## SoC Verification HW4

**1]** (a)

(i) $f = inv(a)$

$\Rightarrow F: f \leftrightarrow a' \equiv (f \to a')(f' \to a) \equiv (f' \vee a') \wedge (f \vee a)$

(ii) $f = and(a, b')$;

$F: f \leftrightarrow a \wedge b' = (f \to a)(f \to b')(f' \to a' \vee b)$

$\qquad = (f' \vee a) \wedge (f' \vee b') \wedge (f \vee a' \vee b)$

(iii) $f = nand(a, b)$

$F: f \leftrightarrow (a \wedge b)' = f \leftrightarrow a' \vee b' = (f \to a' \vee b')(f' \to a \wedge b)$

$\qquad = (f \to a' \vee b')(f' \to a)(f' \to b)$

$\qquad = (f' \vee a' \vee b') \wedge (f \vee a) \wedge (f \vee b)$

(iv) $f = or(a, b)$

$F: f \leftrightarrow a \vee b = (f \to a \vee b)(f' \to a')(f' \to b')$

$\qquad = (f' \vee a \vee b) \wedge (f \vee a') \wedge (f \vee b')$

(v) $f = nor(a, b)$

$F: f \leftrightarrow (a \vee b)' = (f \to a' \wedge b')(f' \to a \vee b)$

$\qquad = (f \to a')(f \to b')(f' \to a \vee b)$

$\qquad = (f' \vee a') \wedge (f' \vee b') \wedge (f \vee a \vee b)$

**b)**

Note here the numbers below refer to the gate numbers i.e. $22 \sim g_{22}$

$F$:23(23 $\leftrightarrow$ 22 $\vee$ 21)(22 $\leftrightarrow$ 20 $\wedge$ 19)(21 $\leftrightarrow$ (20 $\vee$ 18)')(20 $\leftrightarrow$ 17 $\vee$ 16)(19 $\leftrightarrow$ 15 $\vee$ 14)

(18 $\leftrightarrow$ 13 $\vee$ 11)(17 $\leftrightarrow$ (12 $\vee$ 11)')(16 $\leftrightarrow$ 10 $\wedge$ 1')(15 $\leftrightarrow$ 10'$\wedge$ 9 $\wedge$ 2)(14 $\leftrightarrow$ 10 $\wedge$ 2'$\wedge$ 1)

(13 $\leftrightarrow$ (10 $\vee$ 8)')(12 $\leftrightarrow$ 7 $\wedge$ 6)(11 $\leftrightarrow$ (7 $\vee$ 6)')(10 $\leftrightarrow$ (4 $\vee$ 3)')(9 $\leftrightarrow$ (5 $\wedge$ 4)')

(8 $\leftrightarrow$ 2 $\wedge$ 1)(7 $\leftrightarrow$ 4 $\wedge$ 3)(6 $\leftrightarrow$ (2 $\vee$ 1)')(5 $\leftrightarrow$ (3 $\vee$ 1)')


= 23 $\wedge$ (23'$\vee$ 22 $\vee$ 21) $\wedge$ (23 $\vee$ 22') $\wedge$ (23 $\vee$ 21') $\wedge$ (22 $\vee$ 20'$\vee$ 19') $\wedge$ (22'$\vee$ 20) $\wedge$ (22'$\vee$ 19)

$\wedge$ (21 $\vee$ 20 $\vee$ 18) $\wedge$ (21'$\vee$ 20') $\wedge$ (21'$\vee$ 18') $\wedge$ (20'$\vee$ 17 $\vee$ 16) $\wedge$ (20 $\vee$ 17') $\wedge$ (20 $\vee$ 16')

$\wedge$ (19'$\vee$ 15 $\vee$ 14) $\wedge$ (19 $\vee$ 15') $\wedge$ (19 $\vee$ 14') $\wedge$ (18'$\vee$ 13 $\vee$ 11) $\wedge$ (18 $\vee$ 13') $\wedge$ (18 $\vee$ 11')

