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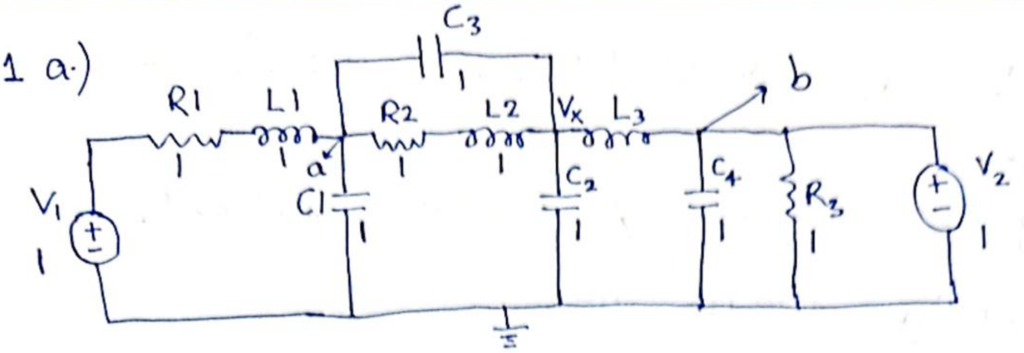
Q1 a.)



All elements are 1

i.e $R_1 = R_2 = R_3 = 1\,\Omega$

$L_1 = L_2 = L_3 = 1H$

$C_1 = C_2 = C_3 = C_4 = 1F$

$\therefore$ MNA Equations

$$G = \begin{bmatrix} G_1+G_2 & -G_2 & 0 \\ -G_2 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1+C_3 & -C_3 & 0 \\ -C_3 & C_2+C_3 & 0 \\ 0 & 0 & C_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} L_1+L_2 & -L_2 & 0 \\ -L_2 & L_2+L_3 & -L_3 \\ 0 & -L_3 & L_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} V_a \\ V_x \\ V_b \end{bmatrix} \quad \& \quad I_s = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\left(G+sC+\frac{1}{s}L^{-1}\right)V = I_s \Rightarrow V = \left(I + sG^{-1}C + \frac{1}{s}G^{-1}L^{-1}\right)G^{-1} I_s}$$

Now we replace inductors by resistors $R_4, R_5, R_6 = 2\,\Omega$

The corresponding matrix are

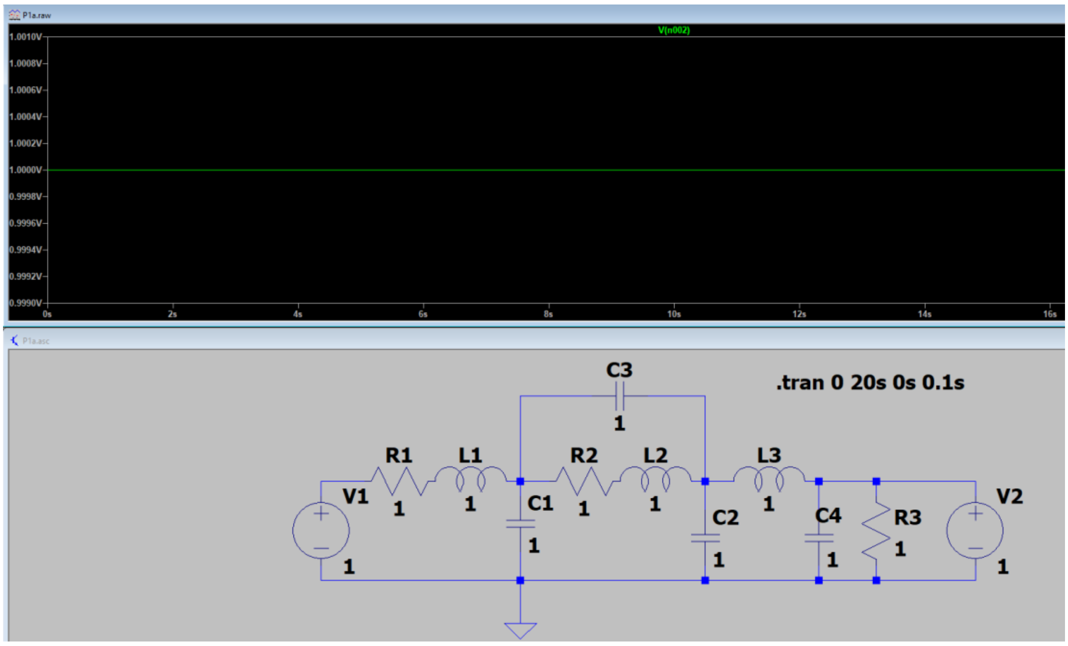
$$G = \begin{bmatrix} \frac{1}{3}+\frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{1}{3}+\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 1+\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2/3 & -\frac{1}{3} & 0 \\ -\frac{1}{3} & 5/6 & -1/2 \\ 0 & -1/2 & 3/2 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1+C_3 & -C_3 & 0 \\ -C_3 & C_2+C_3 & 0 \\ 0 & 0 & C_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_s = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad V = \begin{bmatrix} V_a \\ V_x \\ V_b \end{bmatrix}$$

