
Multi-Agent Reinforcement Learning for Task Offloading in Wireless Edge Networks

Andrea Fox

LIA, Avignon University
Avignon, France

andrea.fox@univ-avignon.fr

Francesco De Pellegrini

LIA, Avignon University
Avignon, France

francesco.de-pellegrini@univ-avignon.fr

Eitan Altman

INRIA, Sophia Antipolis, France
eitan.altman@inria.fr

Abstract

In edge computing systems, autonomous agents must make fast local decisions while competing for shared resources¹². Existing MARL methods often resume to centralized critics or frequent communication, which fail under limited observability and communication constraints. We propose a decentralized framework in which each agent solves a constrained Markov decision process (CMDP), coordinating implicitly through a shared constraint vector. For the specific case of offloading, e.g., constraints prevent overloading shared server resources. Coordination constraints are updated infrequently and act as a lightweight coordination mechanism. They enable agents to align with global resource usage objectives but require little direct communication. Using safe reinforcement learning, agents learn policies that meet both local and global goals. We establish theoretical guarantees under mild assumptions and validate our approach experimentally, showing improved performance over centralized and independent baselines, especially in large-scale settings.

1 Introduction

In decentralized systems, local decisions taken by agents collectively influence the outcome of other agents' actions or the availability of shared resources, with direct impact on individual performance. In mobile edge computing (MEC), for instance, this is the case of edge offloading techniques. They permit multiple devices to independently decide whether to offload computations to a shared edge server or to process them locally. But, while each agent operates based on local observations and limitations, such as battery level, workload, or latency requirements, the collective offloading decisions directly affect server responsiveness and network congestion. Actually, when too many agents offload simultaneously, the resulting overload can degrade performance across the system. In such settings, each agent's objective depends not only on its own decisions but also on the aggregated behavior of others, introducing a coordination challenge. This challenge is further amplified in environments with communication delays or asynchronous agent behavior, where real-time coordination is difficult or infeasible. Designing scalable learning methods that support implicit coordination under these system technical constraints is essential for efficient and robust distributed operation.

¹The code used in our experiments can be found at <https://github.com/Andrea-Fox/multiAgentTaskOffloading>.

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Many existing approaches to coordination in multi-agent reinforcement learning assume additive rewards or rely on shared incentives and communication protocols. However, in practical systems like edge computing and wireless access networks, agent interactions are tightly coupled through shared resources, and rewards are often non-additive due to congestion effects. To address these coordination challenges, we propose a decentralized learning framework based on independent constrained Markov decision processes (CMDPs). In this framework, coordination is achieved implicitly through shared constraints that are updated infrequently. These constraints regulate the maximum frequency at which each agent can offload tasks, effectively serving as a virtual coordination mechanism over shared resources. Each agent optimizes its own policy using techniques from constrained reinforcement learning, ensuring local autonomy while maintaining system-wide alignment through periodic, constraint-driven synchronization. This approach enables fast, scalable, and communication-efficient decision-making in distributed environments like edge computing, where centralized coordination is often impractical.

Our contributions are resumed hereafter:

- We introduce DCC, a general decentralized reinforcement learning framework for multi-agent coordination under shared resource constraints; we apply it to the problem of task offloading in wireless edge computing systems. Each device (agent) solves a local CMDP, and coordination emerges implicitly through the shared constraint vector that regulates offloading behavior across the network.
- We provide a tractable approximation of the global objective via decomposition, establish theoretical guarantees on its validity under mild assumptions and error bounds in the nonlinear case.
- We validate the DCC framework through preliminary numerical experiments in toy environments. While limited in scope, our results highlight the scalability of the approach and the net improvement over centralized and independent baselines. They lay the groundwork for more extensive evaluations in future research.

The paper is organized as follows: section 2 reviews related work, while section 3 introduces the Markov game and the system model under consideration. In section 4, we present the proposed algorithm for the agents’ constrained policy optimization. Numerical results are provided in section 5, followed by concluding remarks in the final section.

2 Related works

Multi-agent reinforcement learning (MARL) has been extensively surveyed in [11, 33, 2]. Centralized Training Distributed Execution (CTDE) approaches are the most widely used in MARL. For example, MADDPG [18], MAPPO [32], and COMA [7] employ centralized critics to guide training, while policies are executed in a decentralized manner. Value-decomposition methods such as VDN [26] and QMIX [23] improve scalability further by factorizing value functions, though they operate under an individual-global-max assumption, which does not hold in settings where agents compete for scarce resources. Independent learners such as IQL [27] and IPPO [6] avoid centralization entirely, but struggle to coordinate in environments with interdependent rewards.

Constrained reinforcement learning (CRL) extends standard RL by requiring policies to satisfy long-term constraints in addition to maximizing rewards. A variety of approaches have been proposed to solve constrained Markov decision processes (CMDPs) [9, 17]. Primal-dual methods, such as RCPO [29], optimize reward and constraints concurrently, while value-based methods [5] adapt Q-learning to the constrained setting. CPO [1] formulates the problem as a constrained trust-region update, offering guarantees but incurring high computational cost. IPO [16] introduces a first-order method using barrier functions, and PCPO [31] projects policies onto the feasible set after unconstrained optimization.

While reinforcement learning has been widely applied to MEC offloading problems—typically optimizing task latency or energy consumption using single-agent algorithms such as DQN [15] and DDQN [28]—multi-agent settings remain comparatively underexplored. Some recent works integrate MARL into this domain: for example, [8] combines QMIX with DQN to jointly minimize average delay and energy cost, while [12] employs decentralized actor-critic agents for user-level

offloading decisions. However, these approaches either rely on centralized training, assume frequent inter-agent communication, or do not explicitly address coordination under shared resource constraints. In contrast, our method formulates offloading as a set of independent constrained MDPs, where coordination emerges implicitly via infrequent updates to shared constraints, enabling scalable, communication-efficient learning tailored to congestible wireless environments.

3 System model

In wireless edge computing, a collection of mobile devices must decide at each time step whether to execute computational tasks locally, offload them to a shared edge server, or defer processing altogether. Each device operates independently based on its local state, which may include factors such as task backlog, latency sensitivity, energy harvesting rate, local processing cost, the time elapsed since the last data processing, and battery level. The wireless channel and the edge server represent shared, capacity-limited resources. When too many devices offload simultaneously, contention and server overload degrade performance for all users. To mitigate this, we impose a constraint on the long-term frequency with which each agent may offload, serving as a virtual coordination mechanism. This constraint *does not* reflect a physical device limitation, but rather a system-level policy that governs fair and efficient resource usage.

Each device is modeled as an agent solving a local constrained MDP (CMDP) [3], optimizing its policy to balance local performance objectives with the global constraint on offloading frequency. Coordination emerges implicitly as agents adapt their policies based on local observations and periodic updates to the shared constraint, enabling decentralized yet system-aware behavior.

3.1 Markovian formulation

We model the system described in section 3 as a decentralized multi-agent problem in which N agents must coordinate their actions to minimize a global objective function. The joint state of the system is denoted by $s = (s_1, \dots, s_N) \in \mathcal{S}$, where s_i is the local observation of agent i . Each agent selects an action a_i from its individual action space, and the joint action is $a = (a_1, \dots, a_N) \in \mathcal{A}$.

We assume independent transition dynamics, i.e., the transition probability of the system factorizes across agents as $p(s' | s, a) = \prod_{i=1}^N p_i(s'_i | s_i, a_i)$, where $s' = (s'_1, \dots, s'_N)$ is the next state, and p_i denotes the local transition kernel for agent i .

Each agent's reward consists of a local component and a global component that depends on the actions of the other agents. This formulation is sufficiently general to model a broad range of mobile edge computing applications.

$$r(s, a) = \sum_{i=1}^N r_i(s_i, a) = \sum_{i=1}^N u_i(s_i) + \mathbb{I}[a_i = a_{\text{crowd}}] \cdot d(N(a)) \quad (1)$$

where $u_i : \mathcal{S}_i \rightarrow \mathbb{R}$ describes the local utility function for agent i , $a_{\text{crowd}} \in \mathcal{A}_i$ is a designated *crowded action*, corresponding to offloading the task to the edge computer, $N(a) = \sum_{j=1}^N \mathbb{I}[a_j = a_{\text{crowd}}]$ is the number of agents choosing the crowded action and $d : [1, \infty) \rightarrow \mathbb{R}$ is a differentiable function with $d(1) = 0$, $d'(n) > 0$, and $d''(n)$ exists and does not change sign.

The function $d(n)$ models congestion at the edge: the more agents select the crowd action, the greater the penalty each receives for choosing it. This structure induces coordination challenges reminiscent of *crowding games* or *singleton congestion games*, where agents compete over a shared, congestible resource. This additive form preserves the separability of local rewards while explicitly modeling the coupling from congestion through $d(N(a))$, which captures the performance degradation when many agents offload simultaneously.

We assume the following misalignment between individual and collective incentives:

Assumption 1 (Individual Incentive for Crowd Action). *Let $a \in \mathcal{A}$ be such that $N(a) = 1$ and $a_i = a_{\text{crowd}}$, i.e., only agent i selects the crowd action. Let $a' \in \mathcal{A}$ be identical to a except $a'_i \neq a_{\text{crowd}}$, with $a'_j = a_j$ for all $j \neq i$. Then, for all states s :*

$$r(s, a) < r(s, a'). \quad (2)$$

That is, the crowd action appears individually beneficial when selected in isolation. However, this creates a coordination dilemma: if every agent independently learns to select the crowd action, the shared penalty $d(N(a))$ increases, degrading the overall system performance. This structure highlights the need for coordination-aware learning methods beyond naive reward minimization.

The objective function we want to minimize is

$$J(\pi, \beta) = \mathbb{E}_{a \sim \pi(s)} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \mid s_0 \sim \beta \right] \quad (3)$$

While the dynamics are independent across agents, the reward function introduces a critical coupling through the crowd-dependent term $d(N(a))$, which depends on the number of agents selecting a specific action a_{crowd} . This structure creates several challenges in analyzing or optimizing $J(\beta)$:

- **Non-decomposability:** The reward $r(s, a)$ is not additive across agents, as the congestion penalty depends on the global joint action a . As a result, standard decomposition techniques used in multi-agent systems with independent rewards (e.g., independent learners) are not applicable.
- **Coupled incentives:** Even if agents act independently, their optimal behavior is interdependent due to the shared penalty. The optimal joint policy may require implicit coordination to avoid overcrowding the sensitive action, a behavior that cannot be captured by decentralized greedy learners.
- **Noisy reward:** Since the congestion term depends on how many agents select a_{crowd} , individual agents receive noisy or misleading feedback about their own contribution to the reward. This complicates both policy gradient and value-based methods.
- **Non-stationarity in decentralized learning:** In decentralized training, each agent’s policy evolves independently. Coupling in the reward makes the environment non-stationary from each agent’s perspective, even though the transition dynamics are stationary.

These properties position the problem beyond standard cooperative multi-agent reinforcement learning (MARL) settings with additive rewards or shared goals. In the next section, we present a learning framework designed to address these challenges.

4 Using MARL for optimizing offloading

We now introduce the **DCC framework** (Decentralized Coordination via CMDPs), a general structure for multi-agent reinforcement learning in shared-resource environments. DCC enables scalable coordination among agents by combining three key ingredients:

1. **Lightweight Communication:** Agents operate independently, without exchanging real-time state or action information. Inter-agent communications do occur, but only infrequently and to share scalar constraint variables, which enables coordination.
2. **Constraint-Based Coupling:** Each agent solves a constrained MDP in which the constraint controls the crowded action. The update of the constraint variable serves as a coordination signal across agents.
3. **System-Level Alignment:** Optimizing the constraint vector steers individual agent behavior toward global objectives, without requiring centralized control or synchronous communication.

