MATH 392: Intro to Neural Networks

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- So, at its core, modern AI falls within the framework of **machine learning** (ML).

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- **Training**: *Fit the model* to the given data to get the best approximation to F within our class.

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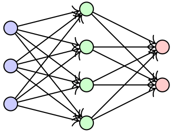
- **Prediction**: Given a house with a certain square footage x, predict the house price to be $y = f(x) = \hat{m}x + \hat{b}$.

What are neural networks?



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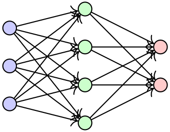

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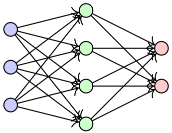

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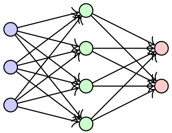

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- The model parameters are called *weights*; they are trained using a process called *backpropagation*.
- LLMs like ChatGPT are based on neural networks with billions of weights!

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  Highly optimized hardware (GPUs) and software (e.g. PyTorch) allow for large-scale parallel computations.

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- Learn how to evaluate NN's (model optimization, regularization, and hyperparameter tuning).
- Demonstrate your learning by completing a final project.

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- **Research-spirit**: Heavy emphasis on asking interesting questions and collaborating with peers on projects.

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- You are a math major looking to fill the "Application Course" credit.

Closing remarks

I hope you will consider enrolling in MATH 392!

Please feel free to reach out to me at *arvindsuresh@arizona.edu* if you have any questions about the course, or simply want to chat about neural networks (or anything else mathematical, for that matter).

Thank you for your time!!