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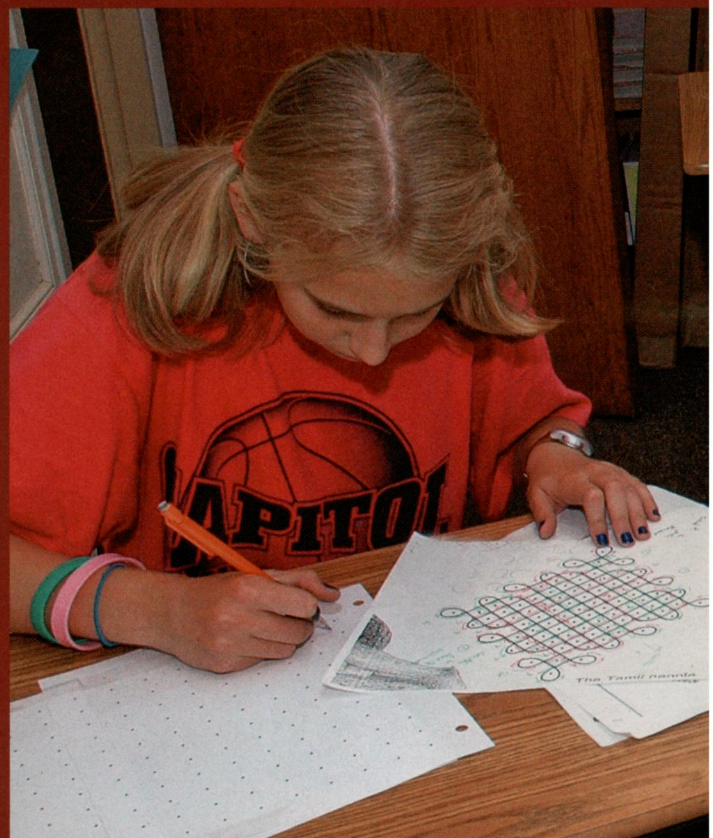

Teaching Mathematics through the ART OF KOLAM

SYAMALA CHENULU

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ONE GOAL OF THE NCTM'S CONNECTIONS Standard is that mathematics instruction pre-K–12 should "enable all students to recognize and apply mathematics in contexts outside of mathematics" (NCTM 2000, p. 64). Art of all kinds provides opportunities to address this goal. Moreover, many mathematics educators, including myself, believe that it is important and beneficial to provide a multicultural perspective in our classrooms. "Knowledge of the ideas of others can enlarge our view of what is mathematical and, in particular, add a more humanistic and global perspective to the history of mathematics. This enlarged view, in which mathematical ideas are seen to play a vital role in diverse human endeavors, provides us with a richer and fuller picture of mathematics" (Ascher 2002, p. 200).

I had given the art department in our school several books about *kolam*, traditional designs placed on the thresholds of homes in Tamil Nadu near my birthplace in southern India (see the sidebar on p. 425). I was thrilled to hear that some of my students were learning about and making these designs in their art classes. As a mathematics teacher, I felt I could not let this opportunity slip by. Thus began a two-week period during which my sixth-grade classes spent a part of each mathematics class exploring the mathematics of kolam.

Tracing Graphs

DURING THE FIRST DAY OF THIS STUDY, I INTROduced the students to some basic definitions of graph theory:

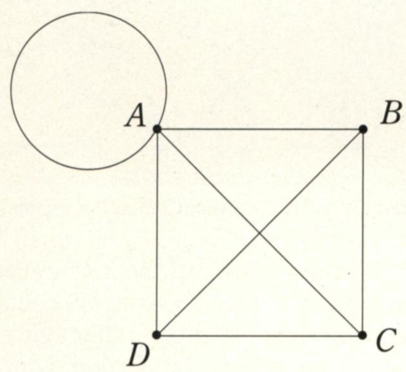
- Graph: A set of vertices (points) connected by edges (line segments or curves)
- Loop: An edge that connects a vertex to itself
- Degree of a vertex: The number of edges connected to a vertex
- Even vertex: A vertex with an even-numbered degree
- Odd vertex: A vertex with an odd-numbered degree
- Path: A sequence of consecutive edges in a graph
- Circuit: A path that begins and ends at the same vertex

The students worked with a few simple graphs to identify these features and clarify their definitions (see **fig. 1**).

I then explained to the students that the Swiss mathematician Leonhard Euler did much of the early work with graph theory. They were familiar with Euler, so this added to their interest. I explained that a graph has an *Euler path* if every edge can be traced without going over the same edge more than once. Also, a graph has an *Euler circuit* if the path starts and ends at the same point.