We remark that DCC is not a single algorithm but a general *framework* compatible with a broad class of reinforcement learning methods for local MDPs, including value-based and policy-gradient approaches. We begin by providing a high-level intuition of the proposed algorithm. A detailed description of each component is then developed in sections 4.1 to 4.5, including the underlying reward approximation, the CMDP formulation, and the multi-timescale optimization process.

To approximate the global objective (3), we reformulate it as a sum of local value functions constrained by agent-specific action frequencies:

$$J(\pi_{\text{global}}^*, \beta) \approx \inf_{\theta} \sum_{i=1}^N J_i(\pi_i^*(\theta_i), \beta_i) \quad (4)$$

Each agent i solves its own CMDP, where the constraint θ_i encodes how often it can select the crowded action. This decomposition decouples learning while preserving system-level coordination through optimization of the shared constraint vector θ .

To facilitate the development of a three-timescale learning algorithm, we define an approximate reward function: it replaces the actual number of agents which choose the congested action with the expected value. It corresponds to number of agents who select the crowded action according to their local constraint. This approximation removes the direct dependency on joint actions: each agent optimizes its policy independently and yet full constraint vector θ accounts for the aggregate system behavior.

Our algorithm uses a three-timescale learning process to solve the approximation in (4):

- **Fast and Intermediate Timescales:** For a fixed constraint vector θ , each agent independently optimizes its own policy by solving a local CMDP using a Lagrangian-based safe reinforcement learning approach. On the fastest timescale, the policy is updated using a shaped reward of the form $r_i^{\text{tot}}(s, a) = r_i(s_i, a) + \lambda_i c_i(s_i, a_i)$, where c_i denotes the cost associated with selecting the crowded action. Since both θ and λ_i are held fixed during this phase, *any standard reinforcement learning algorithm*, such as PPO, DQN, or even Q-learning if the system is small enough, can be used to optimize the local policy. On the intermediate timescale, the Lagrange multiplier λ_i is updated via a primal-dual method to enforce long-term satisfaction of the constraint.
- **Slow Timescale:** At the slowest time-scale, the constraint vector θ is optimized to improve global coordination. Instead of heuristically selecting θ , we treat it as a coordination variable and optimize it via stochastic approximation. Importantly, the structure of the objective allows each component of the gradient $\nabla_{\theta} J$ to be estimated using just three local policy evaluations per agent, regardless of the total number of agents. This property ensures that the algorithm remains scalable and efficient, even in large systems.

While constraint updates are performed synchronously across agents, they occur at a much slower timescale and can be implemented with minimal coordination overhead. In practice, such updates are orders of magnitude less frequent than policy learning steps, preserving the algorithm’s scalability and decentralization. Notably, the computational complexity of DCC is similar to the one of the RL algorithm chosen, with the primary difference being the computation of the gradient—an operation that is computationally inexpensive.

Together, these components yield a principled and practical algorithm for decentralized coordination under shared resource constraints, with theoretical support for its approximation quality and optimization dynamics.

A simplified pseudocode for DCC is found in algorithm 1, while the full pseudocode in in section D. In the remainder of the section, we investigate more deeply the DCC framework through the following steps:

- Introduction of a decomposable approximation of the immediate reward (section 4.1): we propose a decomposable approximation of the immediate reward that approximates the long-term reward with provably bounded approximation error—and exact equivalence when the congestion function is linear.
- CMDP of the system (section 4.2): we introduce the single agent CMDP that will be used by the three timescale algorithm
- Computing the optimal constrained policy (section 4.3): We describe the process of computing the optimal constrained policy for a single agent, given a fixed value of the constraint.
- Differentiability of the objective (section 4.4): We establish that the long term reward of each agent is differentiable with respect to θ , which allows us to efficiently identify the minimum in (4)
- Efficient computation of the gradient (section 4.5): We show how the structure of the gradient can be leveraged to significantly accelerate the computation of the infimum of the objective function.

Algorithm 1 DCC Framework (Simplified Overview)

```
1: Initialize: constraint vector  $\theta^0$ , multipliers  $\lambda_i^0$ , policies  $\pi_i^0$  for each agent  $i$ 
2: function OPTIMIZELOCALCMDP( $i, \theta, n$ )
3:   for  $m = 0, 1, \dots, M(n)$  do ▷ Intermediate timescale: Lagrange multiplier update
4:     for  $t = 0, 1, \dots, T(m, n)$  do ▷ Fast timescale: policy optimization
5:       Update policy  $\pi_i$  via RL step on shaped reward  $r_i^{\text{tot}}$ 
6:     end for
7:     Update multiplier  $\lambda_i$ 
8:   end for
9:    $\hat{J}_i(\theta) \leftarrow$  evaluate final policy  $\pi_i^*$ 
10:  return  $\hat{J}_i(\theta)$ , final policy  $\pi_i^*$ , multiplier  $\lambda_i^*$ 
11: end function
12:
13:
14: for  $n = 0, 1, 2, \dots$  do ▷ Slowest timescale: constraint optimization
15:   for each agent  $i = 1, \dots, N$  do
16:      $\hat{J}_i(\theta), \pi_i^*, \lambda_i^* \leftarrow$  OPTIMIZELOCALCMDP( $i, \theta^n, n$ )
17:     Estimate local gradient  $\nabla_{\theta} \hat{J}_i(\theta^n)$ 
18:   end for
19:   Update shared constraint vector:  $\theta^{n+1} \leftarrow \theta^n + \eta_n \nabla \hat{J}(\theta^n)$ 
20: end for
```

4.1 Approximating the objective function via decomposition

In a general setting general, the value function decomposition is a viable alternative when

$$r(s, a) = \sum_i r_i(s_i, a_i)$$

meaning that each agent can act independently while still achieving the globally optimal outcome [19, 24]. However, in the setting studied in this work, this condition does not hold due to a coupling term that depends on how many agents select the *crowded* action. Specifically, the global reward takes the form

$$r(s, a = \pi_{\text{global}}(s)) = \sum_i f_i(s_i) + \mathbb{I}_{a=a_{\text{crowd}}}(a_i) \cdot d(N(a)) \quad (5)$$

where $N(a)$ denotes the number of agents that choose the crowded action under π_{global} . (as defined in section 3).

To address the challenge of decomposing this reward function, we propose a new (approximated) formulation of the reward :

$$\begin{aligned} \hat{r}(s, a; \theta) &= \sum_i u_i(s_i) + \mathbb{I}_{a=a_{\text{crowd}}}(a_i) \cdot d \left(1 + \sum_{j \neq i} \theta_j \right) \\ &= \sum_i \hat{r}_i(s_i, a_i; \theta_{-i}), \end{aligned} \quad (6)$$

where θ_j denotes the average constraint value for agent j , i.e. the expected frequency with which agent j selects the crowded action, and $\theta_{-i} = \sum_{j \neq i} \theta_j$.

Intuitively, this approximation replaces the randomness in the number of offloading agents ($d(N)$) with its expectation (θ), which reduces variance in the reward signal while still capturing the average congestion effect. This makes the problem more tractable for decentralized learning, while preserving the essential trade-offs.

This amounts to treating the policies of other agents (which are unknown to any given agent) as if they select the congested action as frequently as their individual constraints permit. Although this may not strictly hold in practice, it is justified by Assumption 1, which states that the congested action yields the highest reward when chosen in isolation—implying that agents are likely to fully utilize their constraints.

This approximation brings two key benefits for decentralized learning. First, by replacing the actual number of agents choosing the crowded action with its expected value under the constraint vector θ , it removes the non-stationarity typically introduced by inter-agent coupling. As a result, each agent's reward depends only on its local state, action, and the fixed parameter θ , effectively decoupling the learning dynamics across agents during the fast timescale. Second, the use of an expected congestion term reduces the variance of the reward signal each agent observes, since it no longer depends on the stochastic actions of others. This lower-variance signal leads to more stable updates and can accelerate policy learning. Finally, in practical implementations such as edge computing, this formulation allows an agent to compute its reward without waiting for real-time feedback on how many others offloaded in the same timestep. While this does not reduce communication during deployment, it simplifies both training and simulation, and avoids introducing extra delays into the reward computation.

Error Bound for the Reward Approximation The following result gives an indication of the error that one could have when considering the approximated definition of the reward in (6). In particular, we define $J(\pi, \beta)$ the discounted reward associated to a policy π when considering the reward r and $\hat{J}(\pi, \beta)$ the one obtained when considering the approximated reward \hat{r} defined in (6).

Lemma 1. *Given a global policy π and a vector $\theta \in \mathbb{R}^N$ such that*

$$\mathbb{E}_{a \sim \pi} [N_{i,t}(a)] = \theta_i,$$

where $N_i(a)$ denotes the random variable representing the frequency with which agent i selects the crowded action at time t , it is verified that for a non linear penalty function d

$$|J(\pi, \beta) - \hat{J}(\pi, \beta)| \leq \frac{1}{1 - \gamma} \sum_i \theta_i \left(\frac{\theta_{-i}}{N_{agents} - 1} d(N_{agents}) - d(1 + \theta_{-i}) \right)$$

Moreover, if d is linear, the two reward values coincide exactly, and the approximation becomes exact.

Proof. See section A.1. □

This leads to the following result, which shows that, under certain conditions on the immediate reward function, the approximation in (4) holds exactly. As a result, the decomposition is valid, and the algorithm proposed in this work can reliably recover the optimal policy. Importantly, this also establishes clear conditions under which the method is guaranteed to be optimal.

Proposition 1. *Let assume we know $\theta^* \in \mathbb{R}^N$ such that $\mathbb{E}_{a \sim \pi^*} [N_{i,t}(a_t)] = \theta_i^* \forall i$, for the optimal global policy π^* and assume that $d(\cdot)$ is linear. Then the value function does not depend on the type of reward that we consider and an optimal global policy can be obtained as the combination of the appropriate optimal local policies.*

Proof. From lemma 1, we know that if the function d is linear, then the discounted reward of a policy is invariant under the choice between the original reward r and the approximated reward \hat{r} . Assuming the optimal constraint values θ^* are known, we can compute the locally optimal policy for each agent using \hat{r} . A policy composed by combining these local policies, denoted as $\pi_{composed}^*$, is then a solution to the global minimization problem with respect to \hat{r} , and by the invariance, also with respect to r . This leads to the conclusion:

$$J(\pi_{composed}, \beta) = \hat{J}(\pi_{composed}, \beta) = \sum_i \hat{J}_i(\pi_i^*(\theta^*), \beta)$$

□

4.2 Constrained MDP for Each Agent

As introduced in eq. (4), we approximate the global objective by decomposing it into a collection of independent Constrained Markov Decision Processes (CMDPs), one per agent. Constrained Markov Decision Process (CMDP) [3] extends the standard MDP by incorporating constraints on long-term cost signals. In this section, we describe the structure of the CMDP associated with each agent i .

State space The state space \mathcal{S}_i for agent i consists of its local observation s_i , such that the full joint state is $(s_1, \dots, s_N) \in \mathcal{S}$.

In the task offloading scenario considered, each agent’s state is given by $s_i = (x_i, e_i)$, where x_i denotes the *Age of Information*, i.e., the number of timesteps since the last data processing was completed, and e_i represents the current energy level of the agent’s battery.

Action space The action space $\mathcal{A}_i(s_i)$ includes a designated *crowded action*, a_{crowd} , which represents use of a shared resource. All other actions are independent of shared usage.

For example, in the task offloading scenario described in section 3, we have $\mathcal{A}_i(s_i) = \{\text{“wait”}, \text{“local processing”}, \text{“offload”}\}$, where “offload” corresponds to a_{crowd} .

Reward function The agent receives a reward according to the approximated formulation described in section 4.1, where the influence of other agents is captured through the fixed (in the slow and intermediate timescale) parameter θ_{-i} :

$$\hat{r}_i(s, a; \theta_{-i}) = u_i(s_i) + \mathbb{I}_{a_i = a_{\text{crowd}}} \cdot d(1 + \theta_{-i}).$$

Cost function The cost signal $c_i(s_i, a_i)$ is a naive function active only when the shared resource is used:

$$c_i(s_i, a_i) = \begin{cases} 1, & \text{if } a_i = a_{\text{crowd}} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Constraint The constraint is defined by the parameter θ_i , which limits the long-term frequency with which agent i can select the crowded action.

