NFV Optimization Algorithm for Shortest Path and Service Function Assignment

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***Abstract*—Our paper focuses on the concept of Network Function Virtualization (NFV): the implementation of requests consisting of various service functions on servers located in data centers. This paper attempts to minimize both the cost of routing and service function assignment of requests from source to destination node on a network. This problem falls under the class of Integer Linear Programming (ILP) which is NP-Hard and cannot be solved in polynomial time. Towards developing a solution, we propose to split the problem into two separate optimization subproblems: shortest path routing and service function assignment. We utilize Dijkstra's Shortest Path algorithm and a Greedy method for service function assignment to propose a new heuristic algorithm that minimizes the total cost of routing and service functions assignment. We also analyze the run-time complexity of our proposed heuristic algorithm. Our experimental results suggest that our proposed algorithm matches the optimal ILP solution within acceptable limits.**

***Index Terms*—Network Function Virtualization, Integer Linear Programming, Dijkstra's Shortest Path Algorithm, Greedy Heuristic**

I. Introduction

In the past few years, the concept of network function virtualization (NFV) has gained momentum as an architectural choice for dramatically improving network efficiencies. NFV, at its core, involves the deployment of several network functions such as firewalls, deep packet inspection, CDN servers, etc. in the form of software on high volume shared servers in data centers. This concept allows for the programmability of network control, therefore, flexible and capable of adapting to the fluctuating throng of required functions while also greatly increasing the efficiency of the servers. The process of implementing virtual network functions onto a single server and allocating the necessary resources to carry it out is called service function chaining (SFC). SFC creates a chain of the different services to be carried out in an appropriate order, taking available resources, size, and other factors into consideration. This process may be automated to provide the fastest and most efficient execution of a set of assigned services, and this has been the focus of much of the previous work regarding the topic.

II. Our Contributions

Previous authors have utilized an ILP based solution in order to place virtual network functions into appropriate data centers. These authors have also provided heuristics and several approximation algorithms to solve this problem. However, unlike our proposed model, few of them have taken into consideration the routing cost from source to its destination while assigning and placing network functions.

Blenk et al. [1] delved into the placement of hypervisors, which serves as a layer between an SDN controller and networks, consisting of various necessary functions. The authors stated the importance of a "good placement" of these hypervisors. Amaya et al. [2] explored the specifics of the service chaining, creating a model that allows various orderings to be ranked and evaluated. They also derive a route from their ordering, and apply various constraints to the data centers. Ghasem et al. [3] explored the details of service chaining and the concept of network function virtualization, while Addis et. al [4] focused on the benefits of optimizing the route taken by function virtualization requests. Crichigno et. al [5] consider a routing and placement scheme like our work. However, they propose a different heuristic to solve the problem. Subsequent analysis in this paper suggests that our proposed algorithm has superior run time complexity as compared to Crichingo et. al [5] while providing similar to slightly improved results.

Our paper is unique in attempting to explore a solution toward minimizing cost by splitting the problem into two sub-problems: optimization of both network service function assignment among data centers and the routing cost. Since the problem is ILP, which is NP-Hard and cannot be solved in polynomial time, we search for a solution of lower complexity. Utilizing a Greedy method and Dijkstra's shortest path algorithm [6][7], we create a heuristic algorithm in order to minimize both costs. Our algorithm provides complete source to destination route for all requests along with the assignment of network functions requested. In our work, we also analyze the run-time complexity of our proposed heuristic algorithm.