$\wedge$ (17 $\vee$ 12 $\vee$ 11) $\wedge$ (17'$\vee$ 12') $\wedge$ (17'$\vee$ 11') $\wedge$ (16 $\vee$ 10'$\vee$ 1) $\wedge$ (16'$\vee$ 10) $\wedge$ (16'$\vee$ 1')

$\wedge$ (15 $\vee$ 10 $\vee$ 9'$\vee$ 2') $\wedge$ (15'$\vee$ 10') $\wedge$ (15'$\vee$ 9) $\wedge$ (15'$\vee$ 2)

$\wedge$ (14 $\vee$ 10'$\vee$ 2 $\vee$ 1') $\wedge$ (14'$\vee$ 10) $\wedge$ (14'$\vee$ 2') $\wedge$ (14'$\vee$ 1)

$\wedge$ (13 $\vee$ 10 $\vee$ 8) $\wedge$ (13'$\vee$ 10') $\wedge$ (13'$\vee$ 8') $\wedge$ (12 $\vee$ 7'$\vee$ 6') $\wedge$ (12'$\vee$ 7) $\wedge$ (12'$\vee$ 6)

$\wedge$ (11 $\vee$ 7 $\vee$ 6) $\wedge$ (11'$\vee$ 7') $\wedge$ (11'$\vee$ 6') $\wedge$ (10 $\vee$ 4 $\vee$ 3) $\wedge$ (10'$\vee$ 4') $\wedge$ (10'$\vee$ 3')

$\wedge$ (9 $\vee$ 5 $\vee$ 4) $\wedge$ (9'$\vee$ 5') $\wedge$ (9'$\vee$ 4') $\wedge$ (8 $\vee$ 2'$\vee$ 1') $\wedge$ (8'$\vee$ 2) $\wedge$ (8'$\vee$ 1)

$\wedge$ (7 $\vee$ 4'$\vee$ 3') $\wedge$ (7'$\vee$ 4) $\wedge$ (7'$\vee$ 3) $\wedge$ (6 $\vee$ 2 $\vee$ 1) $\wedge$ (6'$\vee$ 2') $\wedge$ (6'$\vee$ 1')

$\wedge$ (5 $\vee$ 3 $\vee$ 1) $\wedge$ (5'$\vee$ 3') $\wedge$ (5'$\vee$ 1')

**So in total 60 clauses and 140 literals.**

**c)** Note here the numbers below refer to the gate numbers i.e. $22 \sim g_{22}$

$F$: 23(23 → 22 ∨ 21)(22 → **20** ∧ 19)(21 → **20'** ∧ 18')(20 → **17** ∨ **16**)(20' → **17'** ∧ **16'**)

(19 → 15 ∨ 14)(18' → 13' ∧ 11')(17 → **12'** ∧ **11'**)(17' → **12** ∨ **11**)(16 → **10** ∧ 1')

(16' → **10'** ∨ 1)(15 → **10'** ∧ 9 ∧ 2)(14 → **10** ∧ 2' ∧ 1)(13' → **10** ∨ 8)(12 → **7** ∧ **6**)

(12' → **7'** ∨ **6'**)(11 → **7'** ∧ **6'**)(11' → **7** ∨ **6**)(10 → 4' ∧ 3')(10' → 4 ∨ 3)(9 → 5' ∨ 4')

(8 → 2 ∧ 1)(7 → 4 ∧ 3)(7' → 4' ∨ 3')(6 → 2' ∧ 1')(6' → 2 ∨ 1)(5' → 3 ∨ 1)


= 23(23 → 22 ∨ 21)(22 → **20**)(22 → 19)(21 → **20'**)(21 → 18')(20 → **17** ∨ **16**)

(20' → **17'**)(20' → **16'**)(19 → 15 ∨ 14)(18' → 13')(18' → 11')(17 → **12'**)(17 → **11'**)

(17' → **12** ∨ **11**)(16 → **10**)(16 → 1')(16' → **10'** ∨ 1)(15 → **10'**)(15 → 9)(15 → 2)

(14 → **10**)(14 → 2')(14 → 1)(13' → **10** ∨ 8)(12 → **7**)(12 → **6**)(12' → **7'** ∨ **6'**)