While our formulation is based on the discounted reward criterion, the DCC-RL framework is not inherently restricted to this setting. In principle, the same decomposition and coordination structure can be applied in the average reward case, with appropriate adjustments to the underlying reinforcement learning algorithm. The choice between average and discounted formulations depends primarily on the application context and the algorithm used to solve each agent’s constrained MDP. In the discounted case, we define the discounted reward and discounted cost under policy π_i and initial state distribution β_i as follows:

$$J_i(\pi_i, \beta_i; \theta_{-i}) = \mathbb{E}_{\pi, s_0 \sim \beta} \left[\sum_{t=0}^{\infty} \gamma^t \sum_{t=0}^T \hat{r}_i(S_t, A_t; \theta_{-i}) \middle| S_0 = s_0 \right] \quad (8)$$

$$K_i(\pi_i, \beta_i) = \mathbb{E}_{\pi, s_0 \sim \beta} \left[\sum_{t=0}^{\infty} \gamma^t c_i(S_t, A_t) \middle| S_0 = s_0 \right] \quad (9)$$

The optimization problem faced by each agent, assuming fixed constraint parameters θ , is then:

$$\begin{aligned} \text{minimize:} \quad & J_i(\pi_i, \beta_i; \theta_{-i}) \\ \text{subject to:} \quad & K_i(\pi_i, \beta_i) \leq \theta_i \end{aligned} \quad (CMDP_i)$$

4.3 Learning the Optimal Local Constrained Policy

A central advantage of our framework is its flexibility: the local constrained policy optimization can be performed using any safe reinforcement learning (SRL) method suited to the environment. In particular, by adopting a Lagrangian-based approach, we decouple constraint enforcement from reward maximization. This separation enables the use of any standard reinforcement learning algorithm on the fastest timescale, with constraint satisfaction handled independently on a slower timescale. This modularity is crucial in decentralized settings with heterogeneous agents or dynamics, allowing DCC to adapt to a wide range of learning scenarios and observation structures.

Lagrangian methods are a natural fit for our setting, as they allow constraint satisfaction to be enforced without the need for projection or repair, and are compatible with standard RL algorithms. While such methods can exhibit instability or slow convergence—particularly in settings where constraint violations incur severe penalties or pose safety risks—these issues are mitigated in our case.

Here, constraints reflect system-level efficiency or coordination objectives rather than hard safety requirements, so temporary violations during training do not lead to catastrophic outcomes. As a result, we can safely use Lagrangian updates without sacrificing performance or stability, making them a principled and practical choice for decentralized learning under shared constraints.

In particular, we follow the structure of RCPO [29] for constrained optimization, employing either PPO [25], DQN [21] or Q-learning [30], depending on the scale of the system, as the algorithm to handle the fastest timescale

In this framework, in the fastest timescale each RL agent optimizes a shaped reward of the form:

$$\hat{r}_i^{\text{total}}(s_i, a_i; \theta_{-i}) = \hat{r}_i(s_i, a_i; \theta_{-i}) + \lambda_i c_i(s_i, a_i), \quad (10)$$

where \hat{r}_i and c_i have been defined in section 4.2.

The scalar coefficient λ_i serves as a Lagrange multiplier, adaptively tuned during training to ensure constraint satisfaction. To formalize this, we define the agent-specific Lagrangian function:

$$\mathcal{L}_i(\pi_i, \lambda_i; \theta) = \mathbb{E}_{\pi_i} \left[\sum_{t=0}^{\infty} \gamma^t \hat{r}_i(s_t, a_t; \theta_{-i}) + \lambda_i (c_i(s_t, a_t) - \theta_i) \right] \quad (11)$$

where θ_i is the fixed constraint threshold for agent i , and θ_{-i} denotes the sum of the vector of constraint parameters associated with all other agents. In this formulation, θ_{-i} is treated as fixed during the agent's local optimization, capturing the expected influence of other agents on the congestion dynamics. The k -th update of the Lagrange multiplier is then computed as follows:

$$\begin{aligned} \lambda_i^{k+1} &= \Gamma_{\lambda_i} [\lambda_i^k + \eta_i(k) \nabla_{\lambda} \mathcal{L}_i(\pi_i, \lambda_i^k; \theta)] \\ &= \Gamma_{\lambda_i} \left[\lambda_i^k + \eta_i(k) \left(\mathbb{E}_{\pi_i} \left[\sum_{t=0}^{\infty} \gamma^t \sum_{t=0}^T c_i \right] - \theta_i \right) \right] \end{aligned}$$

and Γ_{λ_i} is a projection on the space of possible Lagrange multipliers, i.e. the set of positive real numbers and $\eta_i(t)$ is a descending sequence converging to 0.

Crucially the value of λ_i is optimized for each agent independently, allowing the coordination signal to emerge in a fully decentralized manner. Our method enables agents to learn how the cost function should influence their behavior individually, in order to satisfy long-term constraints while maintaining local performance.

4.4 Coordination via optimization of the constraints

Having described the two faster timescales, we now focus on the slowest one, where the constraint vector θ is optimized to improve global coordination.

In particular, we want to show that we can converge to the local optimal value of the constraint vector $\theta \in \mathbb{R}^N$. The safe RL algorithms described in section 4.3 will then be able to find the optimal policy for that value.

We consider the long term Lagrangian reward with regard to θ , assuming that we can solve the corresponding constrained problem and find the optimal values of π and λ . This corresponds to the quantity we want to minimize for each agent. With abuse of notation, we define the quantity

$$\begin{aligned} \hat{J}_i(\theta) &= \hat{J}(\pi_i^*(\theta)) \\ &= \mathcal{L}_i(\pi_i^*(\theta), \lambda_i^*(\theta); \theta) \\ &= \mathbb{E}_{a \sim \pi_i^*(\theta)} \left[\sum_{t=0}^{\infty} \gamma^t \sum_{t=0}^T \hat{r}_i(s_t, a_t; \theta_{-i}) + \lambda_i^* \cdot (c_i(s_t, a_t) - \theta_i) \right] \end{aligned} \quad (12)$$

and the function we want to minimize is

$$\hat{J}(\theta) = \sum_i \hat{J}_i(\theta) \quad (13)$$

First we show the differentiability of this function with regard to the constraint θ .

Lemma 2. $\hat{J}_i(\theta)$ is differentiable in θ almost everywhere.

Proof. The proof of this result is provided in section A.3, where we show that a variation of the Envelope Theorem [20] can be applied within the context of our setting. \square

In order to discuss the optimization of the virtual constraint we want to use the stochastic gradient ascent methods of the Kiefer-Wolfowitz family [13]. The iteration scheme is

$$\theta^{n+1} = \Pi_{\Theta} (\theta^n - \alpha_n \hat{g}_n) \quad (14)$$

where θ^n is the n th iterate of the parameter, \hat{g}_n represents an estimate of the gradient of the objective function, $\{\alpha_n\}_n$ is a sequence converging to 0 and Π_{Θ} is a projection on the space of possible virtual constraint vectors Θ .

If we drop λ and π for the sake of clarity, the i -th component of the gradient estimate writes

$$(\hat{g}_n)_i = \frac{\hat{J}(\theta + c_n (\Delta_n)_i) - \hat{J}(\theta)}{c_n (\Delta_n)_i} \quad (15)$$

where $\{c_n\}$ is a sequence converging to 0 and $\{\Delta_n\}$ is an i.i.d. vector sequence of perturbations of i.i.d. components $\{(\Delta_n)_i, i = 1, \dots, N\}$ with zero mean and where $\mathbb{E} [|(\Delta_n)_i|^{-2}]$ is uniformly bounded.

The following result defines the conditions on the objective function, step-size sequence α_n , and gradient estimates \hat{g}_n that give the convergence to a local minima.

Theorem 1. Assume that

- $\{\alpha_n\}$ and $\{c_n\}$ be such that $\sum_n \alpha_n = \infty$, $\sum_n \left(\frac{\alpha_n}{c_n} \right)^2 < \infty$
- $\mathbb{E} [|(\Delta_n)_i|^{-2}]$ is uniformly bounded on Θ

Then the algorithm converges to a local optimal value of $\min_{\theta} \sum_i \hat{J}_i(\theta)$.

Proof. Lemma 2 guarantees the differentiability of the objective function. The assumptions on the sequences α_n and c_n allow us to use Proposition 1 in [14] for a subset of Θ which contains the initial vector θ_0 . This proves the convergence of the algorithm to a locally optimal solution. \square

See Proposition 1 in [14] for the statement and the necessary conditions for the convergence to the global optimal point.

4.5 Numerical optimization of the constraints via evaluations of the local policies

We conclude this section by showing how the structure of the gradient can be further simplified, leading to a more efficient computation.

The following result, which is an immediate consequence of the chain rule, reveals how the computation of the gradient of the objective function can be simplified, thereby improving the overall efficiency of the algorithm:

Proposition 2.

$$\frac{\partial}{\partial \theta_i} \hat{J}(\theta_i, \theta_{-i}) = \frac{\partial \hat{J}_i(\theta_i, \theta_{-i})}{\partial \theta_i} + \sum_{j \neq i} \frac{\partial \hat{J}_j(\theta_i, \theta_{-i})}{\partial \theta_{-j}} \quad (16)$$

Proof. After applying the chain rule, it is sufficient to observe that $\frac{\partial \theta_{-j}}{\partial \theta_i} = 1$ when $j \neq i$. \square

Proposition 2 illustrates how the problem structure, together with the form of the constrained value function, can be leveraged to compute the gradient of the approximate objective function efficiently. Specifically, this result allows each component of the gradient at a given point θ to be estimated

using only three stochastic evaluations per agent, rather than $N + 1$, as we would only need to evaluate $\hat{J}_i(\theta_i, \theta_{-i})$, $\hat{J}_i(\theta_i + \epsilon, \theta_{-i})$ and $\hat{J}_i(\theta_i, \theta_{-i} + \epsilon)$.

While Proposition 2 enables an efficient numerical estimation of the gradient, the following result provides a more precise analytical characterization of its components and clarifies their qualitative behavior.

Proposition 3. *The following is verified:*

1. $\frac{\partial \hat{J}_i(\theta)}{\partial \theta_i} = -\frac{\lambda_i^*}{M} \leq 0$ a.e., where λ_i^* is the optimal value of the Lagrange multiplier in θ_i and M is a positive constant,
2. $\frac{\partial \hat{J}_i(\theta)}{\partial \theta_j} = \theta_i \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \geq 0$, $\forall j \neq i$ a.e., where π_i^* is the optimal policy for agent i .

Proof. It is once again a consequence of theorem 2; in particular of the definition of the directional derivatives. The full proof is provided in section A.4. \square

Proposition 3 formalizes the intuition about the sign of the gradient components and provides the exact expression for the local derivatives. In particular, relaxing the constraint for one device decreases its own reward (i.e., benefits it from a selfish perspective) while penalizing the others, as a consequence of the shared congestion effect.

It is worth noting, however, that this expression still relies on the exact values of the optimal multipliers λ_i^* , which are not directly available during learning. The proposed algorithm converges to these values asymptotically, but in practice the gradients are approximated. For this reason, in the numerical experiments we currently rely on finite-difference estimates.

The additional simplification obtained by directly substituting λ_i^* in place of the local finite-difference evaluations could further reduce computation times. This refinement has not yet been tested in the stochastic approximation setting, though we have verified its correctness in small systems where λ_i^* was computed exactly via linear programming (section 5.5). Evaluating the performance of this direct substitution in the approximate setting is left for future work.

