W(\pi) = \sum_{i \in I} U_i(\pi)$$

- A joint policy  $\pi$  is **welfare-optimal** if  $\pi \in \arg \max_{\pi'} W(\pi')$

### Fairness optimality:

$$F(\pi) = \prod_{i \in I} U_i(\pi), \quad U_i(\pi) > 0 \quad \forall i$$

- A joint policy  $\pi$  is **fairness-optimal** if  $\pi \in \arg \max_{\pi'} F(\pi')$

## MARL learning framework

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## Single-Agent RL Reductions

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## Central learning:

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⇒ A central policy is learned over the joint action space
- What is the central reward? Scaling!

## Independent learning:

- Apply single-agent RL algorithms to each agent independently  
⇒ Agents do not explicitly consider or represent each other
- Possibly suboptimal.

# MARL Challenges

## Singe-Agent RL Challenges

- Unknown environment dynamics
- Exploration-exploitation dilemma
- Non-stationarity from bootstrapping
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## Multi-Agent RL Challenges

- Non-stationarity from multiple learning agents
- Equilibrium selection
- Multi-agent credit assignment
- Scaling to many agents

# Modes of Operation in MARL

Modes of operation in MARL:

## **Self-play:**

- *Algorithm self-play*: all agents use the same learning algorithm (and parameters)
- *Policy self-play*: agent's policy is trained directly against itself

## **Mixed-play:**

- Agents use different learning algorithms

## Agent modeling

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## Agent Modeling & Best Response

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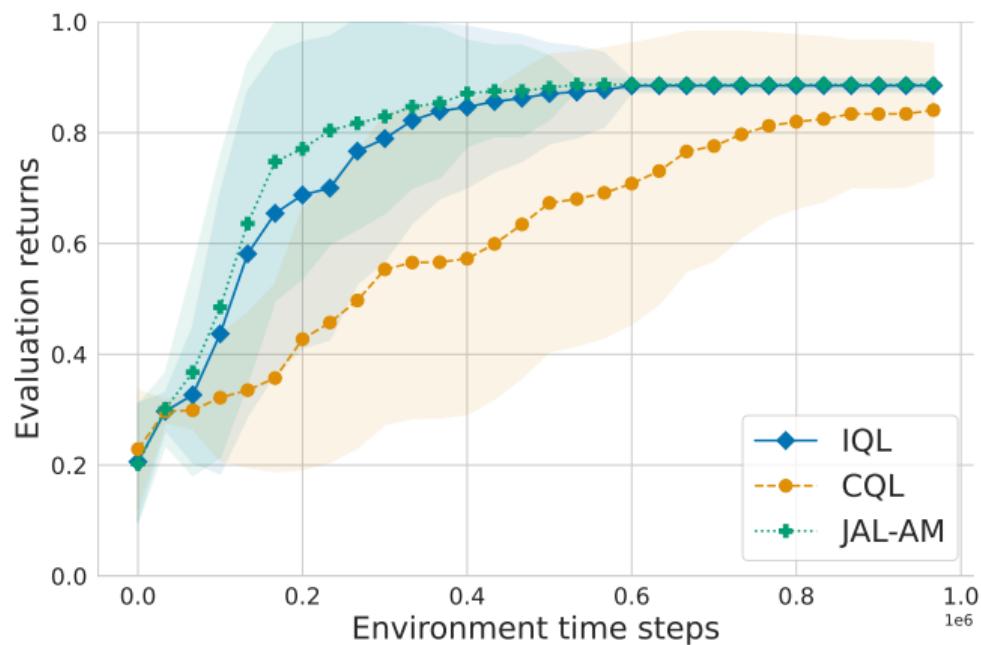
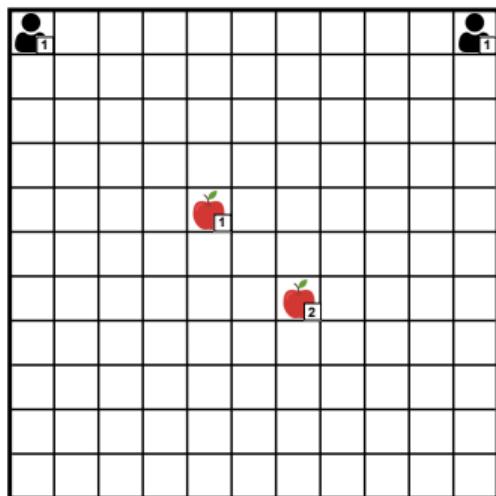
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Other approach: **agent modeling with best response**

- Learn models of other agents to predict their actions
- Compute optimal action (best response) against agent models

# Joint Action Learning with Agent modeling in Level-Based Foraging



## Deep MARL

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# Challenges of Multi-Agent Reinforcement Learning

## Reminder

MARL algorithms suffer from multi-agent specific challenges:

- **Non-stationarity**: exacerbated due to changing policies of all agents
- **Equilibrium selection**: how to converge to a stable equilibrium?
- **Multi-agent credit assignment**: how to attribute rewards to agents' actions?
- **Scaling to many agents**: how to efficiently scale to large numbers of agents?

