

1 Introduction

This project investigates the use of different MDP and reinforcement learning algorithms to solve MDP problems.

1.1 MDP problems

Both of the MDP problems are variants of a maze problem. This is a rectangular board with multiple tiles, and from a certain tile s , the agent can choose to move up, down, left, or right. When choosing an action a from state s , the agent will move to some state s' with a transition probability $T(s, a, s')$. The goal is to find an optimal state-to-action mapping such that the expected reward is maximized over several states.

There are walls on the grid, and bumping into them produces a penalty of 50 (for the sake of this MDP problem, this can be interpreted as a gain of -50 reward). Thus, a way to think of this problem is that we want to maximize reward by minimizing the number of walls that are bumped into.

One of the problems is a maze on a 10 x 10 grid, and the other one is a maze on a 70 x 70 grid. For the purposes of these experiments, this maze problem is interesting, since the two-dimensional state space allows the optimal action from each state to be readable to the human eye. Thus, the arrangement of the states in gridworld allows one to effectively tell how well one algorithm is doing compared to the other just by eyeballing the actions at every state.

Many well-known MDP's like GridWorld involve maximizing some positive amount of expected reward, but it is interesting that reinforcement can alternatively be applied for cost-minimization problems (in this case minimizing the number of walls that are bumped into).

1.2 Experiments

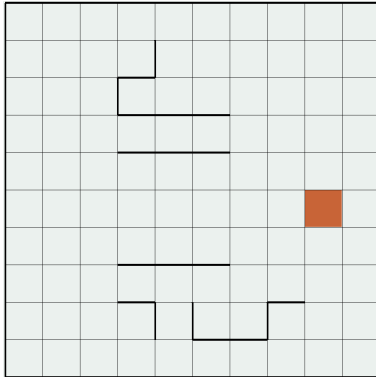
The actions in these experiments have some stochasticity, given a state and action, there is a 70 percent chance that the agent will travel to the intended state and a 30 percent chance that the agent will travel to another random state. In the RL-MDP simulation, this feature is controlled by the value of the PJOG parameter, which is set to .3 (which means there is a $1 - .3 = .7$ chance of the agent going in the intended direction).

2 Results and Analysis

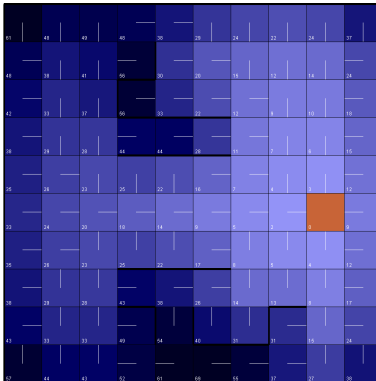
Below are some screenshots of the running of the various experiments. At each tile, there is a line segment, with one endpoint at the center of the tile and the other endpoint in another direction. The direction of each segment at each tile represents the optimal action to take from that particular state. Also, there is a number on each tile, which is proportional to the expected utility that would occur if the particular policy is chosen.

2.1 Maze: Fewer States

Here is the image of the particular maze that was used for the experiment with fewer states:

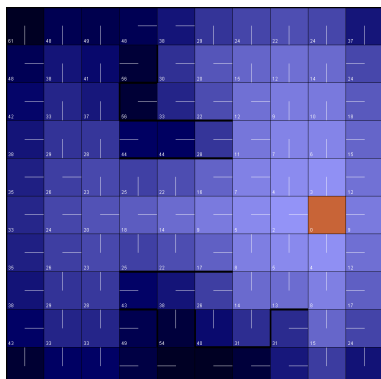


2.1.1 Value iteration



With a precision parameter of 0.001, $PJOG = 0.3$, and wall penalty of 50, this algorithm took 61 steps to converge, taking a total of 69 ms.

2.1.2 Policy Iteration



The iter. limit parameter (the maximum number of times for the policy to be evaluated) was set to 10, and the value limit (the maximum number of value updates within each policy iteration) was set to 5000. The algorithm converged within 6 policy evaluation iterations, taking a total of 19 ms.

Here, policy evaluation worked more quickly than value iteration and still gave similar result on the maze tile values. Of course, each policy iteration took much longer than each value iteration, since the number of steps in the algorithm was equal to $S^2 * 10$, but there were much fewer iterations in the policy iteration algorithm.

For each state in each value iteration, the algorithm needed to consider what the expected utility by moving up, down, left, or right. This was unlike what happened in policy iteration, where the algorithm only considered the result of one action, allowing for fewer computations.