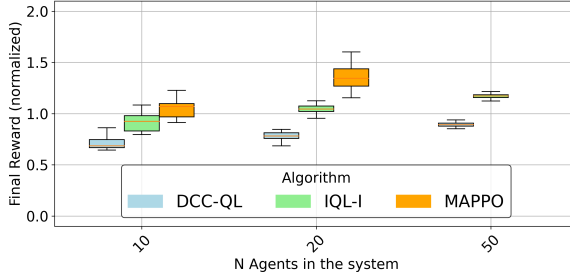
The framework described in this section provides a principled and scalable approach to decentralized coordination in multi-agent systems with limited communication. By decomposing the global objective through constrained MDP formulations and optimizing across three timescales, agents can independently learn behaviors that align with system-level goals. The theoretical guarantees and structural properties of the reward function justify this decomposition and enable efficient learning despite the inherent coupling in agent behavior. These results lay the foundation for the empirical evaluation in the next section, where we assess the algorithm’s performance under realistic edge computing scenarios.

5 Numerical experiments

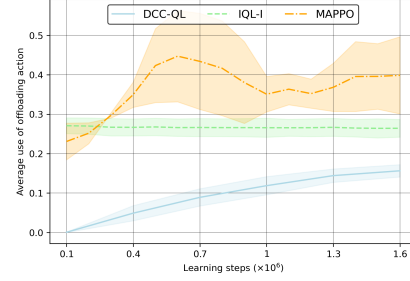
We consider the environment introduced in section 3, with the toy model detailed in section B. Each experiment is averaged over 15 runs on independently generated environments. We focus on small instances where the optimal policy can be determined via Q-learning, enabling clear validation of our approach.

We compare DCC-QL against two representative baselines. First, independent Q-learning (IQL), which represents the fully decentralized setting without communication. Second, MAPPO [32], a CTDE method that has shown strong performance in cooperative MARL benchmarks. We use IQL rather than the more common IPPO because the toy environments are very small, and IPPO’s larger architectures tend to overfit, obscuring the relative benefits of our coordination mechanism.

Finally, we stress that this is an early-stage exploration of the DCC framework. Our experiments on toy environments are intended as proof-of-concept to validate theoretical properties and highlight scalability trends, rather than as comprehensive benchmarks. Extending this evaluation to realistic wireless network simulators constitutes an important direction for future work.



(a) Normalized final average reward across methods and system sizes. Rewards are scaled so that 1 equals DCC-QL’s performance after the first 10^5 steps. Error bars indicate variability across seeds. MAPPO results for 50 devices (average ≈ 3.5) are omitted to avoid distorting the scale.



(b) Evolution of the offloading frequency in a 10-device system. Starting from $\theta = 0$, DCC-QL gradually converges to stable usage, while IQL quickly locks into a suboptimal policy with excessive offloading.

5.1 Scalability properties

The first experiment evaluates the scalability of DCC-QL by comparing its discounted reward after five iterations of constraint improvement—starting from $\theta = 0$, meaning agents are initially prohibited from using the common resource—with that of the baselines as the number of devices increases. All methods are given the same total number of policy-learning steps, and rewards are normalized so that 1 corresponds to the reward achieved by DCC-QL after the first 10^5 learning steps. As shown in Fig. a, DCC-QL consistently outperforms independent Q-learning across all system sizes, while MAPPO, though competitive in small systems, degrades rapidly as the number of devices grows, likely due to its fixed network architecture being unable to handle the enlarged state–action space. We also note that in larger systems, never using the offloading action—i.e., the normalization baseline obtained with $\theta = 0$ —yields higher rewards than IQL, underscoring how IQL’s lack of coordination prevents it from capturing the system dynamics.

5.2 Offloading action frequency

As a second observation, we analyze the frequency of the offloading action in the case with 10 devices (results for $N = 20$ and $N = 50$ are provided in the additional material). Figure b shows that DCC-QL gradually evolves from never using the offloading action (the initial evaluation and normalization baseline) to a moderate and stable usage level, consistent with the constraints derived in section 4. In contrast, IQL quickly converges to a suboptimal policy that overuses the offloading action, as expected given its attractiveness (Assumption 1) and the lack of any coordination mechanism. MAPPO, meanwhile, has not converged within the limited training budget considered here and continues to overuse the offloading action; however, when trained for 10 million steps (six times more than evaluated in this experiment), it eventually achieves performance comparable to DCC-QL, indicating that it can learn the dynamics of the small system but only at significantly higher sample complexity.

5.3 Effect of Initial Constraint

In the main experiments, the environments considered were relatively homogeneous. In such cases, starting from uniform constraints is a natural choice. We therefore repeated the first main experiment from section 5, initializing DCC-QL with larger values of the constraints instead of $\theta = 0$. In particular, we chose as initial constraints values similar to the final values obtained by DCC-QL from naive initial constraints.

As shown in fig. 2, after five iterations of constraint improvement the final discounted reward is essentially the same for both initializations, with uniform initialization yielding slightly higher values. The difference is more pronounced in early iterations: when starting from uniform constraints, the algorithm achieves higher rewards sooner, particularly in systems with fewer devices. This confirms that a better prior on the constraints can accelerate learning, as observed in fig. 3.

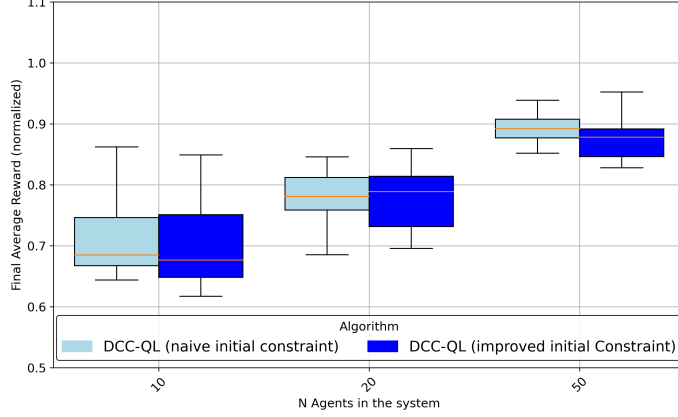


Figure 2: We compare the final normalized reward after 5 iteration of DCC-QL when starting from a naive constraint ($\theta = 0$) and when starting from an optimized value, where the optimized one has been chosen by looking at the final values obtained by DCC-QL when starting from naive initial constraints.

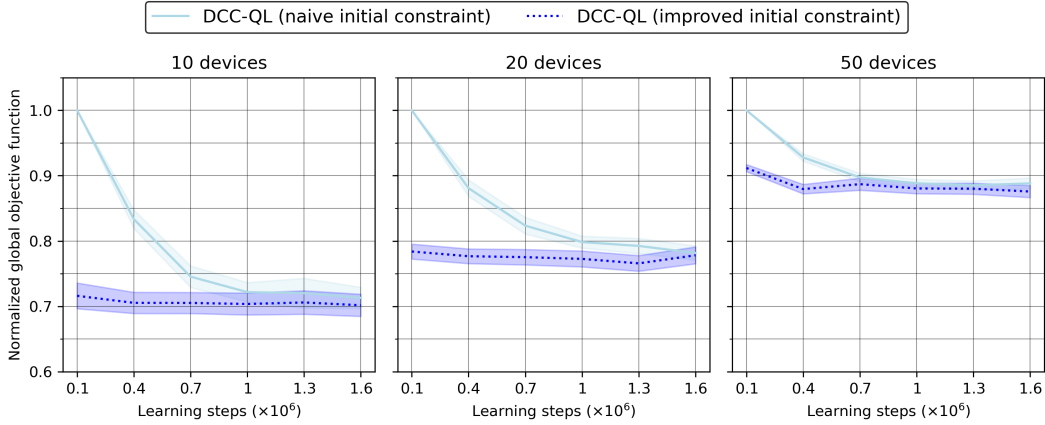


Figure 3: Comparison of the evolution of the reward as we start from optimized initial constraints in settings with a different amount of devices.

5.4 Non linear penalty

We investigate how the frequency of the offloading action changes when varying the exponent α of the penalty function $d(n) = (n - 1)^\alpha$. The experiment was carried out with 20 devices, since in the case of 10 devices the offloading frequency remains essentially unchanged across all values of α . This is likely because, with fewer devices, it is rare for many of them to simultaneously use the shared resource, making the penalty less pronounced. Moreover, the penalty is independent of α when exactly two devices offload at the same time, so differences only emerge when three or more devices compete for the shared resource—a situation that occurs more frequently with 20 devices.

Figure 4 shows that, across all algorithms, the offloading frequency decreases as α increases, consistent with the expectation that higher exponents strengthen the penalty and discourage simultaneous offloading. The effect is particularly pronounced for DCC-QL and MAPPO.

5.5 Evaluation gradient objective function

We conducted an additional experiment to empirically validate the gradient properties derived in Proposition 3. For each value of the exponent of the penalty function, we generated 15 small random environments and solved each CMDP with the linear program to obtain the optimal policy and its

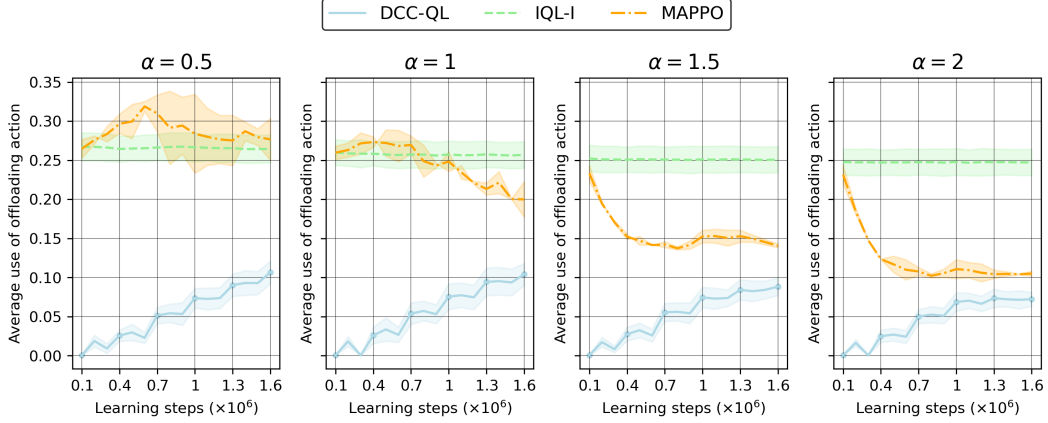


Figure 4: Evolution of the offloading action frequency for different values of the penalty exponent α . Increasing α leads to a consistent decrease in offloading frequency across algorithms, reflecting the stronger penalization of simultaneous offloading.

discounted reward. First we perturbed the constraint vector by a finite noise ϵ and estimated gradients with respect to both the local component θ_i and the coupling term θ_{-i} . The results (Fig. 5) confirm the theoretical prediction and the intuition: the local gradient is consistently negative, while the coupling gradient is positive.