$$\therefore\ (G+sC)V = I_s$$

$$\left(\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & 5/6 & -1/2 \\ 0 & -1/2 & 3/2 \end{bmatrix} + s\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} V_a \\ V_x \\ V_b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

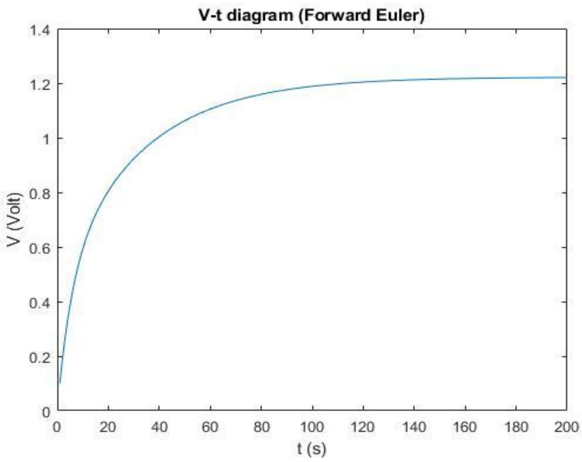


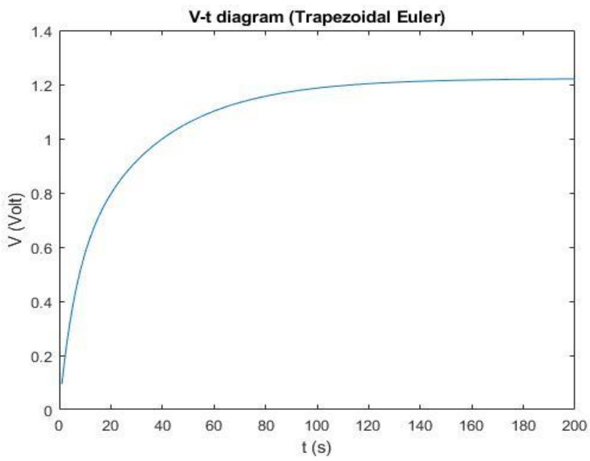
LTSPICE simulation of Part1 a); the files can be found in P1a.asc corresponding simulation output in P1a.raw

Q1) Part b, c, d

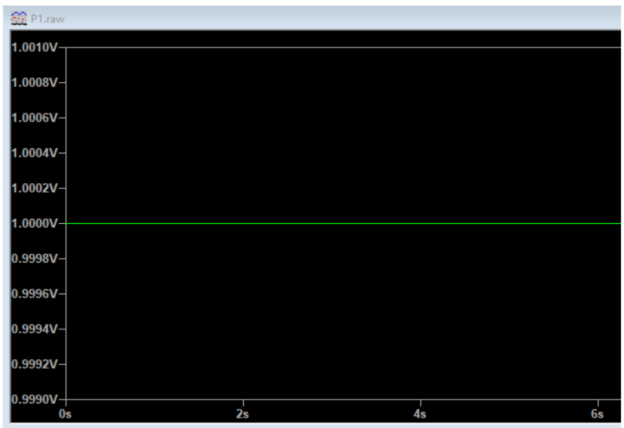
Please refer to P1.m for Forward Euler and backward Euler (parts b and c)

