

INFBIOx121 - Algorithms module

Sequence mapping

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Gigsaw from University of Dundee

- Four people in each group
- Set up the reference sequence: both forward and reverse (reverse complement)
- Each person takes 10 reads. **Notice that reads are reverse complemented on the back. So use the side that is most convenient.**
- Map individually in parallel: two persons can map to forward and two to reverse.
- Then combine the reads to variant call together.
- In 10 mins, I want to know what you have found out

Gigsaw thoughts....

- Did you find some other interesting positions?
- Did all reads map perfectly
- Could this be a diploid organism?
- What if you had only had:
 - one of the references?
 - one side of the read?



MAPPING WITH BWA

Why mapping?

- The biggest difference with Sanger
 - we did not design and use primers for sequence amplification
 - we sonicated
 - >> we do not know where the reads “originate” from
- For each read
 - we need to determine its likely origin
 - how likely it is that we have correctly identified its origin

- The Burrows-Wheeler Transform (BWT) is a way of permuting the characters of a string T into another string $BWT(T)$
- For example for $T = \mathbf{abaaba\$}$
- $\$$ defined to be a character that does **not** appear elsewhere in T and which is **lexicographically prior** to all other characters
- The rotations of T

\$	a	b	a	a	b	a
a	\$	a	b	a	a	b
b	a	\$	a	b	a	a
a	b	a	\$	a	b	a
a	a	b	a	\$	a	b
b	a	a	b	a	\$	a
a	b	a	a	b	a	\$

Lexicographic
sorting of rows

Burrows-Wheeler Matrix BW $M(T)$

\$	a	b	a	a	b	a
a	\$	a	b	a	a	b
a	a	b	a	\$	a	b
a	b	a	\$	a	b	a
a	b	a	a	b	a	\$
b	a	\$	a	b	a	a
b	a	a	b	a	\$	a

BWT(T)

BWT(T) = abba\$aa

Relationship to suffix array

- Suffix array = sorted array of all suffixes of a string
- $T = \text{abaaba\$}$

All suffixes of T

0	a b a a b a \$
1	b a a b a \$
2	a a b a \$
3	a b a \$
4	b a \$
5	a \$
6	\$

Sorted suffixes

6	\$
5	a \$
2	a a b a \$
3	a b a \$
0	a b a a b a \$
4	b a \$
1	b a a b a \$

Matching order

BWM:	SA: Suffixes given by SA:
<u>\$</u> abaaba	6 \$
<u>a</u> \$abaab	5 a \$
<u>a</u> ba\$ab	2 aaba \$
<u>a</u> ba\$aba	3 aba \$
<u>a</u> baaba\$	0 abaaba \$
<u>b</u> a\$abaa	4 ba \$
<u>b</u> aaba\$a	1 baaba \$

The LF mapping property of the BWT

- $\text{BWT}(\text{abaaba}\$) = \text{abba}\$aa$
- Seem like we have lost which **a** in $\text{BWT}(T)$ corresponds to which **a** in T
- BUT we have not thanks to an important property of the BWT called **LF mapping**

\$	a	b	a	a	b	a
a	\$	a	b	a	a	b
a	a	b	a	\$	a	b
a	b	a	\$	a	b	a
a	b	a	a	b	a	\$
b	a	\$	a	b	a	a
b	a	a	b	a	\$	a

$$T = a_0b_0a_1a_2b_1a_3\$$$

The subscript corresponds to the number of times the character has previously been seen in T

We call the subscript the rank
\$ does not require rank as
occurs only once

\$	a_0	b_0	a_1	a_2	b_1	a_3
a_3	\$	a_0	b_0	a_1	a_2	b_1
a_1	a_2	b_1	a_3	\$	a_0	b_0
a_2	b_1	a_3	\$	a_0	b_0	a_1
a_0	b_0	a_1	a_2	b_1	a_3	\$
b_1	a_3	\$	a_0	b_0	a_1	a_2
b_0	a_1	a_2	b_1	a_3	\$	a_0

The LF mapping states: the i^{th} occurrence of a character c in the **Last** column has the same rank as the i^{th} occurrence of c in the **First** column

We will not
prove why!!

How do we reverse BWT? Useful for compression

Skipped

We re-rank them:

- * Before we ranked according how many times occurred previously in T ie T ranking
- * Now we re-rank according to how many times previously seen in BWT(T) ie B ranking

<i>F</i>	<i>L</i>	<i>rank</i>
\$	a	0
a	b	0
a	b	1
a	a	1
a	\$	0
b	a	2
b	a	3

Start with \$

<i>F</i>	<i>L</i>	<i>rank</i>
\$	a	0
a	b	0
a	b	1
a	a	1
a	\$	0
b	a	2
b	a	3

Rows are rotations of T so last col of the first row contains the character to left of \$ in T: a



Rank array tells us this is a with rank 0



How do we get the character to the left of a_0 ?