Then, using very small ϵ , we compared left and right finite-difference estimates, which matched

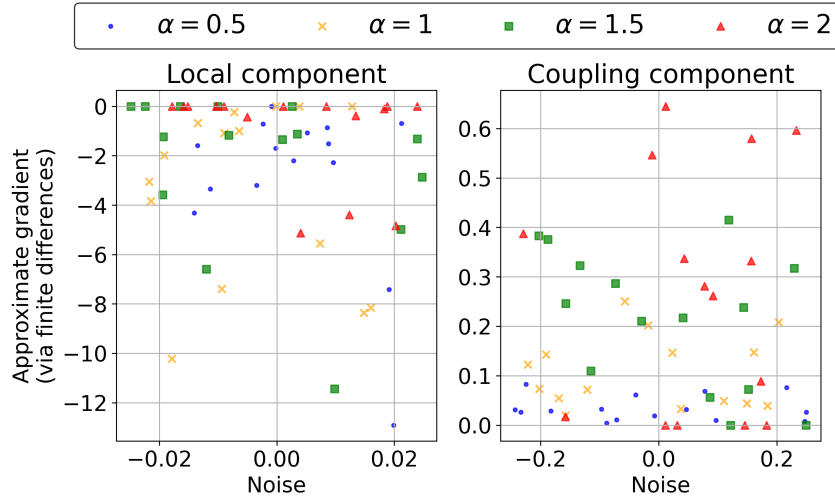
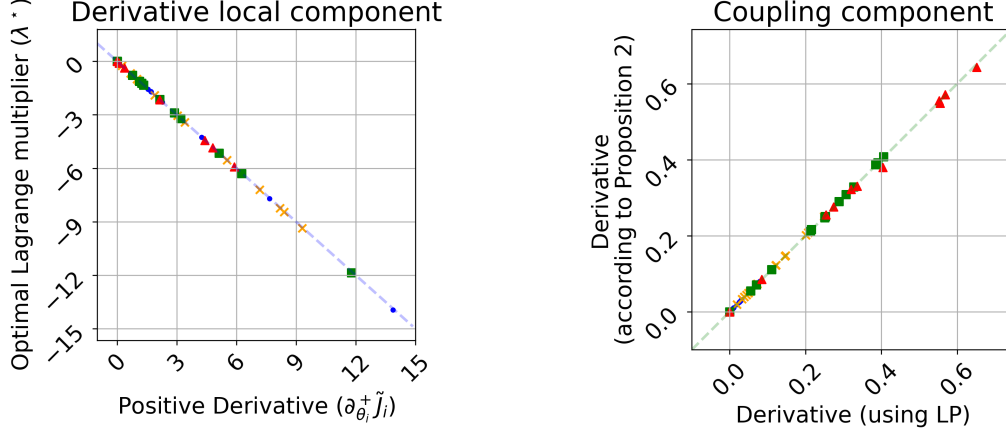


Figure 5: In these figures we evaluate an approximation of the gradient of $\tilde{J}_i^\ell(\theta)$ using the finite difference method. We considered a noise $\epsilon \in (0.01, 0.25)$ for both cases, In the left figure, representing the local component of the noise, we expected negative values, while in the right figure we expected positive values.

closely, confirming differentiability (not reported). Moreover, in fig. 6 we verified that the local derivative equals the negative of the optimal Lagrange multiplier, and that the coupling derivative matches the expression in Proposition 3.

These findings validate proposition 3 and suggest a possible refinement of DCC-QL : instead of estimating local gradients via finite differences, the algorithm could directly substitute the Lagrange multiplier, potentially leading to a more efficient implementation. We leave this extension to future work.



(a) Numerical verification that the finite-difference derivative of the objective with respect to the local noise coincides with the negative of the optimal Lagrange multiplier, confirming exact correspondence up to solver precision.

(b) Numerical verification that the finite-difference derivative of the objective with respect to the local noise coincides with the value given by proposition 3, confirming exact correspondence up to solver precision.

Figure 6: In these figures we evaluate an exact gradient of $\tilde{J}_i^\ell(\theta)$ using the finite difference method with a very small noise $\epsilon \in \{-0.00001, 0.00001\}$.

Additional experimental results are provided in the appendix, including the complete data underlying fig. 9, the evolution of offloading behavior in larger systems, comparisons with alternative baselines, and numerical evaluations of the theoretical results from lemma 1 using linear programming. Together, these supplementary experiments reinforce the conclusions drawn from the main results.

6 Conclusions

This paper introduced the DCC framework, a constraint-based approach to decentralized coordination in multi-agent reinforcement learning, and illustrated its potential in the context of task offloading at the wireless edge. Our focus here has been on laying the theoretical foundations and providing preliminary, proof-of-concept experiments to validate the core ideas. While the empirical evaluation is intentionally limited to toy models, the results suggest that constraint-driven implicit coordination can scale better than centralized methods and consistently outperform independent learners.

Future work includes extending the framework to support asynchronous updates, exploring richer forms of shared constraints, and conducting broader experiments to further validate the effectiveness of our approach. In this early work we abstract away the system-level implementation of constraint updates, but envision that in practice they could be coordinated through lightweight periodic broadcasts from edge servers or distributed consensus protocols among devices. Exploring these mechanisms remains a valuable direction for future work.

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A Theoretical proofs

A.1 Proof of Lemma 1

Proof.

$$\begin{aligned}
J(\pi, \beta) - \hat{J}(\pi, \beta) &= \mathbb{E}_{a_t \sim \pi} \left[\sum_t \gamma^t \sum_i r_i(s_t, a_t) \mid s_0 \sim \beta \right] - \mathbb{E}_{a_t \sim \pi} \left[\sum_t \gamma^t \sum_i \hat{r}_i(s_t, a_t) \mid s_0 \sim \beta \right] \\
&= \mathbb{E}_{a_t \sim \pi} \left[\sum_t \gamma^t \sum_i \mathbb{I}_{a_t, i = a_{crowd}} (d(N(a_t)) - d(1 + \theta_{-i})) \right] \\
&\leq \frac{1}{1 - \gamma} \left(\sum_i \theta_i \mathbb{E}_{a_t \sim \pi} [d(N(a_t)) - d(\mathbb{E}[N(t)]) \mid a_i = a_{crowd}] \right)
\end{aligned}$$

In the case with linear penalty function d , we can easily conclude that $R_\pi(\beta) = \hat{R}_\pi(\beta)$. For the nonlinear case, first we show that

$$\mathbb{E}_{a_t \sim \pi} [d(1 + \theta_{-i}) \mid a_i = a_{crowd}] = d(1 + \theta_{-i})$$

When d is convex, we can notice that

$$\begin{aligned}
\mathbb{E}_{a_t \sim \pi} [d(N(a_t)) \mid a_i = a_{crowd}] &\leq \left(1 - \frac{\mathbb{E}[N_t \mid a_i = a_{crowd}] - 1}{N_{agents} - 1} \right) d(1) + \\
&\quad + \frac{\mathbb{E}[N_t \mid a_i = a_{crowd}] - 1}{N_{agents} - 1} d(N_{agents}) \\
&= \frac{\mathbb{E}[N_t \mid a_i = a_{crowd}] - 1}{N_{agents} - 1} d(N_{agents}) \\
&= \frac{1 + \theta_{-i} - 1}{N_{agents} - 1} d(N_{agents}) \\
&= \frac{\theta_{-i}}{N_{agents} - 1} d(N_{agents}) \tag{17}
\end{aligned}$$

Note that $d(1) = 0$ due to its definition. This implies that, for convex penalty function d it is verified that

$$J(\pi, \beta) - \hat{J}(\pi, \beta) \leq \frac{1}{1 - \gamma} \sum_i \theta_i \left(\frac{\theta_{-i}}{N_{agents} - 1} d(N_{agents}) - d(1 + \theta_{-i}) \right)$$

Finally, for concave penalty function d , we can easily prove that

$$\mathbb{E}_{a_t \sim \pi} [d(N(a_t)) \mid a_i] \geq \frac{\theta_{-i}}{N_{agents} - 1} d(N_{agents})$$

and therefore conclude that, for every nonlinear penalty function d it is verified that

$$|J(\pi, \beta) - \hat{J}(\pi, \beta)| \leq \frac{1}{1 - \gamma} \sum_i \theta_i \left(\frac{\theta_{-i}}{N_{agents} - 1} d(N_{agents}) - d(1 + \theta_{-i}) \right) \tag{18}$$

□

A.2 Theoretical background for the proof of lemma 2

Before proving the main result, we first recall the relevant theoretical background, including the statement of Theorem 2. For clarity, we briefly review the envelope theorem, which applies broadly to optimization problems with parameterized constraints and objective functions defined over choice sets with arbitrary topology. A full treatment can be found in Section 3.5 of [20].

Consider the following maximization program with k parameterized inequality constraints:

$$\begin{aligned}
V(t) &= \sum_{x \in X: g(x, t) \geq 0} f(x, t), \quad \text{where } g : X \times [0, 1] \rightarrow \mathbb{R}^k \tag{19} \\
X^*(t) &= \{x \in X : g(x, t) \geq 0, f(x, t) = V(t)\}.
\end{aligned}$$

In the reference framework used by [20], X is a convex set, f and g are such that zero duality gap holds, and the Slater constraint qualification is verified at some $\hat{x} \in X$, i.e., $g_h(\hat{x}, t) > 0$ for all $h = 1, \dots, k$.

The set of saddle points of the Lagrangian over $(x, y) \in X \times \mathbb{R}_+^k$ at parameter value t takes the form $X^*(t) \times Y^*(t)$, where $X^*(t)$ is the set of solution to (19) and $Y^*(t)$ is the set of the solutions of the dual program

$$Y^*(t) = \arg \min_{y \in \mathbb{R}_+^k} \left(\sup_{x \in X} L(x, y, t) \right)$$

When the zero duality gap condition holds, the value $V(t)$ of the constrained maximization problem equals the saddle of the Lagrangian with parameter t , i.e., $V(t) = \min_{y \in \mathbb{R}_+^k} \max_{x \in X} L(x, y, t) = \max_{x \in X} \min_{y \in \mathbb{R}_+^k} L(x, y, t)$

The following result, which is denoted as Corollary 5 in [20], is crucial to prove the differentiability of (13).

Theorem 2. *Suppose that X is a convex compact set in a normed linear space, f and g are continuous in x , for the Lagrangian zero duality condition holds, $f_t(x, t)$ and $g_t(x, t)$ are continuous in (x, t) , and there exists $\hat{x} \in X$ such that $g(\hat{x}, t) \gg 0$ for all $t \in [0, 1]$. Then:*

1. V is absolutely continuous and for any selection $(x^*(t), y^*(t)) \in X^*(t) \times Y^*(t)$,

$$V(t) = V(0) + \int_0^t L_t(x^*(s), y^*(s), s) ds$$

2. V is directionally differentiable, and its directional derivatives equal:

$$\begin{aligned} V'(t+) &= \max_{x \in X^*(t)} \min_{y \in Y^*(t)} L_t(x, y, t) = \min_{y \in Y^*(t)} \max_{x \in X^*(t)} L_t(x, y, t) \quad \text{for } t < 1 \\ V'(t-) &= \min_{x \in X^*(t)} \max_{y \in Y^*(t)} L_t(x, y, t) = \max_{y \in Y^*(t)} \min_{x \in X^*(t)} L_t(x, y, t) \quad \text{for } t > 0 \end{aligned}$$

The result holds since optimal dual variables are proved bounded, and hence from Theorems 4 and 5 in [20] can be applied.

A.3 Proof of lemma 2

Proof. In order to prove the desired result for the function $\hat{J}_i(\theta)$ we need to use theorem 2 and make a distinction between two separate cases, $t = \theta_i$ and $t = \theta_j$.

We consider

$$V(t) = \min_{y \in \mathbb{R}_+^k} \max_{x \in X} L(x, y, t) = \max_{x \in X} \min_{y \in \mathbb{R}_+^k} L(x, y, t)$$

Next we define how all the quantities involved in the two different cases.

A.3.1 $t = \theta_i$

In this case, all the values of θ_j for $j \neq i$ are fixed and simply define the function f . Consider the following:

- $X = L^\gamma(\beta)$: this is the set of stationary distributions given an initial distribution β and the discount factor γ
- it is safe to assume the existence of a vector $\theta_{max} \in \mathbb{R}$ such that, $\theta_i < \theta_{max}$. This allows us to normalize the values of the parameter θ . A possible value of θ_{max} could be the cost obtained when always choosing the crowded action. In general, as long as the cost is a bounded function, this property is easily verified
- $f(x, t) = f_{\theta-i}(\rho) = \sum_{s,a} \rho_i(s, a) \hat{r}_i(s, a; \theta_{-i})$, with $\hat{r}_i : \mathcal{S} \times \mathcal{A} \times [0, 1]^{N-1} \rightarrow \mathbb{R}$ defined by the fixed parameters θ_{-i}
- $g(x, t) = g(\rho, \theta_i) = \sum_{s,a} \rho(s, a) c(s, a) - \frac{\theta_i}{\theta_{max}}$, with $c : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^N$

- the Lagrangian is explicitly written as

$$L_i(\rho, \lambda, \theta_i) = f_{\theta_{-i}}(\rho) + \lambda_i g(\rho, \theta_i)$$

assuming we are studying the differentiability wrt θ_i

Now we show that the hypotheses of Theorem 2 are verified.