For part d, Ltspice simulation can be found in P1.asc and the corresponding simulation output in P1.raw





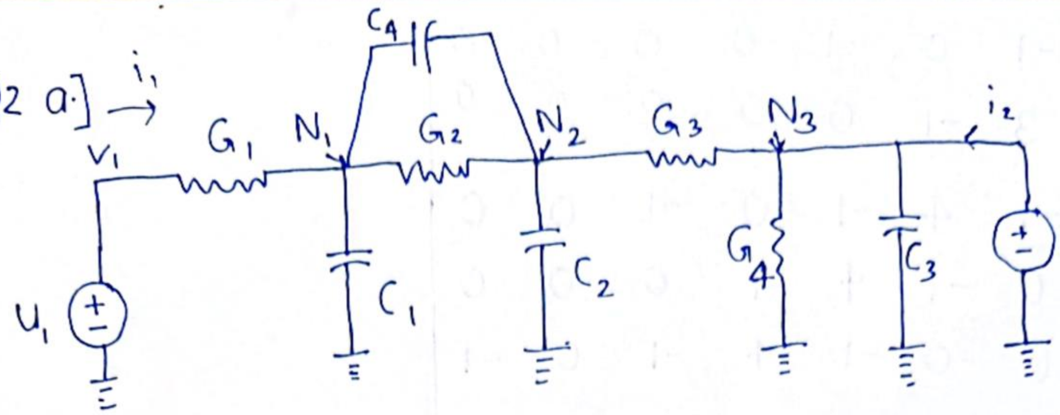
Ltspice simulation of Part 1 d



It can be seen that the simulation result of forward Euler and trapezoidal are identical but the steady voltage when compared with Ltspice does not match exactly. The possible reasons I could think of are as follows:

1) Since the given Voltage source is DC and we are doing a transient analysis without any specifications of the Voltage source.
2) The values we assume (for example C = 1 Farad and L = 1 Henry) are too large.

Q2 a]



MNA for 3 nodes N_1, N_2, N_3

$$G = \begin{bmatrix} G_1+G_2 & -G_2 & 0 \\ -G_2 & G_2+G_3 & -G_3 \\ 0 & -G_3 & G_3+G_4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

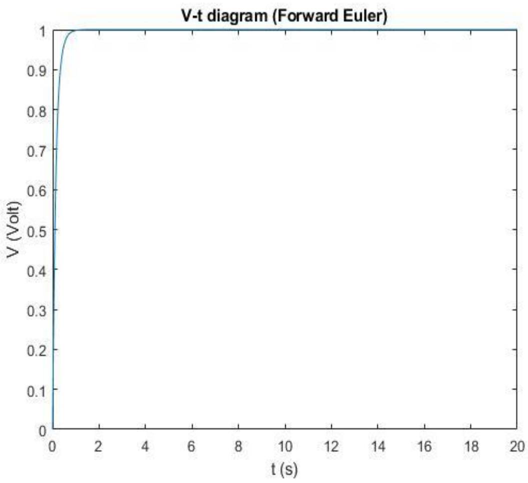
$$C = \begin{bmatrix} C_1+C_4 & -C_4 & 0 \\ -C_4 & C_2+G_4 & 0 \\ 0 & 0 & C_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