Each student received a copy of the graphs shown in **figure 2**. Their tasks were to see whether the edges of each graph could be traced without lifting their pencils and to make conjectures about the characteristics of graphs that are Euler paths and/or circuits.

After the students had worked for a few minutes, I asked whether they had any suggestions for characteristics that might be important in determining whether a graph contains an Euler path or an Euler circuit. The characteristics suggested by students included the symmetry of the graph and "something about the vertices." The students talked in small groups and then decided on the headings for a table in which they would record



Vertices: A, B, C, D
Edges: $AB, BC, BD, AD, AA, AC, CD$
Degrees of the vertices: A-5; B-3; C-3; D-3. All are odd vertices.
A path: A-B-D
A circuit: A-B-D-C-A

Fig. 1 A simple graph

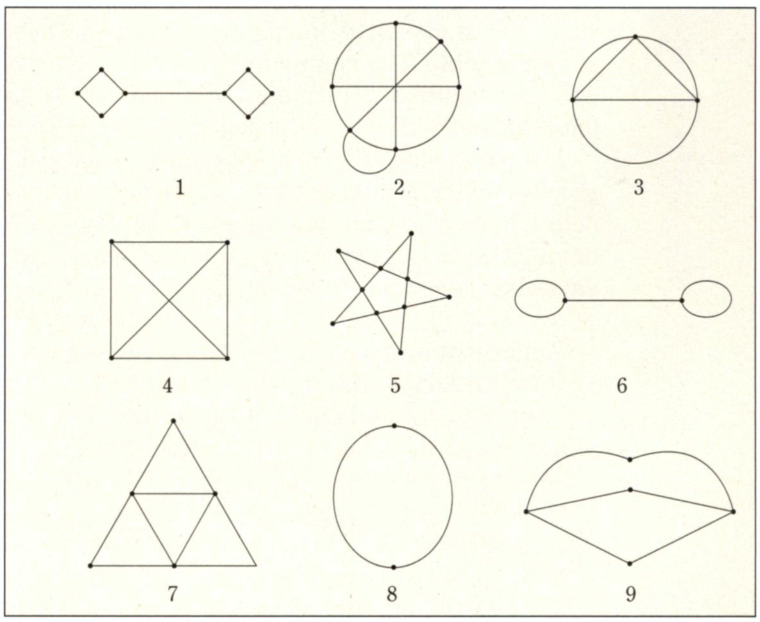


Fig. 2 Do the graphs contain an Euler path or an Euler circuit?

TABLE 1
Students Record Their Findings about Euler Paths and/or Circuits

Figure Number	Number of Odd Vertices	Number of Even Vertices	Euler Circuit?	Euler Path?	Number of Lines of Reflection	Smallest Angle of Rotation
1	2	6	no	yes	2	180°
2	4	3	no	no	0	none
3	0	3	yes	yes	1	none
4	4	1	no	no	4	90°
5	0	10	yes	yes	5	72°
6	2	0	no	yes	2	180°
7	0	4	yes	yes	3	120°
8	0	2	yes	yes	2	180°
9	2	3	no	yes	1	none

their findings (see **table 1**). Originally, some groups wanted to include the categories "Does the graph have reflection symmetry?" and "Does the graph have rotational symmetry?" but they were persuaded by classmates that these categories were unnecessary.

When students arrived the next day, they were excited to share their conjectures. They first verified the information they had recorded in their tables. For several of the graphs, this step required a student coming to the board, drawing the graph, and demonstrating his or her method of tracing. In each class, the students eventually agreed on the data shown in **table 1**. A common problem for students was their failure to remember that a loop counts as one edge but adds two to the degree of its vertex.

Many conjectures were offered about vertices in graphs and the characteristics that are important in determining whether a graph contains an Euler path or an Euler circuit. After much discussion over two days, the classes concurred on the following:

- You cannot have an odd number of odd vertices. Gabe explained, "Start with a graph that has all even vertices. Make one of the vertices odd by adding another edge. At the other end of that edge, there will always be another odd vertex."
- If all vertices of a graph are even, it has an Euler path and an Euler circuit. Emma offered, "Since each vertex has an even number of edges connected to it, for every edge that enters each vertex, there is a 'partner edge' to leave by."