X is a convex compact set in a normed linear space This choice of the space of the probability distributions, allows us to prove that X is a convex compact set in a normed linear space, as proved in Corollary 10.1 in [3].

f and g are continuous and concave in x The continuity and concavity in x of the functions f and g is an immediate consequence of the definition given above.

$f_t(x, t)$ and $g_t(x, t)$ are continuous in (x, t) The immediate reward function does not depend on θ_i , therefore $f_t(\cdot) = 0$. On the other hand, $g_t(\cdot) = \frac{1}{\theta_{max}}$

Existence of $\hat{x} \in X$ such that $g(\hat{x}, t) \gg 0$ for all $t \in [0, 1]$ The existence of a point $\hat{x} \in X$ such that $g(\hat{x}, t) \gg 0$ for all $t \in [0, 1]$ corresponds to the existence of a stationary distribution that strictly satisfies all constraints. This condition is met, for example, by a policy that never selects the crowded action—ensuring all constraints are strictly satisfied—provided that $\theta_i > 0$.

A.3.2 $t = \theta_j$

In this case, all the values of θ_k for $k \neq j$ are fixed; they will be considered fixed parameters in the definition of f and g . Consider the following:

- $X = L^\gamma(\beta)$ this is the set of stationary distributions
- it is safe to assume the existence of a vector $\theta_{max} \in \mathbb{R}$ such that, $\theta_i < \theta_{max}$. This allows us to normalize the values of the parameter θ . A possible value of θ_{max} could be the cost obtained when always choosing the crowded action. In general, as long as the cost is a bounded function, this property is easily verified
- $f(x, t) = f_{\theta_{-i,j}}(\rho, \theta_j) = \sum_{s,a} \rho_i(s, a) \hat{r}_i(s, a; \theta_{-i,j}, \theta_j)$, with $\theta_{-i,j} = \sum_{k \neq i,j} \theta_k$
- $g(x, t) = g_{\theta_i}(\rho) = \sum_{s,a} \rho(s, a) c(s, a) - \frac{\theta_i}{\theta_{max}}$, with $c : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}^N$
- the Lagrangian is explicitly written as

$$L_i(\rho, \lambda, \theta_j) = f_{\theta_{-i,j}}(\rho, \theta_j) + \lambda_i g_{\theta_i}(\rho)$$

assuming we are studying the differentiability wrt θ_i

To conclude our proof, it suffices to show that the hypotheses of Theorem 2 are verified for both choices of the parameter.

X is a convex compact set in a normed linear space This choice of the space of the probability distributions, allows us to prove that X is a convex compact set in a normed linear space, as proved in Corollary 10.1 in [3].

f and g are continuous and concave in x The continuity and concavity in x of the functions f and g is an immediate consequence of the definition given above.

$f_t(x, t)$ and $g_t(x, t)$ are continuous in (x, t) The continuity of $f_t(x, t)$ in (x, t) is an immediate consequence of the definition of the reward function r_i and the assumption that it is continuous with regard to θ_{-i} and therefore it is differential wrt θ_j , $\forall j \neq i$. On the other hand, $g_t(\cdot) = 0$

Existence of $\hat{x} \in X$ such that $g(\hat{x}, t) \gg 0$ for all $t \in [0, 1]$ The existence of a point $\hat{x} \in X$ such that $g(\hat{x}, t) \gg 0$ for all $t \in [0, 1]$ corresponds to the existence of a stationary distribution that strictly satisfies all constraints. This condition is met, for example, by a policy that never selects the crowded action—ensuring all constraints are strictly satisfied—provided that $\theta_j > 0$.

A.3.3 Conclusion of the proof

Thanks to what is showed in sections A.3.1 and A.3.2 we know that the objective function $V(t)$ is absolutely continuous with regard to t , for all choices of t . Consider now the function

$$\hat{J}_i^\rho(\theta) = \mathbb{E}_{s,a \sim \rho_i^*} [\hat{r}_i(s, a; \theta_{-i})]$$

where ρ_i^* depends on θ .

When we fix all coordinates θ_k for $k \neq j$, the mapping

$$\theta_j \mapsto \hat{J}_i^\rho(\theta)$$

coincides with $V(\theta_j)$, which is known to be absolutely continuous.

It then follows that $\hat{J}_i^\rho(\theta)$ is differentiable a.e.

Finally, note how we can use Theorem 3.3 in [3] to show how an optimal solution ρ^* for LP is such that the stationary policy $\pi(\rho^*)$ is optimal for the original constrained problem. This implies that, from the optimal stationary distribution we can also retrieve the optimal policy.

This allows us to conclude that the function $\hat{J}_i(\theta)$ is differentiable wrt every component of θ and therefore that theorem 1 is verified for the problem studied.

□

A.4 Proof of proposition 3

Proof. Theorem 2 states that V is directionally differentiable, and its directional derivatives equal:

$$\begin{aligned} V'(t+) &= \max_{x \in X^*(t)} \min_{y \in Y^*(t)} L_t(x, y, t) = \min_{y \in Y^*(t)} \max_{x \in X^*(t)} L_t(x, y, t) \quad \text{for } t < 1 \\ V'(t-) &= \min_{x \in X^*(t)} \max_{y \in Y^*(t)} L_t(x, y, t) = \max_{y \in Y^*(t)} \min_{x \in X^*(t)} L_t(x, y, t) \quad \text{for } t > 0 \end{aligned}$$

When considering the parameter $t = \theta_i$ and θ_j fixed for $j \neq i$, we know that

$$\begin{aligned} L(\rho, \lambda, \theta_i) &= f_{\theta_{-i}}(\rho) + \lambda_i g(\rho, \theta_i) \\ &= \sum_{s,a} \rho_i(s, a) \hat{r}_i(s, a; \theta_{-i,j}, \theta_j) + \lambda_i \left(\sum_{s,a} \rho(s, a) c(s, a) - \frac{\theta_i}{\theta_{max}} \right) \end{aligned}$$

It is easily verified that $\frac{\partial L(\theta_i)}{\partial \theta_i} = -\frac{\lambda}{\theta_{max}}$ and therefore when $\theta_i \in (0, \theta_{max})$ it is verified

$$\begin{aligned} V'(\theta_i+) &= -\max_{\rho \in L^\gamma(\beta)} \min_{\lambda \in \mathbb{R}} \frac{\lambda}{\theta_{max}} \\ V'(\theta_i-) &= -\min_{\rho \in L^\gamma(\beta)} \max_{\lambda \in \mathbb{R}} \frac{\lambda}{\theta_{max}} \end{aligned}$$

As noted in [20], this is a special case for which it is verified that

$$V'(t) = V'(\theta_i+) = V'(\theta_i-) = -\frac{\lambda^*}{\theta_{max}} \leq 0 \text{ a.e.}$$

This yields the corresponding result, with $M = \theta_{max}$.

On the other hand, when $t = \theta_j$, with $j \neq i$, we have

$$\begin{aligned} L(\rho, \lambda, \theta_j) &= f_{\theta_{-i,j}}(\rho, \theta_j) + \lambda_i g_{\theta_i}(\rho) \\ &= \sum_{s,a} \rho_i(s, a) \hat{r}_i(s, a; \theta_{-i,j}, \theta_j) + \lambda_i \left(\sum_{s,a} \rho(s, a) c(s, a) - \frac{\theta_i}{\theta_{max}} \right) \end{aligned}$$

It then follows that

$$\begin{aligned} \frac{\partial}{\partial \theta_j} L(\rho, \lambda, \theta_j) &= \frac{\partial}{\partial \theta_j} \sum_{s_i, a_i} \rho_i(s_i, a_i) \hat{r}_i(s_i, a_i) \\ &= \frac{\partial}{\partial \theta_j} \sum_{s_i, a_i} \rho(s_i, a_i) (u_i(s_i, a_i) + \mathbb{I}_{a=a_{crowd}}(a_i) d(1 + \theta_{-i})) \\ &= \frac{\partial}{\partial \theta_j} \sum_{s_i} \rho(s_i, a_{crowd}) d(1 + \theta_{-i}) \\ &= \sum_{s_i} \rho(s_i, a_{crowd}) \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \geq 0 \quad \forall \rho, \lambda \end{aligned}$$

where the final inequality follows from the assumption that the function d is strictly increasing, i.e. $d' > 0$. Moreover, not how the derivative is equal to 0 if and only if the action a_{crowd} is never chosen.

Therefore, we can further conclude that

$$\begin{aligned} \frac{\partial}{\partial \theta_j} \hat{V}(\theta_j) &= \max_{\rho} \min_{\lambda} \sum_{s_i} \rho(s_i, a_{crowd}) \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \\ &= \sum_{s_i} \rho^*(s_i, a_{crowd}) \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \\ &= \mathbb{P}(a_i = a_{crowd} \mid a_{i,t} \sim \pi_i^*) \cdot \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \\ &= \theta_i \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i}) \end{aligned}$$

where the optimal stationary distribution ρ^* is a function of the constraint vector θ . □

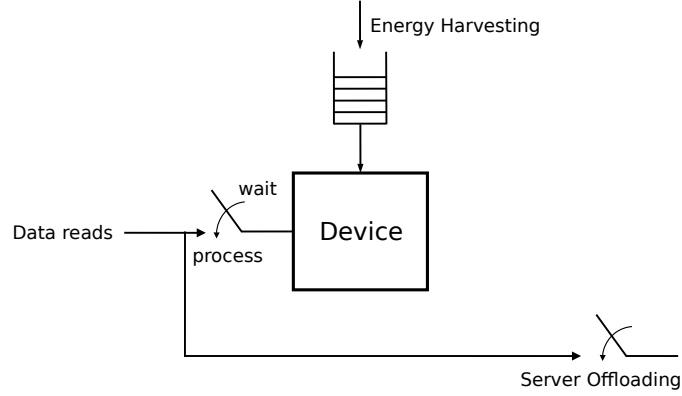


Figure 7: Device performing local processing of data batches with energy harvesting and offloading.

B Toy model considered

Description of the toy model The device-server scheme is represented in Fig. 7 for a single device: the device can read data batches, process them locally or offload them to the edge server. Thus, data batches are either read and processed locally on the device or read and offloaded. Time steps are discrete with index $t = 1, 2, \dots$. To process data on the device at time t , a certain amount of energy units, denoted as C_t , is required. C_t is modeled as a Markov chain. When the device offloads to the edge server, the data processing task is offloaded and it has zero energy cost for the device. However, sending and retrieving processed data from the server may take some time, which depends on how many devices have offloaded the data. This will result in a reward that accounts for the delay in receiving the processed data. The energy to transmit data to the server is assumed negligible compared to that spent for local data processing. The device is equipped with a battery of capacity $B > 0$ energy units. Additionally it can harvest a certain number of energy units H_t per timestep t . As for the processing cost, the energy fetched per timestep, namely the *harvesting rate*, is described by a Markov chain.

Description of the CMDP For each device we can introduce a Constrained Markov Decision Process (MDP) to model the system. The state of the device i at time t is denoted as $s_{i,t} = (x_{i,t}, e_{i,t})$, where $x_{i,t}$ represents the age of information (AoI), $e_{i,t}$ is the device battery level.

Note that $x_{i,t}$ is the AoI of the last data batch processed by a tagged device and the AoI is measured at the end of each time slot. It holds $x_{i,t} = 1$ when the device has just processed fresh data. Afterwards, the AoI increases of one timestep at every time slot t until either the device fetches a new data batch or the offloading occurs. In both cases, at the end of the data processing the AoI is reset to 1.