(11 → **7'**)(11 → **6'**)(11' → **7** ∨ **6**)(10 → 4')(10 → 3')(10' → 4 ∨ 3)(9 → 5' ∨ 4')

(8 → 2)(8 → 1)(7 → 4)(7 → 3)(7' → 4' ∨ 3')(6 → 2')(6 → 1')(6' → 2 ∨ 1)(5' → 3 ∨ 1)


= 23 (23' ∨ 22 ∨ 21)(22' ∨ **20**)(22' ∨ 19)(21' ∨ **20'**)(21' ∨ 18')(20' ∨ **17** ∨ **16**)

(20 ∨ **17'**)(20 ∨ **16'**)(19' ∨ 15 ∨ 14)(18 ∨ 13')(18 ∨ 11')(17' ∨ **12'**)(17' ∨ **11'**)

(17 ∨ **12** ∨ **11**)(16' ∨ **10**)(16' ∨ 1')(**16** ∨ **10'** ∨ 1)(15' ∨ **10'**)(15' ∨ 9)(15' ∨ 2)

(14' ∨ **10**)(14' ∨ 2')(14' ∨ 1)(13 ∨ **10** ∨ 8)(12' ∨ **7**)(12' ∨ **6**)(12 ∨ **7'** ∨ **6'**)

(11' ∨ **7'**)(11' ∨ **6'**)(11 ∨ **7** ∨ **6**)(10' ∨ 4')(10' ∨ 3')(10 ∨ 4 ∨ 3)(9' ∨ 5' ∨ 4')

(8' ∨ 2)(8' ∨ 1)(7' ∨ 4)(7' ∨ 3)(7 ∨ 4' ∨ 3')(6' ∨ 2')(6' ∨ 1')(6 ∨ 2 ∨ 1)(5 ∨ 3 ∨ 1)


**So now in total 44 clauses and 100 literals.**

**Therefore : Percentage reduction: 26.67% of clauses, 28.57% of literals**

d) Note here the numbers below refer to the gate numbers i.e. $22 \sim g_{22}$

| Gate | Literal count | Gate | Literal count |
|---|---|---|---|
| 23 | (1,1) | 11 | (4,2) |
| 22 | (2,1) | 10 | (4,4) |
| 21 | (2,1) | 9 | (1,1) |
| 20 | (2,3) | 8 | (2,1) |
| 19 | (1,1) | 7 | (4,3) |
| 18 | (1,2) | 6 | (4,3) |
| 17 | (3,2) | 5 | (1,1) |
| 16 | (3,2) | 4 | (3,2) |
| 15 | (3,1) | 3 | (2,3) |
| 14 | (3,1) | 2 | (2,3) |
| 13 | (1,1) | 1 | (2,5) |
| 12 | (3,2) | | |

e) Note here the numbers below refer to the gate numbers i.e. $22 \sim g_{22}$

Decision score = sum of negative (0) and positive literal (1) counts

| Decision score (D) | Gates |
|---|---|
| 8 | 10 |
| 7 | 1,6,7 |
| 6 | 11 |
| 5 | 2,3,4,17,20 |

Since $g_{10}$ has decision score=8, it will be the first in decision order.

Here for D=7, we look into gates 1,6,7

According to the rules, For same decision score, the smaller difference between (0, 1)-literal counts wins. If tied again, compare their IDs (bigger ID wins).