Also, each value iteration involved observing what happens when moving either direction only once, while policy iteration involved observing what happens when moving in a fixed direction several times. This made the value iteration algorithm more prone to noise from taking each action from a state, since each action is taken only once; since policy iteration takes that fixed action ten times from that state, it is able to disregard stochastic irregularities for a particular action.

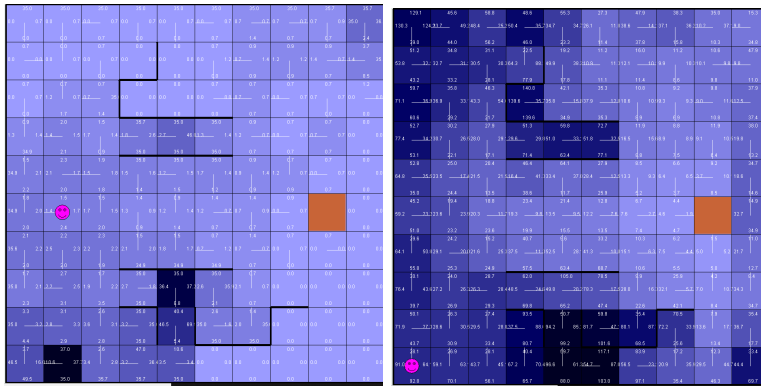
The policy iteration experiment discussed here mentions a 53 ms run that is caused by 10 inner iterations in policy evaluation. However, this can be sped up even more by decreasing the number of inner iterations. The total number of policy iterations doesn't increase here, since the expected number of tiles that random noise would affect is smaller with this small maze state.

The faster time for fewer inner policy iterations and the fixed number of total policy iterations can be confirmed by the table below (which measures policy iterations and time for varying numbers of inner iterations):

Inner iterations	Policy iterations	Time
1	6	4ms
2	8	8ms
5	8	12ms
10	6	19ms

2.1.3 RL

On this MDP, Q-learning was done for 1000 episodes. The learning rate was set to 0.7, and it was made such that it decays.



There were much more iterations done for Q-learning than for the other two algorithms, but the policy is less clear (as can be seen as the values here are not as dark, implying that the Q-learning agent has been trained much less). For example, consider the block that is at the fourth column from the left and on the second row from the top. This block is more lightly shaded, implying that the RL algorithm has less confidence about what the expected reward would be starting from that state. RL Sim did not include an option to run the algorithm until convergence, but it is obvious because of this difference in tile shadings that this Q-learning algorithm did not converge even after 1000 entire iterations.

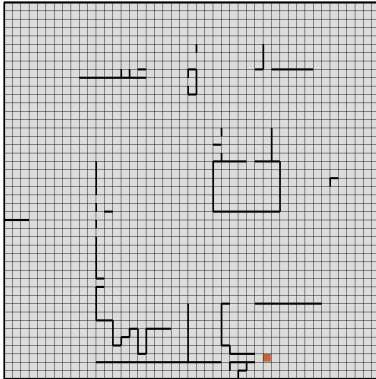
The value iteration algorithm had only taken 61 steps to converge, each step which took $O(s^2)$ time. Each of the 1000 steps in the Q-learning algorithm took $O(s)$ time (a week lower bound on the time, since the agent initially travels much more since it is new to the environment).

It makes sense that the Q-learning algorithm would take much longer to converge. Reinforcement learning algorithms have to undergo an exploration/exploitation tradeoff, which is not present in the MDP algorithms, since the MDP algorithms already have perfect information about the rewards and the transition functions. Thus, the reinforcement learning algorithms ended up taking a significantly longer time than the MDP algorithms. Even in the 10 x 10 maze, where the state

space is quite small, there are still 100 total states and almost 400 possible actions, which the agent needs to try one at a time in order to achieve the information that an MDP has.

2.2 Maze: More States

Here is the image of the particular maze used for a large number of states. The end state is represented by the orange square.



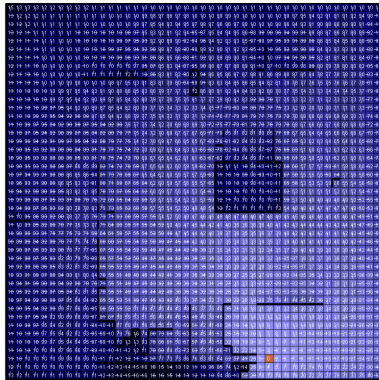
2.2.1 Value iteration

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
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64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
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10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38			

The image above is the final distribution of the values and policies

The value iteration converged after 181 steps, and the algorithm took 10,871 ms.