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Centralised training with decentralised execution (CTDE) can help address some of these challenges.

# The Multi-Agent Policy-Gradient Theorem

## Solution

In MARL, the expected returns of agent  $i$  under its policy  $\pi_i$  depends on the policies of all other agents  $\pi_{-i}$  → the multi-agent policy gradient theorem defines an expectation over the policies of **all** agents ( $h_i$ : individual observation history):

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But can we do better? Perhaps by leveraging more information?

## Centralized Critics

### Note

In actor-critic algorithms, only the policy/actor is used during execution and the critic is used only during training → the critic can be conditioned on centralised information  $z$  without compromising decentralised execution.

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This might include:

- Global state  $s$
- Joint action  $a$
- Joint observation history  $h$
- ...

## Agent Modeling with Deep Learning

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## Agents Modeling – Motivation

In MARL, agents need to consider the policies of other agents to coordinate their actions.

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Approaches presented so far account for the action selection of other agents through:

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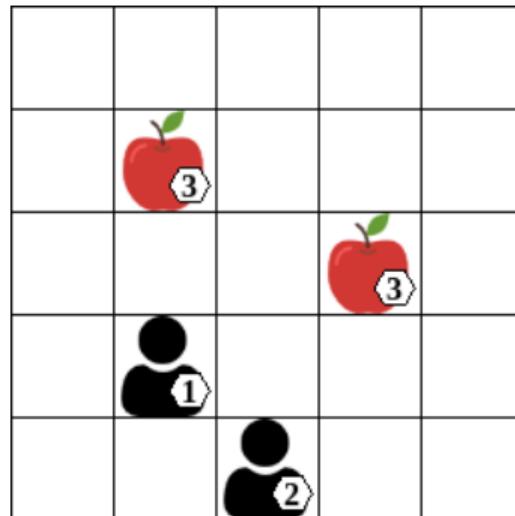
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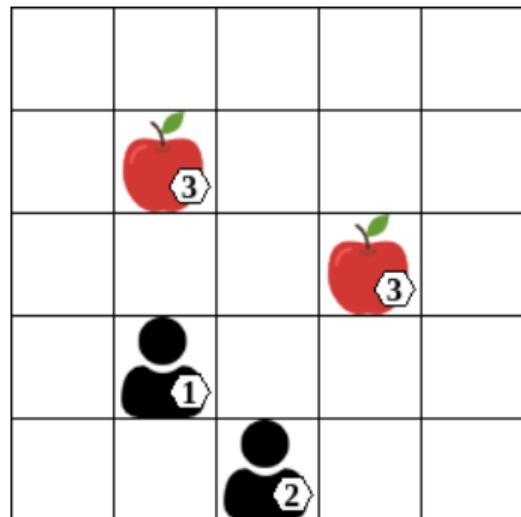
Can we provide agents with more **explicit** information about the policies of other agents so they can learn to coordinate better, e.g. by learning best-response policies?