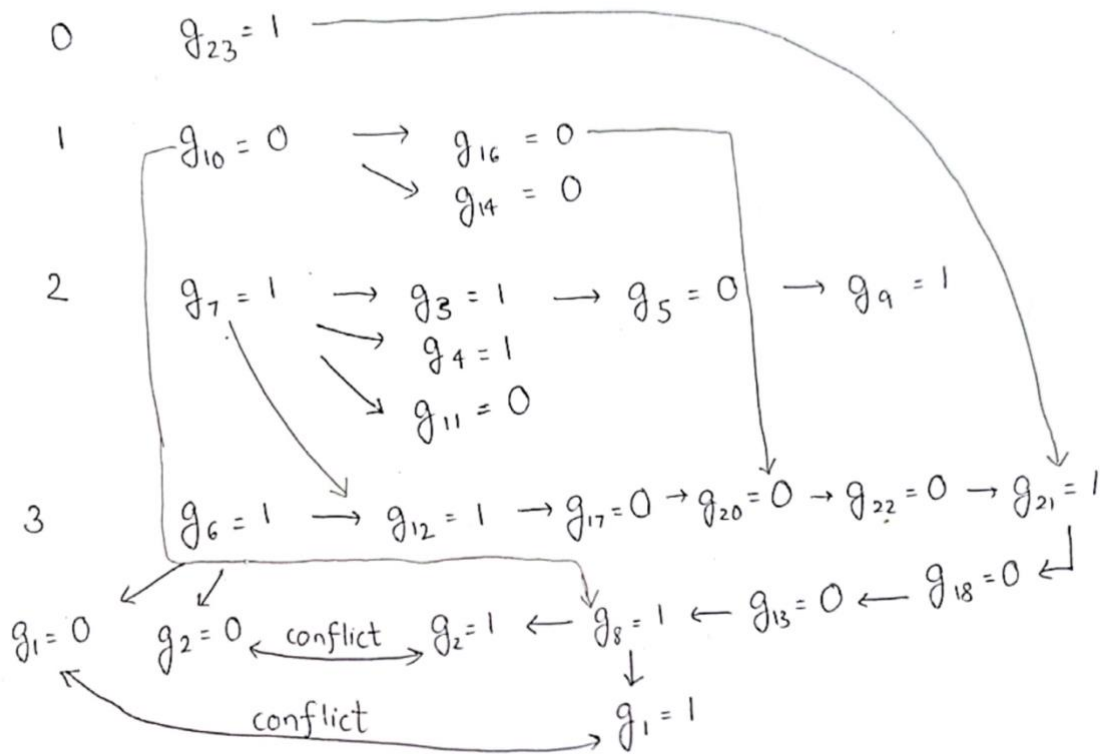Referring to the rules and the literal count table in [d], we have

So, the order is $g_{10} > g_7 > g_6 > g_1 > g_{11} > g_{20} > g_{17}$

And the decision values are

$g_{10} = 0 > g_7 = 1 > g_6 = 1 > g_1 = 0 > g_{11} = 1 > g_{20} = 0 > g_{17} = 1$

(f)  From (e), getting the decision orders

Level

0      $g_{23} = 1$

1      $g_{10} = 0$  →  $g_{16} = 0$
                        $g_{14} = 0$

2      $g_7 = 1$  →  $g_3 = 1$  →  $g_5 = 0$  →  $g_9 = 1$
                     $g_4 = 1$
                     $g_{11} = 0$

3      $g_6 = 1$  →  $g_{12} = 1$  →  $g_{17} = 0$  →  $g_{20} = 0$  →  $g_{22} = 0$  →  $g_{21} = 1$

       $g_1 = 0$   $g_2 = 0$  ← conflict →  $g_2 = 1$  ←  $g_8 = 1$  ←  $g_{13} = 0$  ←  $g_{18} = 0$

                    conflict                              $g_1 = 1$

Referring to the C++ code to find 1st UIP cut

   First UIP cut : $g_6 = 1$

The conflict source   : $\{(g_6 = 1), (g_{10} = 0), (g_{23} = 1), (g_{16} = 0), (g_7 = 1)$
is identified as