III. Mathematical Formulation of the Problem

A. Model

We represent the network using a graph $G = (V, E)$, where V represents the set of nodes on the graph and E represents the links that connect each node. In this set V of nodes, we define derived subset $D \subseteq V$ as the set of data centers where each network function will be virtualized. Each network function is denoted by $f \in F$. A request $r \in R$ from source node src_r to destination node dst_r contains a group of n functions $f_{r,1}, f_{r,2}, \ldots, f_{r,n} \in F_r$ to be implemented by the network. For a pair of nodes $(i, j) \in E$, $c_{i,j}$ is the predetermined

cost of traversing that link, from node i to node j. $x^r_{i,j}$ is a binary variable (either 0 or 1) that denotes whether that link is traversed or not by request r. For each data center $d \in D$, W_d is the maximum capacity available on data center d to virtualize the network function. There are many types of resources that may be denoted by this variable, including storage, memory, and CPU cores. We denote the resource required to virtualize network function $f \in F$ from data center $d \in D$ as $w_{d,f}$, and the cost of virtualizing the network function on the data center is denoted $c_{d,f}$. We define $y^r_{d,f}$ as another binary variable, specifying whether or not a function in f is virtualized or not on a specific data center d on a certain request r.

B. Objective

First, we aim to minimize the total Service Function Assignment cost across the given request. That is, the sum, across all services $f \in F$ and data centers $d \in D$, of the cost of implementing a given function times by whether or not the service is implemented ($c^r_{d,f} \cdot y^r_{d,f}$).

$$\min \sum_{r \in R} \sum_{f \in F} \sum_{d \in D} c^r_{d,f} \cdot y^r_{d,f} \tag{1}$$

Next, we aim to minimize the routing cost for all requests $r \in R$, given as the sum across all pairs $(i,j) \in E$ of the product of associated cost and whether that link is traversed or not ($c_{i,j} \cdot x^r_{i,j}$).

$$\min \sum_{r \in R} \sum_{(i,j) \in E} c_{i,j} \cdot x^r_{i,j} \tag{2}$$

The overall objective function is to minimize both the service function assignment cost as well as total routing cost.

$$\min \sum_{r \in R} \sum_{(i,j) \in E} c_{i,j} \cdot x^r_{i,j} + \min \sum_{r \in R} \sum_{f \in F} \sum_{d \in D} c^r_{d,f} \cdot y^r_{d,f} \tag{3}$$

C. Constraints

We first define a decision variable $x^r_{i,j}$ as a binary variable to represent whether or not a link is traversed from node i to node j in a route r:

$$x^r_{i,j} = \begin{cases} 1 \text{ if traversed} \\ 0 \text{ else} \end{cases} \tag{4}$$

We then define another decision variable $y^r_{d,f}$ as binary to represent whether or not a service function is assigned to the data center d in a route r:

$$y^r_{d,f} = \begin{cases} 1 \text{ if assigned} \\ 0 \text{ else} \end{cases} \tag{5}$$

We add a constraint on the virtualization of each network function f for a route r, allowing each to be implemented only once on a data center.

$$\sum_{d \in D} y^r_{d,f} \leq 1, \forall r \in R, f \in F \tag{6}$$

Additionally, we must ensure the balance of inflow and outflow for each node. Thus, we create a flow constraint for each node traversed in a particular route. The source node must have an outflow of 1, while the destination has an outflow of -1. Every node in between must have a net flow of 0, where the inflow equals outflow.

$$\sum_{(i,j) \in E} x^r_{i,j} - \sum_{(i,j) \in E} x^r_{j,i} = \begin{cases} 1, \ i = src_r \\ -1, \ i = dst_r \\ 0, \ \text{otherwise} \end{cases} \tag{7}$$

Each data center has a maximum ability to implement functions, and we attempt to prevent possible overuse of a data center d by adding a capacity constraint to each d. This constraint prevents each data center from virtualizing functions whose total resource requirement exceeds the given capacity of the center.

$$\sum_{f \in F} w_{d,f} \cdot y^r_{d,f} \leq W_d, \forall r \in R, d \in D \tag{8}$$

We then must ensure that there is an inflow and outflow out of an assigned data center. Thus, we require the outflow at each data center to be greater or equal to the binary variable denoting its activation. A data center d that virtualizes a function f for route r, for example, will have a $y^r_{d,f}$ value of 1; this constraint requires its outflow to also be at least 1. A data center that is unused, meanwhile, may carry any outflow.