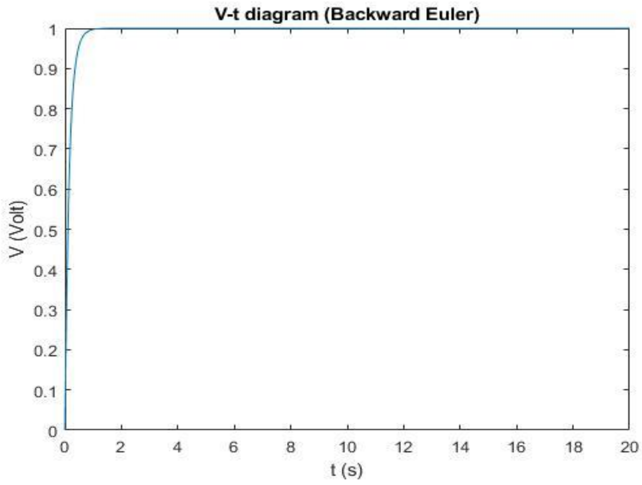
$$I = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad V = \begin{bmatrix} V_{N_1} \\ V_{N_2} \\ V_{N_3} \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

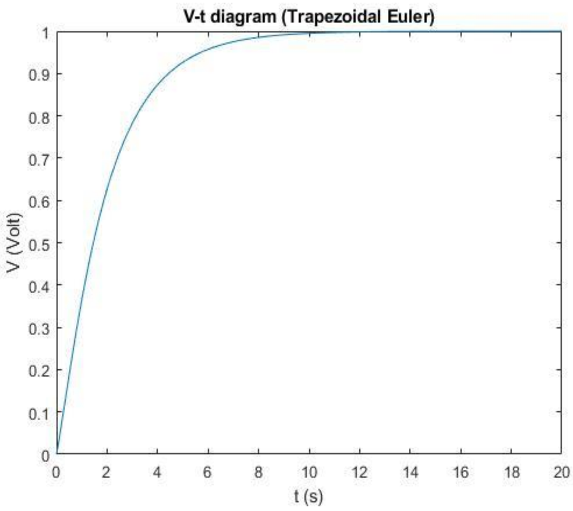
$$\boxed{\therefore \text{MNA is } (G+sC)V = I_s}$$

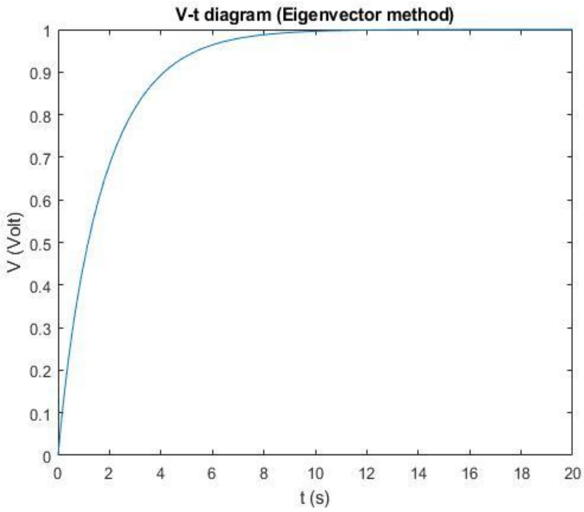
$$\left(\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} + s\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

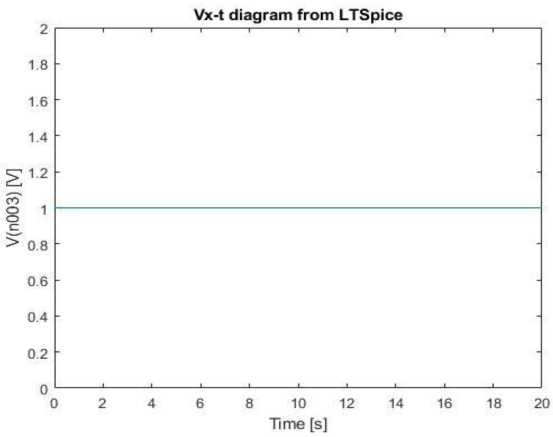
Q2) Parts b,c,d











As we can see these are the plots of all 5 simulations and they are as expected. The slight variation in the initial time can be explained as the transient state trying to reach the steady state. I observed that if we observe for longer duration say t= 50 seconds, then the plot is almost constant as expected.