# Joint-Action Learning with Deep Agent Models in LBF

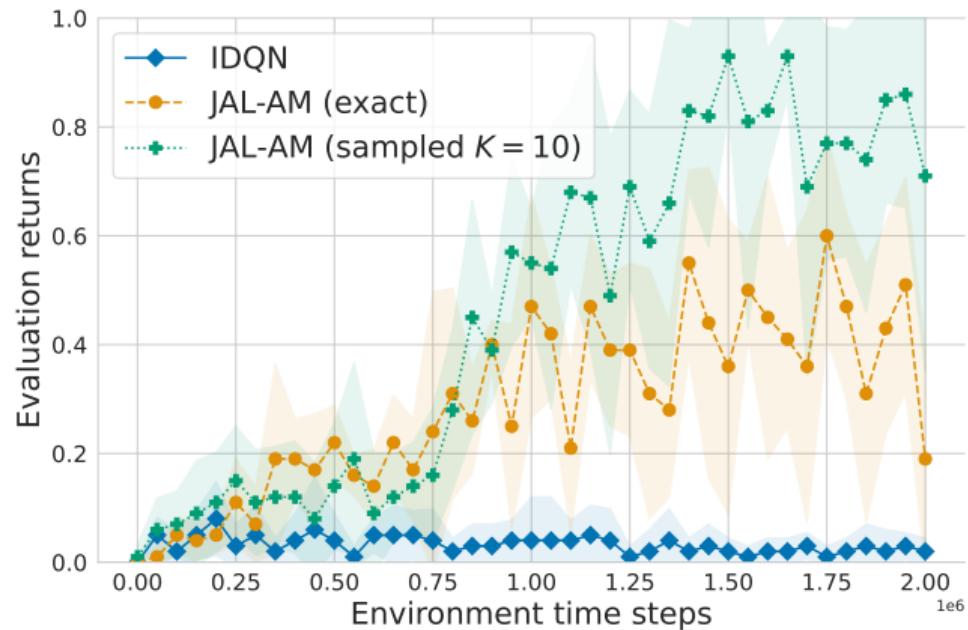


(a) Environment

# Joint-Action Learning with Deep Agent Models in LBF



(a) Environment



(b) Learning curve

## Self-Play Monte Carlo Tree Search

In zero-sum games with symmetrical roles and egocentric observations, agents can use the same policy to control both players

# Self-Play Monte Carlo Tree Search

In zero-sum games with symmetrical roles and egocentric observations, agents can use the same policy to control both players → learn a policy in **self-play**



(a) Agent 1 perspective



(b) Agent 2 perspective

## Population-Based Training – Self-Play for General-Sum Games

### Problem

With MCTS, we focused on policy self-play in **two-agent zero-sum** games. Can we extend the idea of self-play to **general-sum** games with **more than two agents**?

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Population-based training is a generalisation of self-play to general-sum games:

- Maintain a **population of policies** representing possible strategies of the agent
- Evolve populations so they become more effective against the populations of other agents
- We denote the population of policies for agent  $i$  at generation  $k$  as  $\Pi_i^k$ .

# The MARL Book

This lecture was based on

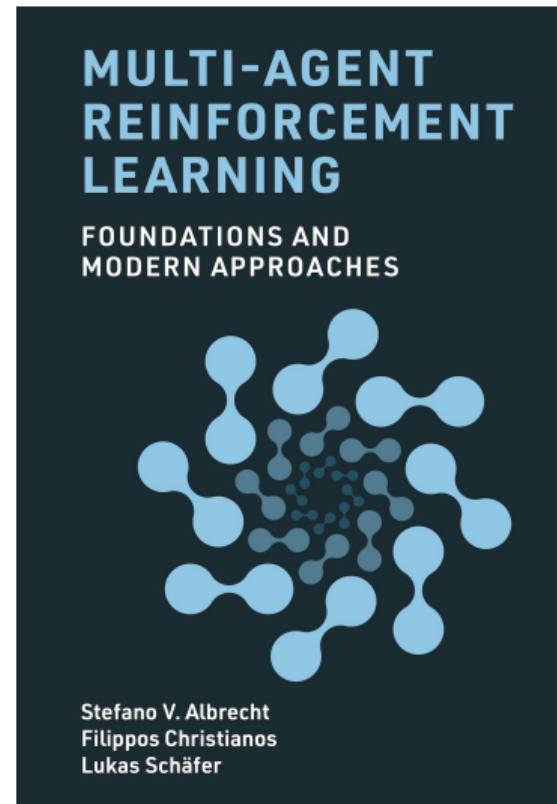
## **Multi-Agent Reinforcement Learning: Foundations and Modern Approaches**

by Stefano V. Albrecht, Filippos Christianos and  
Lukas Schäfer

MIT Press, 2024

Download book, slides, and code at:

[www.marl-book.com](http://www.marl-book.com)



## **Outlook: Whither RL?**

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## Current challenges in RL

- DRL, POMDP, MARL, MORL, IRL, ...
- Sample complexity, exploration, stability, robustness, generalisation, reproducibility
- Problem characteristics, Meta-RL, transfer learning, reward shaping, RL + LLM
- Data-based (off-line) RL, sim-to-real gap, latency, learning from many policies
- Biological RL, population-based methods, naturalistic RL (Wise e.a., 2024)
- Safe RL, human-in-the-loop RL, grounded RL, interdisciplinary problems
- Applications: Industry size of RL: £100B (2025) growth rate 65% (rough estimates!).