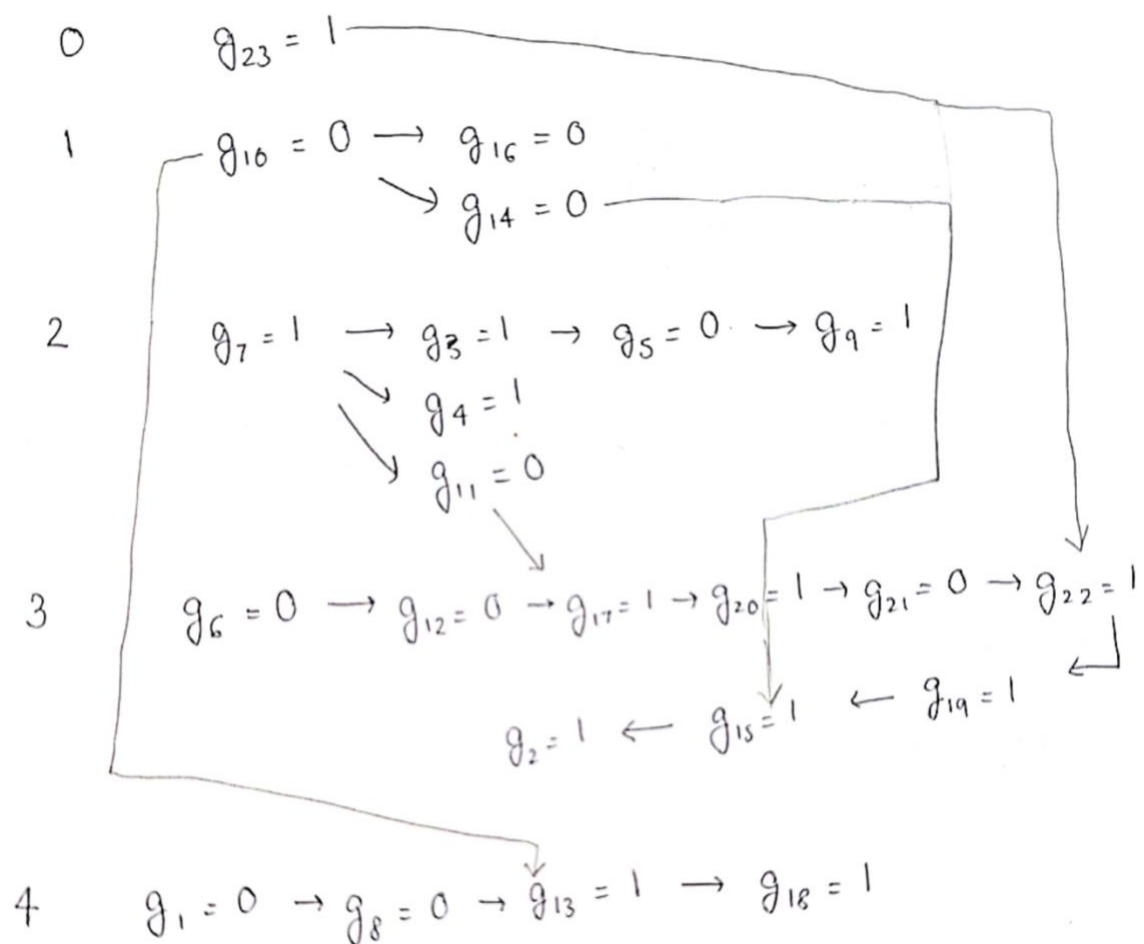The corresponding learned implication is :

$g_{10}' \wedge g_{23} \wedge g_{16}' \wedge g_7 \rightarrow g_6'$

i.e  $(g_{10} + g_{23}' + g_{16} + g_7' + g_6')$

g.) Witness generation

Backtracking to $g_6$

Level

0      $g_{23} = 1$

1      $g_{16} = 0 \longrightarrow g_{16} = 0$
                     $\searrow g_{14} = 0$

2      $g_7 = 1 \longrightarrow g_3 = 1 \rightarrow g_5 = 0 \longrightarrow g_9 = 1$
                $\searrow g_4 = 1$
                  $\searrow g_{11} = 0$

3      $g_6 = 0 \longrightarrow g_{12} = 0 \rightarrow g_{17} = 1 \rightarrow g_{20} = 1 \rightarrow g_{21} = 0 \rightarrow g_{22} = 1$

                    $g_2 = 1 \leftarrow g_{15} = 1 \leftarrow g_{19} = 1$

4      $g_1 = 0 \rightarrow g_8 = 0 \rightarrow g_{13} = 1 \longrightarrow g_{18} = 1$

All gates have been implied

$\therefore$ By backtracking to $g_6$

SAT with $g_1 = 0, \, g_2 = 1, \, g_3 = 1, \, g_4 = 1$