The freshness function, denoted as $u_i(x)$, represents the utility of processing a data batch x time units after the processing of the last batch. The function $u_i(\cdot)$ is bounded and non-increasing which means that beyond a certain value of AoI, any further processing delay would have minimal impact on the utility derived from the data. Consequently, it is assumed that there exists a positive constant M such that $u_i(M+k) = u_i(M)$ for any positive value of k , as it allows us to focus on the relevant time window $\{1, \dots, M\}$ where the utility function exhibits meaningful changes in value in relation to the freshness of the data.

Finally, it is possible that the battery available at timeslot t is not sufficient to terminate the computation immediately, namely, $e_t + H_t - C_t < 0$. In this case, a delay is incurred in order to harvest a sufficient amount of energy and complete the processing of the current batch. The corresponding timeslot has a random duration, due to the stochastic nature of energy harvesting.

The state space of agent i is denoted $\mathcal{S}_i = \{1, \dots, M\} \times \{0, \dots, B\}$.

The action set is $\mathcal{A}_i = \{\text{"read"}, \text{"local processing"}, \text{"offload"}\}$. The action taken by agent i at time t is denoted $a_{i,t}$. If $a_{i,t} = 1$, device i fetches a new data batch, which is processed at energy cost

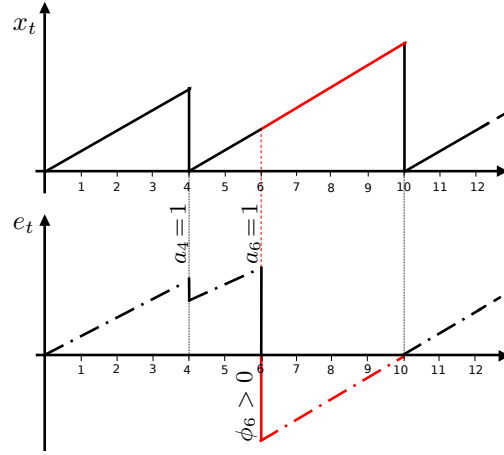


Figure 8: A sample path of the process $s_t = (x_t, e_t)$ for $H \equiv 1$ for a single agent: in red the vacation periods where the battery is empty. After the battery level gets negative at time $t = 6$ a total of 4 timesteps of recharging are needed to complete the execution of the task.

$C_t > 0$. Finally, the dynamics of the AoI for data batches is

$$x_{t+1} = \begin{cases} 1 & \text{if } a_t \in \{\text{"local processing"} \text{ and } e_{i,t} + H_{i,t} - C_{i,t} \geq 0, \text{"offloading"}\} \\ \min\{x_t + 1, M\} & \text{if } a_t \in \{\text{"read"}, \text{"local processing"} \text{ and } e_{i,t} + H_{i,t} - C_{i,t} < 0\} \end{cases}$$

The AoI at the renewal instants is also called *peak AoI* ([4]) in the literature.

As described in section 3 the reward function is composed of two components: a local utility function and a component which represents the congestion. For the local components, we consider the reward function which penalizes having negative battery level:

$$u_i(s_i, a_i) = \begin{cases} x_i - e_i & e_i < 0 \\ x_i & e_i \geq 0 \end{cases} \quad (20)$$

For the sake of simplicity, for this work we consider a simple function also for the penalty component which depends on the congestion. In particular,

$$d(n) = n^\alpha \quad (21)$$

with $\alpha \in \mathbb{R}$. Clearly, for $\alpha = 1$ we are considering the case with linear penalty.

Finally, the cost function is defined as

$$c_i(s_i, a_i) = \begin{cases} 1 & a_i = \text{"offloading"} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

C Additional numerical experiments

We introduce some additional experiments. Whenever reported, the reward is always normalized so that 1 corresponds to the discounted reward obtained by DCC-QL after 10^5 learning steps.

C.1 Evolution scalability comparison: additional data

We plot the reward evolution corresponding to the boxplot in section 5. This highlights why MAPPO results were excluded from the main figure to avoid distortion. The plot also shows that IQL converges quickly due to the small system size, while DCC-QL continues to improve over time as the constraint approaches its optimal value.

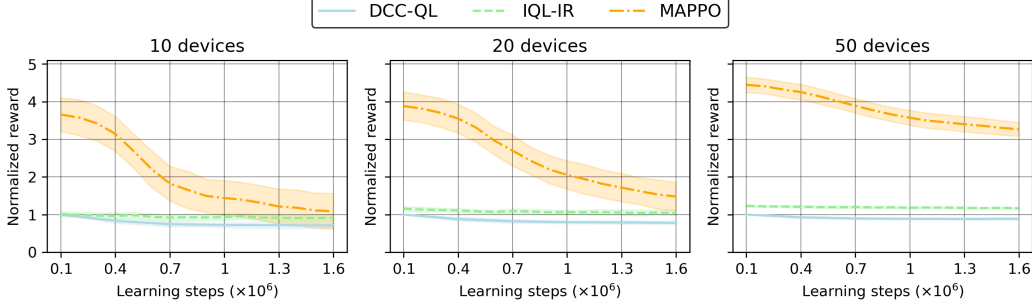


Figure 9: Evolution of the normalized reward over training episodes for different methods (corresponding to the boxplot in section 5).

C.2 Evolution offloading action: additional data

For completeness, we also plot the corresponding results in fig. 1b for $N = 20$ and $N = 50$.

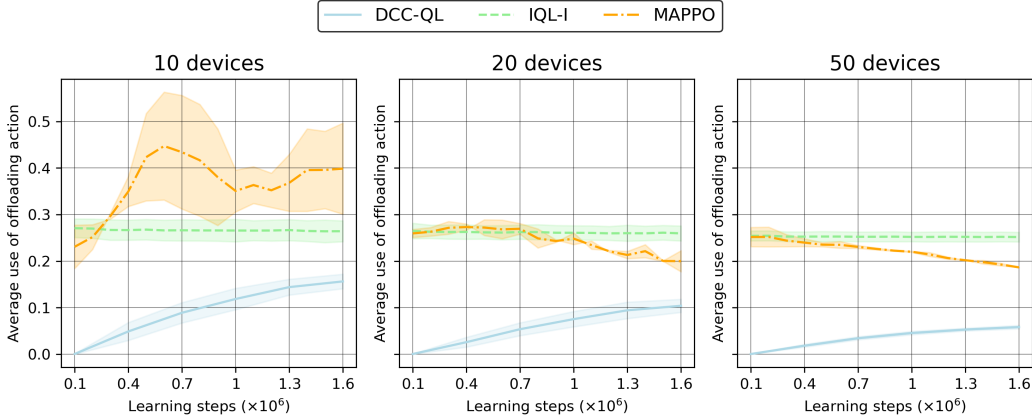


Figure 10: Evolution of the frequency of the offloading action in systems with 10, 20 and 50 devices.

C.3 Comparison with MAPPO over longer timeframes

We include this comparison in Figure 11 to illustrate how MAPPO improves with extended training. While MAPPO achieves lower rewards after 10^7 learning steps, this comes at the cost of substantially more samples. In contrast, DCC-QL makes better use of available data thanks to its Q-learning foundation, achieving strong performance with far fewer interactions. Even with extended training, DCC-QL remains consistently better across system sizes, highlighting its sample efficiency and robustness.

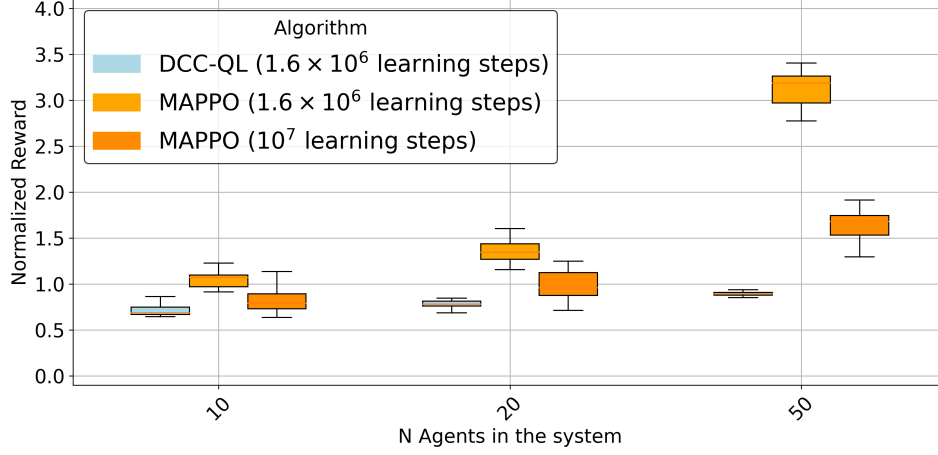


Figure 11: Comparison of DCC-QL with MAPPO after 1.6×10^6 learning steps (as in section 5) and after 10^7 learning steps.

C.4 Additional algorithms for section 5

For completeness, fig. 12 also reports results for IQL with a common reward and for a centralized baseline using A2C. Sharing a global reward signal can, in principle, improve coordination among agents by aligning their incentives and reducing the risk of selfish behavior. However, this comes at the cost of increased variance in the learning signal, and in practice the benefits are limited in our setting. The centralized A2C results are obtained for small system sizes using the Stable Baselines implementation [22] with standard hyperparameters. This corresponds to a single agent observing the entire system and making joint decisions for all devices. To simplify the setup, we did not explicitly restrict illegal actions (e.g., processing with negative energy levels), but instead imposed large penalties, which is generally a suboptimal approach. While this method performs reasonably well with 10 devices, it fails to scale: for 20 devices, the average rewards were already about 15 times higher than the normalization factor, making meaningful comparison impossible. Thus, centralized methods serve here primarily as conceptual baselines rather than practical solutions.

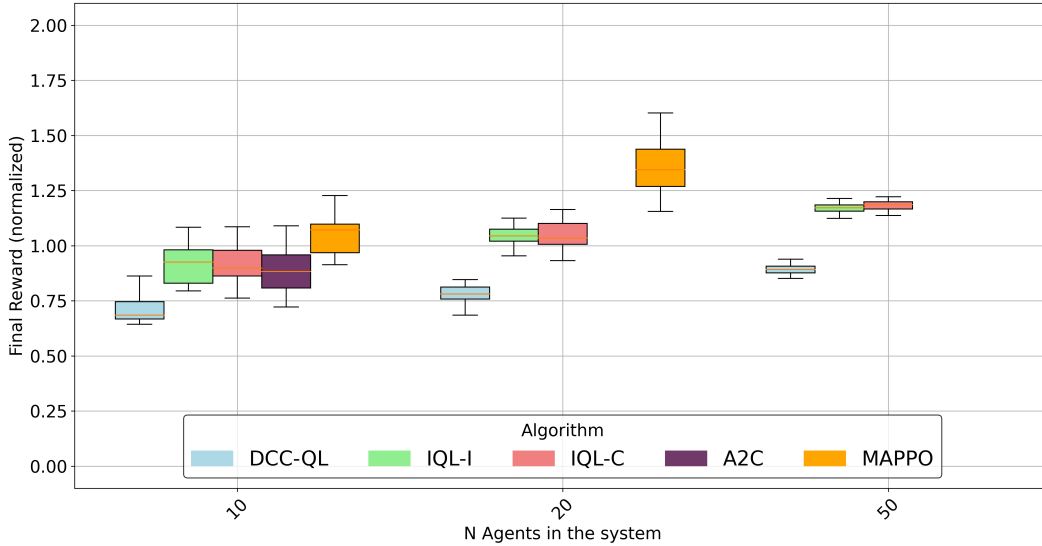


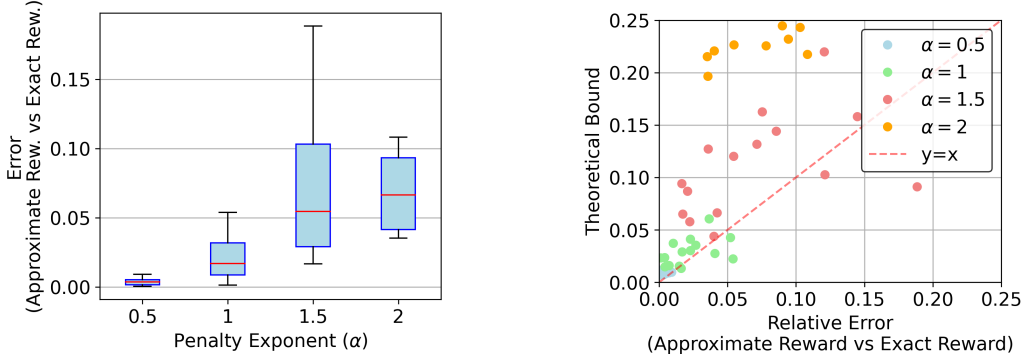
Figure 12: Performance comparison as in section 5, including IQL with common reward and centralized A2C (only for small system sizes).