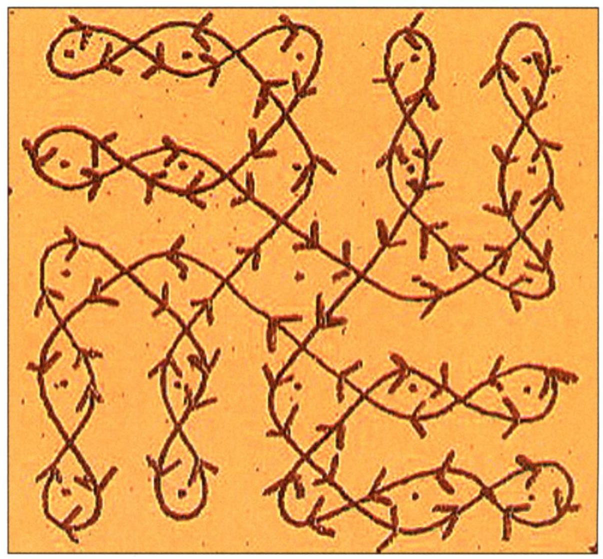


Fig. 3 An example of kolam without reflection symmetry

- Symmetry does not seem to be a characteristic that determines whether a graph will contain an Euler path or an Euler circuit, although all the graphs found so far have reflection symmetry. (Note: Later in their work with kolam, the students found many examples of graphs with both Euler paths and circuits that did not have reflection symmetry. See the example in **fig. 3**.)
- If a graph has exactly two odd vertices, it has an Euler path. James said that he thought of this characteristic because he noticed this pattern in the table's data. He then "tried drawing some

Kolam: A Living Art of India

Kolam (in Tamil), *muggu* (in Telugu), or *rangoli* (in Hindi) is a decorative design drawn by women in front of their homes as a welcome mat. It is a line drawing containing loops drawn around patterns of dots that is generally symmetrical. The designs are drawn using rice powder and, for longevity, diluted rice paste or even paint. In addition, some people in India use limestone and red brick powder to produce a contrasting color scheme.

Traditionally, every morning before sunrise, the ground in front of the house is cleaned with water and swept to give a smooth surface. The water may be mixed with a little cow dung, and the kolam is drawn while the surface is still damp. Although the kolam is a decoration, the rice powder it is drawn with also serves as a meal for ants and birds. In addition, kolam is a welcoming sign to all who enter the home, including Lakshmi, the goddess of wealth and prosperity. The story goes that the lines must be completed so as to prevent evil spirits from entering the shapes, thus preventing their entry into the home.

Modern kolam are sometimes drawn with chalk and augmented with vinyl stickers, but this practice takes away from the art form and loses the physical advantage of an artist bending and working at the patterns.

other graphs and figured out that you have to start at one of the odd vertexes and end at the other one to trace the graph completely without going over any edge twice." Lily added, "This makes sense, since the odd number of edges at those vertices allows you to 'come in and go out of' the vertex a certain number of times with one left over—one odd vertex for the starting point and the other odd vertex for the ending point."

- If a graph has more than two odd vertices, it cannot be traced without repeating an edge and, therefore, it does not have an Euler path or an Euler circuit. The students based this conjecture on their exhaustive efforts to trace graphs 2 and 4 in **figure 2** plus additional graphs they created themselves. They were thrilled when this conjecture continued to hold throughout their remaining investigations.

I was pleased that the students had learned a little graph theory and had also used algebraic thinking in finding and verifying patterns and mathematical reasoning to support their conjectures. They also worked together and communicated their ideas both orally and in writing.

The Mathematics of Kolam

TO BEGIN THE INVESTIGATIONS OF THE MATHEmatics of kolam, we first reviewed what the students had learned about the art (see the sidebar at left). I then asked each class to brainstorm about some mathematical questions we might investigate related to these designs.

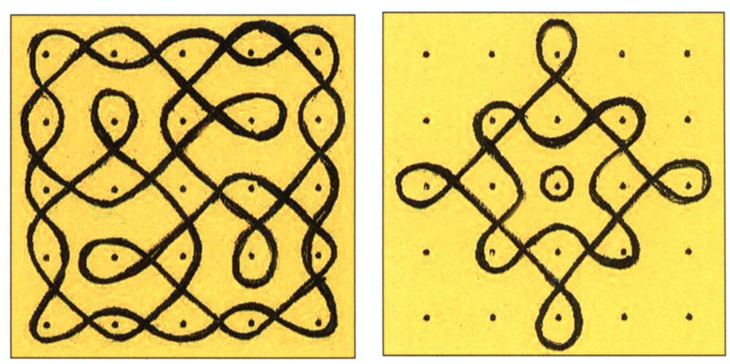
Since all the kolam that my students had created in art class were essentially a geometric design comprising straight or curved lines against a framework of dots (called *pulli)*, they posed a number of questions that led to several engaging mathematical investigations, as described below.

What are the different arrangements of pulli that are used? Is there a way to find the total number of pulli in kolam without counting them one by one? Although the students had worked with several kolam, they decided to restrict this investigation to two simple types that included most of the designs they produced (with a few variations). An example of each type of kolam is shown in **figure 4**.

One type is a square array, most often designed with an odd number of rows and columns. For this type of kolam, the total number of pulli is, of course, $n \cdot n$, or n^2, where n is the number of pulli in one row.