$$\sum_{j \in V} x^r_{i,j} \geq y^r_{d,f}, \forall i \in V, d \in D, r \in R \tag{9}$$

D. Proposed Solution

The above problem belongs to the class of Integer Linear Programming (ILP) problems which are NP-Hard problems that cannot be solved in polynomial time. Hence, the optimal ILP solution may not be practical to implement in real-world larger size network systems. In order to develop a heuristic solution to the problem, we propose splitting the overall problem into two separate optimization sub-problems: a) find the shortest route from a source to a destination node, and b) assignment of the network functions through appropriate data center from source to destination node. This novel approach leads us to propose a heuristic that will utilize Dijkstra's shortest path algorithm and a Greedy network function assignment algorithm to solve the problem.

First, we have an assignment problem, with an objective function given by Equation 1. We also utilize Constraint 5 governing data center use, Constraint 6 limiting service functions to be implemented just once, and Constraint 8 that applies the capacity constraint.

We can then model the remainder of the problem as a pure shortest path problem, using Dijkstra's algorithm to develop a solution. For this, we have an objective function given by Equation 2, and constraint equations 4 and 7.

Finally, we can tie the two problems together using Constraint 9, giving us a solution to the overall problem.

IV. Algorithm for Modified Routing and Greedy Assignment

Based on the above problem split, we propose an algorithm using Dijkstra's shortest path and a Greedy heuristic algorithm to assign network functions to solve the problem.

A. Proposed Modified Dijkstra's Algorithm with Greedy Service Function Assignment

Our proposed algorithm works on a graph $G = (V, E)$ and returns values for $x^r_{i,j}$ for each route that dictate the route taken by its network functions, as well as $y^r_{d,f}$ which denotes the data centers that are assigned to implement each network function. Our algorithm also assumes the use of an adjacency matrix to implement the network graph and allow us to easily find the neighbors of a node in the graph through lookup.

We begin by going through each of the requests in a set R. Using the source and the destination node of this request, we use Dijkstra's algorithm to discover the shortest path for the request, returning the list of nodes that make up the path as SP_r. From here, we create two separate lists: the first, PDC_r, will store data centers found on the shortest path given by Dijkstra's, while the second list, NDC_r, will store data centers that are neighbors along the shortest path. The shortest path is iterated through, and data centers found on the path are added to the first list. Using an adjacency matrix, we define a function $neighbors(n)$ to return all nodes that are linked to node n. Any data centers found among these neighbors are added to NDC_r.

Next, we provide network function assignment for the functions requested by r. For each, we use a Greedy heuristic algorithm in order to select a data center from all those detected by the previous scan, using the list comprised of the union of PDC_r and NDC_r. This is done by comparing the different costs of $c^r_{d,f}$ for each data center and ensuring that the necessary resources are available for implementation. Once the lowest cost data center d has been located, we set the corresponding value of $y^r_{d,f}$ to equal 1, signifying the use of that data center for the function. We then update the available resources of the data center by reducing its capacity of W_d by $w_{d,f}$. If the data center is on the path, the shortest path SP_r does not need to be updated. However, if the data center is found from the neighbors list, we insert a one-link detour to that node into SP_r. This process is repeated for every function in the request.

From there, we expect a majority of the network functions requested to have been assigned and virtualized at an appropriate data center. However, there is a possibility that the request still has unassigned functions, in which case we propose the use of a breadth-first search (BFS) to locate a capable data center. Once a data center is located by BFS, if it contains the necessary resources, we assign it the remaining functions and update its capacity, $y^r_{d,f}$, and SP_r accordingly. This is the worst-case scenario of the algorithm.

Once all functions have been assigned, we use SP_r to update the values of $x^r_{i,j}$ for the route, setting the variable to 1 when the link is traversed. After this process has been repeated for every request, the algorithm return $x^r_{i,j}$ and $y^r_{d,k}$, for all $r \in R$.