C.5 Verification of the Approximate Reward Bound

In this experiment, we considered small environments with 10 devices, where the linear program (LP) can be applied reliably to compute the optimal policy. For each device, we compared the discounted reward obtained using the approximated reward definition in (6) with the corresponding component of the true global reward defined in (5). This comparison allows us to quantify the error introduced by the approximation and to assess its alignment with the theoretical analysis.

To this end, we measured the relative error between the two evaluations and compared it with the theoretical bound established in lemma 1. The results confirm that the observed error is consistently within the predicted range, up to small numerical deviations due to stochastic evaluation. Importantly, the special case $\alpha = 1$, where the penalty function is linear and the theoretical bound reduces to zero, was verified exactly in our experiments. For all nonlinear values of α , the empirical errors remained strictly below the bound, thereby validating the theoretical result in practice.

These findings provide additional evidence for the robustness of the approximation framework, showing that the theoretical guarantees extend to practical instances solved via the LP in small environments.



(a) Boxplot of the relative error as a function of the penalty exponent α

(b) Relative error compared with the theoretical bound from lemma 1, showing that the error remains within the predicted range in most cases.

Figure 13: Evaluation of the approximation error between the discounted reward computed with the exact reward and with the approximated reward.

D Extended Algorithm

The following pseudocode corresponds to the version of the algorithm that uses finite differences to compute the gradient of the objective function. This is the version used in the numerical experiments.

Algorithm 2 Three-Timescale Decentralized Constrained MARL

```

1: Initialize: Initial constraints  $\theta_i^0$ , multipliers  $\lambda_i^0$ , policies  $\pi_i^0$ 
2: function EVALUATE_OPTIMAL_CONSTRAINED_POLICY( $i, \theta_i, \theta_{-i}, n$ )
3:   for  $m = 0, 1, \dots, M(n)$  do ▷ Intermediate timescale —  $\lambda$  update
4:     for  $t = 0, 1, \dots, T(m, n)$  do ▷ Fastest timescale — policy update
5:       Collect trajectories using policy  $\pi_i^{t,m,n}$ 
6:       Compute total reward:  $r_i^{\text{total}}(s_i, a_i; \theta_{-i}) = \hat{r}_i(s_i, a_i; \theta_{-i}) + \lambda_i c_i(s_i, a_i)$ 
7:
8:       Update policy:  $\pi_i^{t+1,m,n} \leftarrow \text{RL step on } r_i^{\text{total}}$ 
9:     end for
10:     $\hat{K}_i \leftarrow K_i(\pi_i^{m,n})$  ▷ Estimate long term cost of current policy
11:     $\lambda_i^{m+1,n} \leftarrow \lambda_i^{m,n} + \alpha_k(\hat{K}_i - \theta_i)$ 
12:  end for
13:  return  $\hat{J}_i(\theta_i, \theta_{-i}) \leftarrow \text{evaluate local policy } \pi_i^{T(M(n),n),M(n)}$ 
14: end function
15:
16:
17: for  $n = 0, 1, 2, \dots$  do ▷ Slowest timescale — constraint update
18:   for each agent  $i = 1, \dots, N$  do
19:      $\hat{J}_i(\theta_i^n, \theta_{-i}^n) \leftarrow \text{EVALUATE\_OPTIMAL\_CONSTRAINED\_POLICY}(i, \theta_i^n, \theta_{-i}^n, n)$ 
20:   end for
21:    $\epsilon \leftarrow \text{RandomNoise}(0, \sigma)$ 
22:   for each agent  $i = 1, \dots, N$  do
23:      $\hat{J}_i(\theta_i^n + \epsilon_i, \theta_{-i}^n) \leftarrow \text{EVALUATE\_OPTIMAL\_CONSTRAINED\_POLICY}(i, \theta_i^n + \epsilon_i, \theta_{-i}^n, n)$ 
24:      $\hat{J}_i(\theta_i^n, \theta_{-i}^n + \epsilon_i) \leftarrow \text{EVALUATE\_OPTIMAL\_CONSTRAINED\_POLICY}(i, \theta_i^n, \theta_{-i}^n + \epsilon_i, n)$ 
25:   end for
26:   for each agent  $i$  do
27:      $\frac{\partial}{\partial \theta_i} \hat{J}_i(\theta_i^n, \theta_{-i}^n) \leftarrow \left( \hat{J}_i(\theta_i^n + \epsilon_i, \theta_{-i}^n) - \hat{J}_i(\theta_i^n, \theta_{-i}^n) \right) / \epsilon_i$ 
28:      $\frac{\partial}{\partial \theta_{-i}} \hat{J}_i(\theta_i^n, \theta_{-i}^n) \leftarrow \left( \hat{J}_i(\theta_i^n, \theta_{-i}^n + \epsilon_i) - \hat{J}_i(\theta_i^n, \theta_{-i}^n) \right) / \epsilon_i$ 
29:      $\nabla \frac{\partial}{\partial \theta_i} \hat{J}(\theta) = \frac{\partial \hat{J}_i(\theta)}{\partial \theta_i} + \sum_{j \neq i} \frac{\partial \hat{J}_j(\theta)}{\partial \theta_{-j}}$ 
30:      $\theta_i^{n+1} \leftarrow \theta_i^n + \eta_n \nabla_{\theta_i} \hat{J}_i(\theta_i^n)$ 
31:   end for
32: end for

```

E Extended Algorithm with exact evaluations of λ_i^*

The following pseudocode corresponds to the version of the algorithm that assumes access to the exact values of λ_i^* , obtained through the learning process, for computing the gradient.

Algorithm 3 DCC Framework

```

1: Initialize: Initial constraints  $\theta_i^0$ , multipliers  $\lambda_i^0$ , policies  $\pi_i^0$ 
2: function OPTIMIZELOCALCMDP( $i, \theta, n$ )
3:   for  $m = 0, 1, \dots, M(n)$  do ▷ Intermediate timescale —  $\lambda$  update
4:     for  $t = 0, 1, \dots, T(m, n)$  do ▷ Fastest timescale — policy update
5:       Collect trajectories using policy  $\pi_i^{t,m,n}$ 
6:       Compute total reward:  $r_i^{\text{total}}(s_i, a_i; \theta_{-i}) = \hat{r}_i(s_i, a_i; \theta_{-i}) + \lambda_i c_i(s_i, a_i)$ 
7:
8:       Update policy:  $\pi_i^{t+1,m,n} \leftarrow \text{RL step on } r_i^{\text{total}}$ 
9:     end for
10:     $\hat{K}_i \leftarrow K_i(\pi_i^{t,m,n})$  ▷ Estimate long term cost of current policy
11:     $\lambda_i^{m+1,n} \leftarrow \lambda_i^{m,n} + \alpha_k(\hat{K}_i - \theta_i)$ 
12:  end for
13:   $\hat{J}_i(\theta) \leftarrow \text{evaluate long term reward of policy } \pi_i^{T(M(n),n),M(n)}$ 
14:  return  $\hat{J}_i(\theta), \lambda_i^{M(n),n}, \pi_i^{T(M(n),n),M(n),n}$ 
15: end function
16:
17:
18: Compute  $\theta_i^{MAX}$  for each agent  $i$ :  $\theta_i^{MAX} \leftarrow K_i(\pi_i^{\text{always offload}})$ 
19: for  $n = 0, 1, 2, \dots$  do ▷ Slowest timescale — constraint update
20:   for each agent  $i = 1, \dots, N$  do
21:      $\hat{J}_i(\theta^n), \lambda_i^*, \pi_i^* \leftarrow \text{OPTIMIZELOCALCMDP}(i, \theta^n, n)$ 
22:   end for
23:   for each agent  $i$  do
24:      $\frac{\partial}{\partial \theta_i} \hat{J}_i(\theta^n) \leftarrow -\frac{\lambda_i^*}{\theta_i^{MAX}}$ 
25:     for each agent  $j \neq i$  do
26:        $\frac{\partial}{\partial \theta_j} \hat{J}_i(\theta^n) \leftarrow \theta_i \frac{\partial}{\partial \theta_j} d(1 + \theta_{-i})$ 
27:     end for
28:   end for
29:   for each agent  $i = 1, \dots, N$  do
30:      $\nabla \frac{\partial}{\partial \theta_i} \hat{J}(\theta) = \frac{\partial \hat{J}_i(\theta)}{\partial \theta_i} + \sum_{j \neq i} \frac{\partial \hat{J}_j(\theta)}{\partial \theta_i}$ 
31:      $\theta_i^{n+1} \leftarrow \theta_i^n + \eta_n \nabla_{\theta_i} \hat{J}_i(\theta_i^n)$ 
32:   end for
33: end for

```

F Numerical experiments hyperparameters

Parameters of the environment To keep a simple system, we considered both harvesting probability and processing cost probability, in an interval respectively $[min_H, max_H]$ and $[min_C, max_C]$. The following table describes the values among which we sampled the values for each experiment.

| Parameter | Sample set |
|-----------|----------------|
| M | 15 |
| B | 15 |
| min_H | $\{0, 1\}$ |
| min_H | $\{1, 2, 3\}$ |
| min_H | $\{1\}$ |
| max_C | $\{5, 7, 10\}$ |
| γ | 0.95 |

F.1 Learning parameters

Table 1 reports the learning parameters used in the experiments for IQL and DCC-QL . We tested several learning and exploration rates, but the values shown here yielded the best performance. Symbols in parentheses correspond to those used in algorithm 3. For DCC-QL , the parameters α_k , σ , and η_n denote the initial values of sequences that decay according to the convergence conditions of the algorithm; these are not applicable to IQL.

| Parameter | DCC-QL | IQL |
|--------------------------------------------------|--------|------|
| Learning rate Q-learning | 0.5 | 0.05 |
| Exploration rate Q-learning | 0.05 | 0.05 |
| Exploration decaly Q-learning | 0.95 | 0.95 |
| LM learning rate (α_k) | 1 | |
| Standard deviation constraint noise (σ) | 0.05 | |
| Constraint learning rate (η_n) | 0.25 | |

Table 1: Learning parameters for DCC-QL and IQL

For MAPPO, we used the implementation in [10] for the custom environment described in section B. l—c—

| | MAPPO |
|-----------------|-------|
| LR | 0.001 |
| NUM_ENVS | 8 |
| TOTAL_TIMESTEPS | 1.6e7 |
| UPDATE_EPOCHS | 2 |
| NUM_MINIBATCHES | 64 |
| GAMMA | 0.95 |
| GAE_LAMBDA | 0.95 |
| CLIP_EPS | 0.2 |
| ENT_COEF | 0.01 |
| VF_COEF | 0.5 |
| MAX_GRAD_NORM | 0.5 |
| ACTIVATION | relu |

Table 2: Learning parameters MAPPO