These plots are saved in Q2/ folder with their names along with LTSpice simulation Q2d.asc and their waveforms

To run my Matlab code simply run P2.m. Inside that I also call the functions simulatemodel.m and Ltspice2Matlab.m

THEME NO. DATE :

Q3.] By definition, $A = LU$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = a_{11}$, $u_{12} = a_{12}$, $u_{13} = a_{13}$

$l_{21} = \frac{a_{21}}{u_{11}}$ $\quad u_{22} = a_{22} - l_{21}u_{12}$ $\quad u_{23} = a_{23} - l_{21}u_{13}$

$l_{31} = \frac{a_{31}}{u_{11}}$ $\quad l_{32} = (a_{32} - l_{31}u_{12})/u_{22}$ $\quad u_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23}$

Generalize ⇒

$$u_{ij} = \begin{cases} a_{ij} - \sum_{k=1}^{i-1} l_{ik}u_{kj}, & \text{if } i \le j \quad (u_{ij} = a_{ij} \text{ when } i = 1) \\ 0, & \text{otherwise} \end{cases}$$

$$l_{ij} = \begin{cases} (a_{ij} - \sum_{k=1}^{j-1} l_{ik}u_{kj})/u_{jj}, & \text{if } i > j \\ 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

(I) In the j^{th} iteration of Gaussian Elimination, we subtract each row below with the j^{th} row $\times m$ to make their i^{th} column value $= 0$. Therefore this m should be $\frac{a_{ij}}{a_{jj}}$, for $i = j+1 : n$. Note that a_{ij} is not the original a_{ij}, but the one after previous iterative subtractions. Define it as $u_{ij} = a_{ij}'$, which is exactly the same as our definition:

$$u_{ij} = a_{ij} - \underbrace{\sum_{k=1}^{i-1} l_{ik}u_{kj}}_{\text{subtraction in the previous elim.}}, \quad i \le j$$

THEME

$\therefore$ It's obvious that $m = \frac{a_{ij}'}{a_{jj}'} = \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj}\right) / u_{jj}$, $i > j$

which is exactly the same as our derivation of l_{ij}

(II) Solving $Ax = b$ with LU decomposition: $Ly = b$, $Ux = y$
we want to show that y is the vector b after Gaussian Elimination
$Ly = b \Rightarrow y_1 = b_1$, $l_{21} y_1 + y_2 = b_2$, $l_{31} y_1 + l_{32} y_2 + y_3 = b_2$
$\Rightarrow y_1 = b_1$, $y_i = b_i - \sum_{k=1}^{i-1} l_{ik} y_k$, $i = 2:n$

In the k^{th} iteration of Gaussian elimination, we also subtract b_i by $l_{ik} y_k$ for $i = k+1:n$. ~~After all iterations, vector b would be $b_i - \sum_{k=1}^{i-1} l_{ik} y_k$ for $i = 2:n$~~

After all iterations, vector b would be $b_i - \sum_{k=1}^{i-1} l_{ik} y_k$ for $i = 2:n$
and b_1 for $i = 1$, which is exactly the same as the vector y derived from $Ly = b$

(III) With the conclusions in I, II, it's obvious that $Ux = y$ is the same as the result of Gaussian Elimination (waiting for backward substitution to solve x)

$\therefore$ (I), (II), (III) imply LU Decomposition & Gaussian Elimination are equivalent.