Algorithm 1 Modified Dijkstra's Algorithm with Greedy Service Function Assignment

Input: $G(V, E); c_{i,j} \forall (i,j) \in E$; $W_d \forall d \in D$; $w_{d,f}, c_{d,f} \forall d \in D$
Output: $y^r_{d,f}, x^r_{i,j} \forall r, d, f$
for all $r \in R$ **do**
  src_r = source node of request
  dst_r = destination node of request
  $SP_r = Dijkstra(src_r, dst_r)$
  initialize PDC_r (set of data centers found on shortest path SP_r) as ϕ
  initialize NDC_r (set of data centers found among path neighbors) as ϕ
  for node $n \in SP_r$ **do**
    if $n \in D$ **then**
      $PDC_r = PDC_r \cup \{n\}$
    end if
    for node $m \in neighbors(n)$ **do**
      if $m \in D$ **then**
        $NDC_r = NDC_r \cup \{m\}$
      end if
    end for
  end for
  for all $f \in F_r$ **do**
    Select $d \in PDC_r \cup NDC_r$, the data center that implements function f at lowest cost, contains necessary resources (Greedy Assignment)
    set $y^r_{d,f} = 1$
    update resources for d appropriately
    if $d \in NDC_r$ **then**
      update SP_r, insert d
    end if
  end for
  if any $f \in F_r$ remains unimplemented **then**
    run breadth-first search (BFS) starting from the most connected node $\in SP_r$
    if data center d is found by BFS **then**
      implement $f \in F_r$ (provided d has necessary resources available)
      update $y^r_{d,f}$
      insert path to d into SP_r
    end if
  end if
  update all $x^r_{i,j}$ using SP_r
end for
return $x^r_{i,j}, y^r_{d,f}$ $\forall r, d, f$

B. Run Time Complexity of Proposed Heuristic Algorithm

Our proposed heuristic algorithm utilizes Dijkstra's shortest path algorithm and a Greedy method in order to calculate an optimal route for a set of requests R. The first component

of the algorithm uses the shortest path algorithm to find a path SP_r from the source node src_r to destination node dst_r. The second uses a Greedy method to find an optimal data center $d \in D$ along the previously discovered path for each service function $f \in F_r$. The Greedy method runs with a linear complexity, while Dijkstra's algorithm implemented with a binary heap will run in logarithmic time [7]. Additionally, the neighbor search is implemented as a simple table lookup due to adjacency matrix implementation of the network and is $O(|V|)$. Therefore, we may conclude that our total algorithm is dominated by the first stage Dijkstra's algorithm run-time complexity. The worst case running time of our implementation of Dijkstra's algorithm with a binary heap is given by $O(|E|log|V|)$, and we call the Dijkstra's once for each request R in a scenario. Our total run time for the proposed heuristic algorithm is $O(|R||E|log|V|)$.

Notably, as a comparison, the algorithm of Crichigno et al. [5] utilizes Dijkstra's algorithm twice, resulting in a worst-case run-time of $O(|R||D||E|log|V|)$, D is the total number of data-centers contained in the topology. Our algorithm is therefore able to run at a faster run time by a factor of $|D|$.

V. Experimental Results

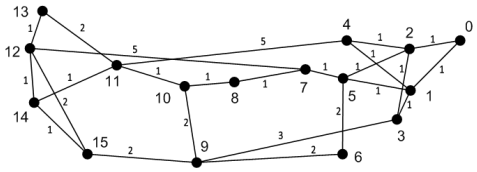
A. Simulation Setup

We tested our algorithm using two different network topologies. The first is the NSF-Net graph, forming an approximate outline of the United States of America and placing nodes at major cities, including Chicago, New York, Atlanta, and Los Angeles. The second is a fully connected hexagon graph with a central node connecting the 6 vertices. The graphs of the same topology differ from each other in network configuration and data center specifications. For example, Figure 1 features an NSF-Net graph, but with various link costs. Figure 2 features the same topology, however, each of its links have a cost of 1, the graph denotes different data centers.