2.2.2 Policy Iteration



The policy iteration converged after 18 steps for 10 inner policy iterations, and the algorithm took 2,841 ms, which is much faster than the value iteration algorithm.

However, using a smaller number of value updates for policy iteration here produces a different result when compared to using a smaller number of updates in the small maze, as shown below:

Inner iterations	Policy iterations	Time
1	104	3516ms
2	55	2908ms
5	8	12ms
10	18	2841ms

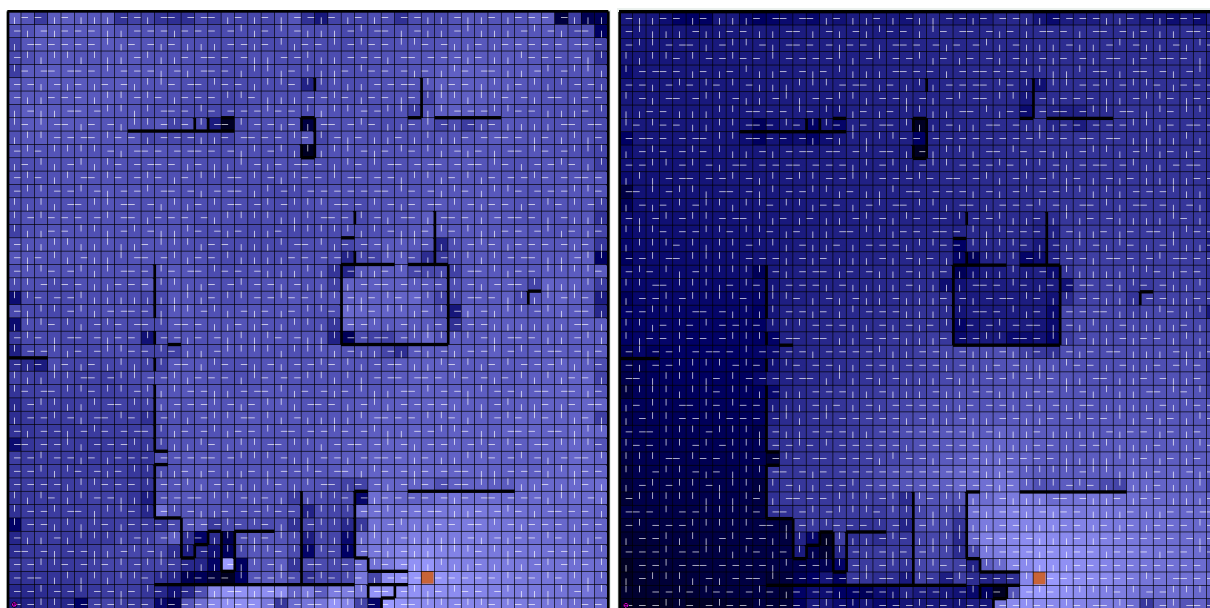
First of all, the number of policy iterations decrease as the number of inner iterations increases. This makes sense since there exist several squares that are very distant (at least 5 squares away) from any walls. Thus, for fewer inner iterations like one iteration, it takes multiple value updates for the information that a wall exists to propagate to certain squares; thus, more policy iterations are needed for fewer inner iterations.

Also, the algorithm takes more time to run with fewer inner iterations, unlike what occurred with a smaller maze. Even though each policy iteration is faster due to fewer inner iterations, there are so many policy iterations that the total time is ultimately increased.

Another reason for the longer runtime with fewer inner iterations is that there is a very large number of tiles, so more tiles are expected to be sensitive to noise due to action stochasticity. As explained

previously, having fewer value updates causes certain policy estimates to be vulnerable to noise, so there would be a delay in the runtime of the algorithm.

2.2.3 RL



The above images show the direction of the policy, and the shading of each tile represents the value at that tile. The image on the left shows the algorithm after 20 episodes, and the one on the right shows the completed algorithm after running for 1000 episodes.

One trend that is noticeable is that the Q-learning algorithm has produced very accurate policies at the bottom left corner of the grid. Unlike in value iteration and policy iteration, the value tiles at the bottom left corner of the grid are very dark, indicating a strong confidence of the algorithm about what needs to be done from the states of that region to maximize the reward.

However, even after 1000 cycles (which involves significantly more computations than policy iteration and value iteration, for similar reasoning as was mentioned in the discussion of the Q-learning algorithm for the smaller maze), the policies at the top right corner of the grid are very inconsistent with each other. Several of the tiles have policies that indicate that the optimal action is upward, but this is obviously not true.

3 Sources

[1] <https://www.cs.cmu.edu/~awm/rlsim/>