Q4 a)

$A = LDL^T$ [Cholesky Decomposition]

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ l_{21} & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ l_{n1} & \cdots & l_{n-1} & 1 \end{bmatrix} \begin{bmatrix} d_{11} & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & d_{nn} \end{bmatrix} \begin{bmatrix} 1 & l_{21} & \cdots & l_{n1} \\ \vdots & 1 & l_{32} & \\ \vdots & & \ddots & l_{n\,n-1} \\ 0 & \cdots & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & 0 \\ l_{21} & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ l_{n1} & \cdots & l_{n\,n-1} & 1 \end{bmatrix} \begin{bmatrix} d_{11} & d_{11} l_{21} & \cdots & d_{11} l_{n1} \\ & d_{22} & d_{22} l_{32} & \\ & & & \\ & & & \end{bmatrix}$$

$a_{11} = d_{11} \qquad a_{21} = l_{21} d_{11} \qquad a_{31} = l_{31} d_{11}$

$a_{22} = l_{21} d_{11} + d_{22} \qquad a_{32} = l_{21} d_{11} l_{31} + d_{22} l_{32}$

$a_{nn} = \sum_{k=1}^{n-1} l_{nk} d_{kk} + d_{nn}, \quad a_{ij} = \sum_{k=1}^{j-1} l_{ik} d_{kk} l_{jk} + l_{ij} d_{jj} \quad (i > j)$

$$\therefore d_{ij} = \begin{cases} a_{ij} - \sum_{k=1}^{i-1} l_{ik} d_{kk} & , \text{if } i = j \quad (d_{ij} = a_{ij} \text{ if } i = j = 1) \\ 0 & , \text{otherwise} \end{cases}$$

$$l_{ij} = \begin{cases} \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} d_{kk} l_{jk}\right) / d_{jj} & , \text{if } i > j \quad (l_{ij} = a_{ij} / d_{jj} \text{ if } j = 1) \\ 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Q4 b] Please refer LDLT.m

Q4b) The code can be found at LDLT.m

Please pass a matrix A to test it

Sample Output

Command Window

```
>> A = [ 1 2 3; 4 5 6; 7 8 9]

A =

     1     2     3
     4     5     6
     7     8     9

>> [L D] = LDLT(A)

L =

    1.0000         0         0
    4.0000    1.0000         0
    7.0000    1.8182    1.0000


D =

    1.0000         0         0
         0  -11.0000         0
         0         0   -3.6364
```

Q5 a.) We know $GV = I$

$$G = \begin{bmatrix} R_2^{-1}+R_3^{-1}+R_5^{-1} & R_2^{-1} & 0 & R_5^{-1} & 0 & 0 & 0 & 0 \\ R_2^{-1} & R_1^{-1}+R_2^{-1}+R_4^{-1} & R_4^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_4^{-1} & R_4^{-1}+R_6^{-1}+R_7^{-1}+R_9^{-1} & R_7^{-1} & 0 & R_9^{-1} & 0 & 0 \\ R_5^{-1} & 0 & R_7^{-1} & R_5^{-1}+R_7^{-1}+R_8^{-1}+R_{10}^{-1} & R_{10}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{10}^{-1} & R_{10}^{-1}+R_{12}^{-1}+R_{13}^{-1}+R_{15}^{-1} & R_{12}^{-1} & 0 & R_{15}^{-1} \\ 0 & 0 & R_9^{-1} & 0 & R_{12}^{-1} & R_{11}^{-1}+R_9^{-1}+R_{12}^{-1}+R_{14}^{-1} & R_{14}^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{14}^{-1} & R_{14}^{-1}+R_{16}^{-1}+R_{17}^{-1} & R_{17}^{-1} \\ 0 & 0 & 0 & 0 & R_{15}^{-1} & 0 & R_{17}^{-1} & R_{15}^{-1}+R_{17}^{-1}+R_{18}^{-1} \end{bmatrix}$$

Substituting all R_i as 1, we get

$$G = \begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 3 \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_x \end{bmatrix}^T$$