For each topology, we generated 4 requests $r \in R$ consisting of group of functions $f \in F$ to be virtualized. These requests each had a different source and destination node found across the topology that the route must traverse between. For every request within each topology, we only changed the source and destination nodes (src_r, dst_r) of the route with respect to the various data centers. These costs of implementing the network function on the data centers were randomly distributed among a set of target values to provide a more reliable testing set. The testing results compares the costs of the routes given by the proposed heuristic algorithm and that of the optimal ILP solution. We graphed to compare the routing cost, assignment cost, and the more importantly the overall cost, given by the sum of the routing and assignment cost, for both the algorithmic and optimal solution.

B. Simulation Results and Discussion

The first test involved the NSF-Net 1 with routing costs randomly generated from 1 through 5. Data centers were placed at Nodes 12, 8, 7, and 5, with implementation costs given by Figure 1. As shown in Figure 6, the algorithm was



	Data Center 14	Data Center 10	Data Center 5	Data Center 0
Function1	5	1	10	14
Function2	9	7	6	10
Function3	3	14	9	3

Fig. 1. NSF 1 Network Graph

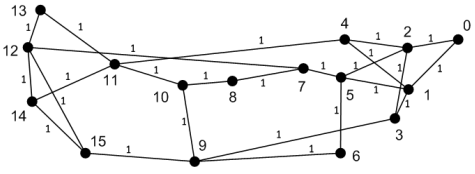
able to compute a solution that equals the optimal total cost in Route 1, and was within 23% over the optimal cost in the remaining 3 routes. Notably, in certain cases such as Route 3 in NSF-Net 1, the optimal ILP solution did not necessarily traverse the shortest path between source and destination nodes unlike in our proposed algorithm. Therefore, the results seem to suggest that the proposed algorithm returns a lower routing cost than the optimal algorithm. However as it should, the overall cost of the algorithm is higher than that of the optimal solution since the assignment costs are much lower for the optimal solution as compared to the proposed algorithm.

In the second test using the NSF-Net 2, we set each route link to 1 and increased the weights of the data center cost, simulating an assignment cost-dominated scenario. Data centers were moved to 14, 10, 5, and 0 as given by Figure 2. As shown in Figure 7, our algorithm matched the optimal net cost in two of the routes and was able to come closer to reaching the optimal overall cost, within 22%.

The third, fourth and fifth tests involved the fully connected hexagonal network as shown in Figures 3-5, adding successively more data centers in each test case. Results shown in Figures 8-10 suggest that our algorithm comes within acceptable range of 20%, 24% and very close 2.9% to the optimal ILP solution in each test case.

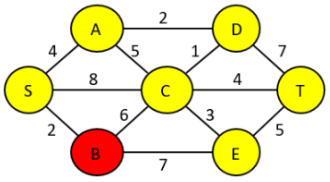
As a comparison with prior work, Crichingo et al.[5] performed their tests on a similar NSF-Net topology and suggested that their greedy heuristic has a gap of up to 30% and 33% from the optimal ILP solution for NF deployment cost and routing cost respectively. In comparison, as seen above, our results seem similar if not slightly superior with respect to gap from the optimal ILP solution, while offering a superior run-time performance as noted in the complexity analysis section.

Overall, we find that our proposed heuristic algorithm produces solutions over the five different configurations that matches the optimal ILP solutions within an acceptable range, while providing an efficient practical solution which is superior in running complexity from existing solutions with similar to slightly improved results. In our future work, we propose to expand the scope of our testing to validate the algorithm against larger network configurations, using more complex optimization packages such as Gurobi, CPLEX, etc.