We can do this if we know the row that begins with a_0



The LF mapping states: the i^{th} occurrence of a character c in the last column has the same rank as the i^{th} occurrence of c in the first column



The LF tells us which row begins with a_0 : the first row containing an "a"

Assuming first row is 0, we visit rows: 0, 1, 5, 3, 2, 6, 4

← $a_3 b_1 a_1 a_2 b_0 a_0 \$$

Reversible means useful for compression

FM index – key to searching a string

- Ferragina and Manzini: BWT can be used as a space-efficient index of T
- Searching for occurrences of P = aba
- The BWM is sorted, so any rows having P as a **prefix** will be **consecutive**

First find the rows beginning with the shortest **suffix** of P i.e. “a”

>> rows 1,2,3 and 4

1

F						L	rank
\$	a	b	a	a	b	a	0
a	\$	a	b	a	a	b	0
a	a	b	a	\$	a	b	1
a	b	a	\$	a	b	a	1
a	b	a	a	b	a	\$	0
b	a	\$	a	b	a	a	2
b	a	a	b	a	\$	a	3

2 Second find all rows begin with the **next** longest **suffix** of P i.e. “ba”

>> these are rows 1 and 2 (**rows are rotations of T**)

3 The **LF mapping** and rank array tell us which rows have “ba” as a prefix: rows beginning with b0 and b1

The LF mapping states: the i^{th} occurrence of a character c in the last column has the same rank as the i^{th} occurrence of c in the first column

Searching T

<i>F</i>						<i>L</i>	<i>rank</i>
\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	0
<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	0
<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	1
<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	1
<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	0
<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	2
<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	3

The LF mapping and rank array tell us which rows have “ba” as a prefix: row beginning with *b*₀ and *b*₁

<i>F</i>						<i>L</i>	<i>rank</i>
\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	0
<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	0
<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	1
<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	1
<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	0
<i>b</i>	<i>a</i>	\$	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	2
<i>b</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	\$	<i>a</i>	3

Occurrences of “ba” are preceded by *a*₂ and *a*₃

Backward matching: apply LF mapping repeatedly to find range of rows **prefixed** by successively longer proper **suffixes of P** until **range is empty or run out of suffixes**

With only 3 backward matching (LF mapping and rotations) operations we have found the rows prefixed with occurrences of “aba”

Looking up offsets

So far we have discussed how to use the FM Index to determine whether and how many times a substring P occurs within T , but we have not discussed how to find *where* P occurs, i.e. the substring's offset into T .

If our index included the suffix array $SA(T)$, we could simply look this up in $SA(T)$. For example, here was the range we ended up with after searching for $P = aba$ within $BWT(abaaba\$)$:

SA and BWM have a corresponding order

F		L	$SA(T)$	BWM:	SA: Suffixes given by SA:
\$	a b a a b	a	6	\$abaaba	6 \$
a	\$ a b a a	b	5	a\$abaab	5 a\$
a	a b a \$ a	b	2	aaba\$ab	2 aaba\$
a	b a \$ a b	a	3	aba\$aba	3 aba\$
a	b a a b a	\$	0	abaaba\$	0 abaaba\$
b	a \$ a b a	a	4	ba\$abaa	4 ba\$
b	a a b a \$	a	1	baaba\$a	1 baaba\$

$SA(T)$ tells us these matches occurred at T offsets 0 and 3.

T=abaaba
012345

Maybe that did not seem impressive.....

<i>F</i>						<i>L</i>	<i>rank</i>
\$	a	b	a	a	b	a	0
a	\$	a	b	a	a	b	0
a	a	b	a	\$	a	b	1
a	b	a	\$	a	b	a	1
a	b	a	a	b	a	\$	0
b	a	\$	a	b	a	a	2
b	a	a	b	a	\$	a	3

I could have found the offsets of “aba” in T. Yes, BUT:

- **either** you then scan T with “aba” >> exhaustive search
- **or** you have to store the whole BWM matrix

When the string you are searching is 3 billion bases long that means:

- **either** you check for a match at every 3 bn positions
- **or** you need to store a matrix that is 3bn x 3bn

If you use the BWT and suffix array:

- You only need to store the first and last columns of the BWM (not the blue section)
- You need to do **X** backward matching operations to locate a string of **length X**

There are many optimisation to:

- Decrease space requirements
- Decrease time requirements
- Allow for mismatches

Application to read mapping

- You need to build an index of the genome you wish to search
- Reads are 100 bp long
- But, the longer the string searched for the longer the search time (**longer backtracking**), thus an advantage to search with a shorter string.
- There are 4 different nucleotides and 3.0×10^9 bases of genome
 - $4^{16} = 4.0 \times 10^9$
 - $4^{17} = 17.0 \times 10^9$
- Assuming complex sequence, 16 to 17 bp should be enough for a unique match
- BUT
 - variation in sample relative to reference
 - base call errors
 - **THUS** 17 bp search string is not enough
- BWA searches with a “seed” of 32 and allows 2 differences in the seed
- **Reads with seed matches are aligned using Smith-Waterman**

Mapping exercise with BWA

- Log into the virtual machine
- Open the exercise file in a text editor:
algorithms_mappingExercise.bash
- Open a terminal and carry out the exercise