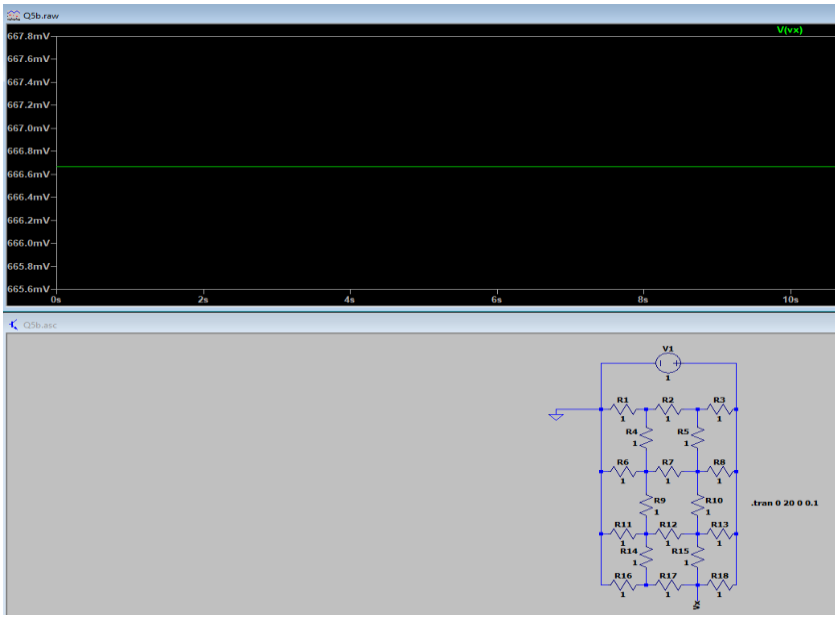
$$I = \begin{bmatrix} V_1/R_3 & 0 & 0 & V_1/R_8 & V_1/R_{13} & 0 & 0 & V_1/R_{18} \end{bmatrix}^T$$

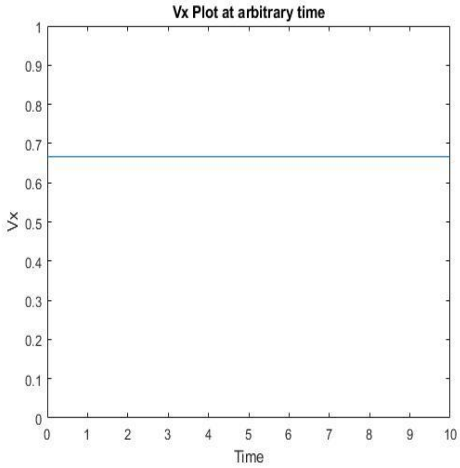
Substituting all as 1

$$I = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}^T$$

Solving by Matlab $V_x = 0.6667$ V

Q5 Summary





Vx = 0.667 from LT SPICE at arbitrary time

Vx = 0.667V as from Matlab

* C:\Users\ArvindNeeraj\Downloads\cad optimization\R10943157_finalexam\Q5b.asc

```
       --- Operating Point ---

V(n002):        0.333333       voltage
V(n003):        0.666667       voltage
V(n001):        1              voltage
V(n004):        0.333333       voltage
V(n005):        0.666667       voltage
V(n006):        0.333333       voltage
V(n007):        0.666667       voltage
V(n008):        0.333333       voltage
V(vx):          0.666667       voltage
I(R18):         0.333333       device_current
I(R17):         0.333333       device_current
I(R16):         0.333333       device_current
I(R15):         0              device_current
I(R14):         0              device_current
I(R13):         0.333333       device_current
I(R12):         0.333333       device_current
I(R11):         0.333333       device_current
I(R10):         0              device_current
I(R9):          0              device_current
I(R8):          -0.333333      device_current
I(R7):          -0.333333      device_current
I(R6):          -0.333333      device_current
I(R5):          1.11022e-016   device_current
I(R4):          0              device_current
I(R3):          0.333333       device_current
I(R2):          0.333333       device_current
I(R1):          0.333333       device_current
I(V1):          -1.33333       device_current
```

Exported Voltage data at arbitary time

The file can be found as LTSPICE exported voltage data.txt

And the simulations and circuit can be found as Q5b.raw, Q5b.asc and the Matlab code can be found as Q5.m