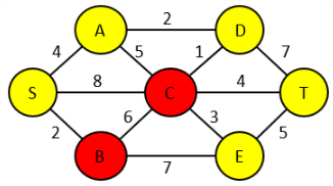
	Data Center 12	Data Center 8	Data Center 7	Data Center 5
Function1	12	8	7	5
Function2	6	5	1	15
Function3	6	10	4	7
Function4	6	3	6	8

Fig. 2. NSF 2 Network Graph



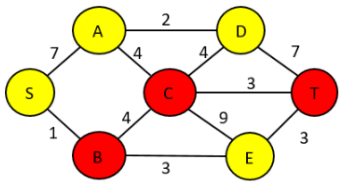
	Data Center B
Function1	3
Function2	6
Function3	1

Fig. 3. HEXAGON 1 Network Graph



	Data Center B	Data Center C
function1	3	6
function2	2	1
function3	4	4

Fig. 4. HEXAGON 2 Network Graph



	Data Center B	Data Center C	Data Center T
Function1	12	12	20
Function2	30	30	20
Function3	60	60	100
Function4	50	50	45

Fig. 5. HEXAGON 3 Network Graph

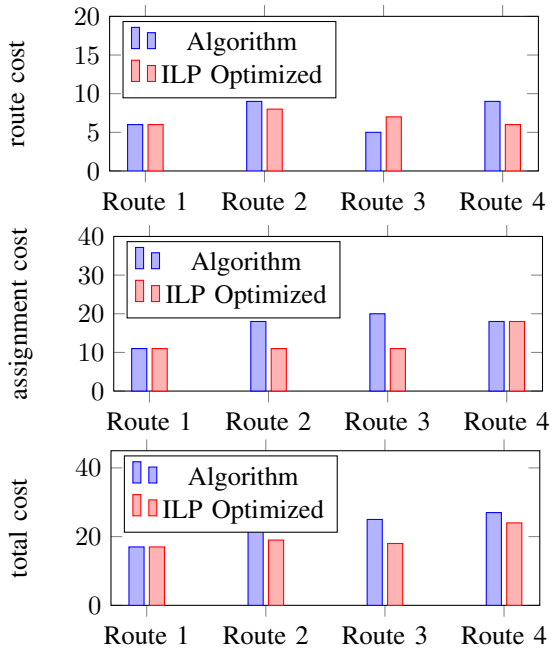


Fig. 6. NSF-Net 1 route, assignment and total cost comparison for Algorithm vs ILP Optimized

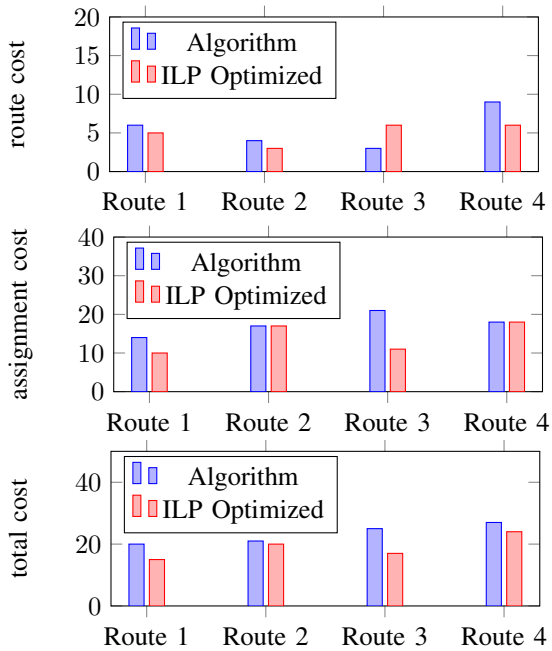


Fig. 7. NSF-Net 2 route, assignment and total cost comparison for Algorithm vs ILP Optimized

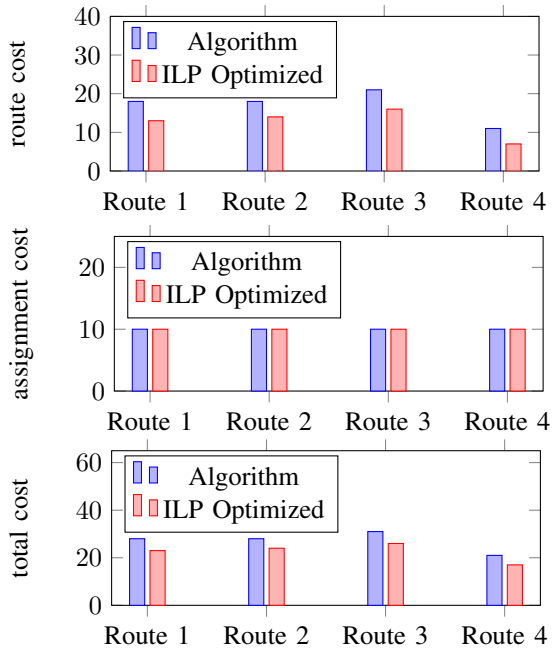


Fig. 8. HEXAGON 1 route, assignment and total cost comparison for Algorithm vs ILP Optimized

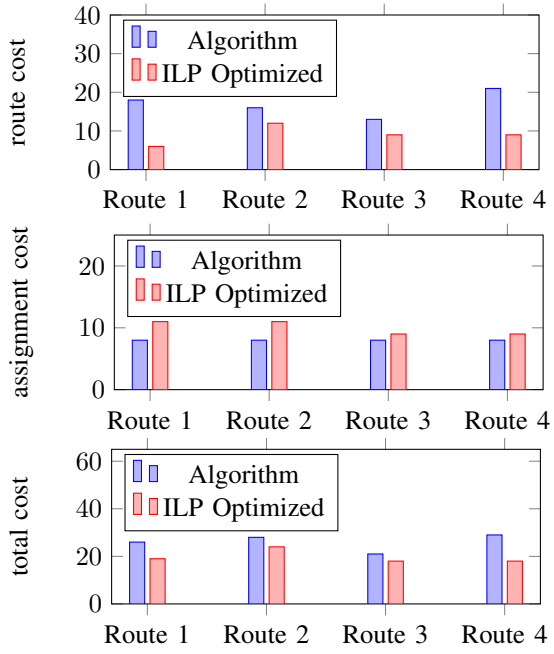


Fig. 9. HEXAGON 2 route, assignment and total cost comparison for Algorithm vs ILP Optimized

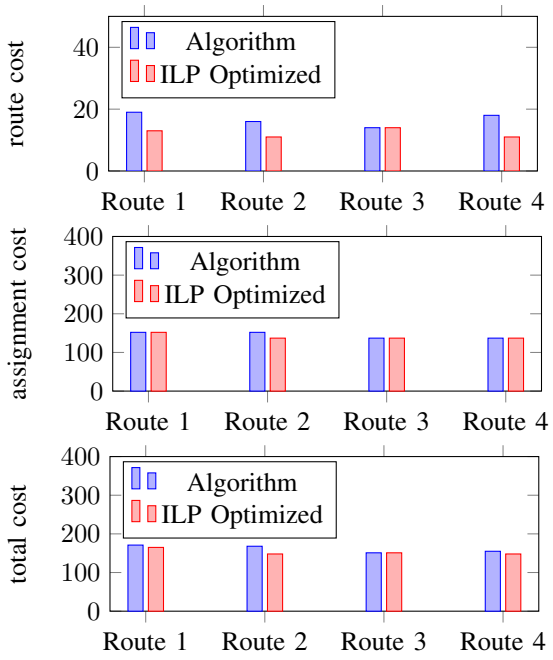


Fig. 10. HEXAGON 3 route, assignment and total cost comparison for Algorithm vs ILP Optimized

VI. Conclusions

We proposed an optimization problem of service function assignment and shortest path for network function virtualization. Since the problem as defined falls in the class of ILP problems, it is NP-Hard and cannot be solved in polynomial time. Therefore, we propose a solution that splits the problem into two separate optimization subproblems: shortest path and service function assignment. This split lends itself into our proposed heuristic, involving the combination of a modified Dijkstra's shortest path algorithm and Greedy heuristic algorithm for network function assignment. We also analyze the run-time complexity of our proposed heuristic algorithm. Experimental results over different network topologies suggest that our proposed algorithm matches the optimal ILP solution within acceptable limits.

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