The second type of arrangement has a center row that always contains an odd number of pulli. The number of pulli in each successive row, above and below the center row, decreases by two until the top and bottom rows each contain only one pulli. Working in pairs or small groups and sharing among groups, the students worked diligently to discover the following:

1. The sum of any group of increasing odd numbers is always a square number (e.g., $1 + 3 + 5 = 9$ and $1 + 3 + 5 + 7 = 16$).
2. The sum of r rows of increasing odd numbers is r^2 (e.g., for $1 + 3 + 5$, there are 3 rows, so the sum is 3^2, or 9).



Pulli are shown here in a square array.

This hand-drawn figure has an odd number of pulli (1, 3, 5).

Fig. 4 Two types of kolam

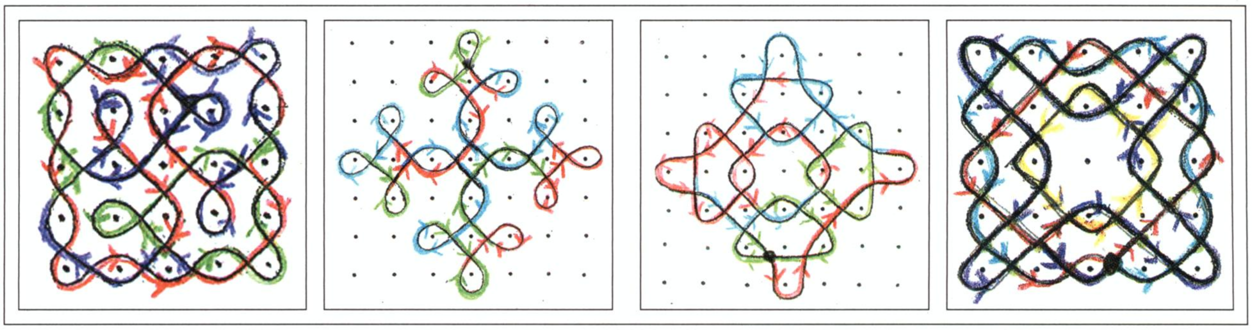
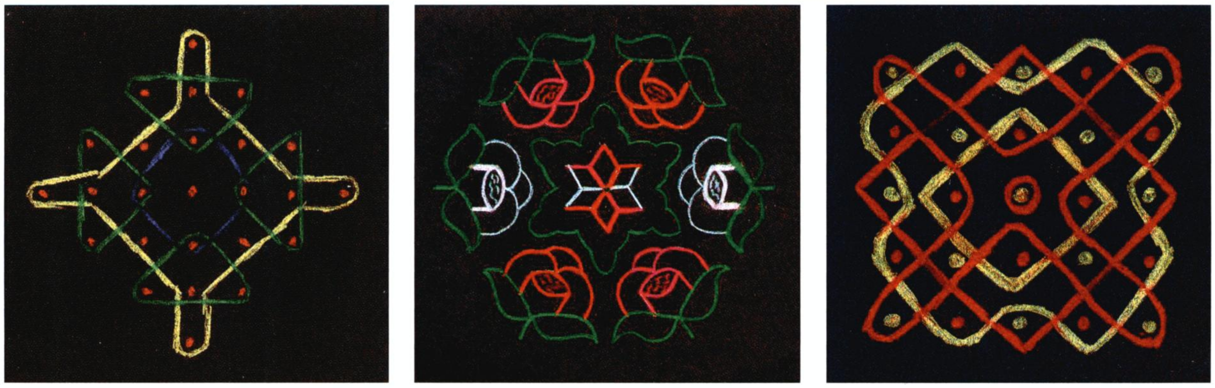


Fig. 5 Kolam with tracings by students to show Euler circuits



Four lines of reflection; the smallest angle of rotation is 90 degrees. | Six lines of reflection; the smallest angle of rotation is 60 degrees. | Four lines of reflection; the smallest angle of rotation is 90 degrees.

Fig. 6 Students' symmetrical kolam

3. If n is the number of pulli in the center row of this type of kolam, then $(n + 1)/2$ is the number of rows in half of the kolam, including the center row (e.g., for kolam with rows containing 1, 3, 5, 3, 1 pulli, $(5 + 1)/2 = 3$ rows).
4. Then $((n + 1)/2)^2$ is the sum of those rows (from step number 2).
5. So $2((n + 1)/2)^2$ is the sum of all the rows; however, this expression counts the center row of pulli twice, so $2((n + 1)/2)^2 - n$ is the total number of pulli. Another class, using similar reasoning, found the rule

$$\text{total pulli} = 2((n - 1)/2)^2 + n,$$

when n is the number of pulli in the center row.

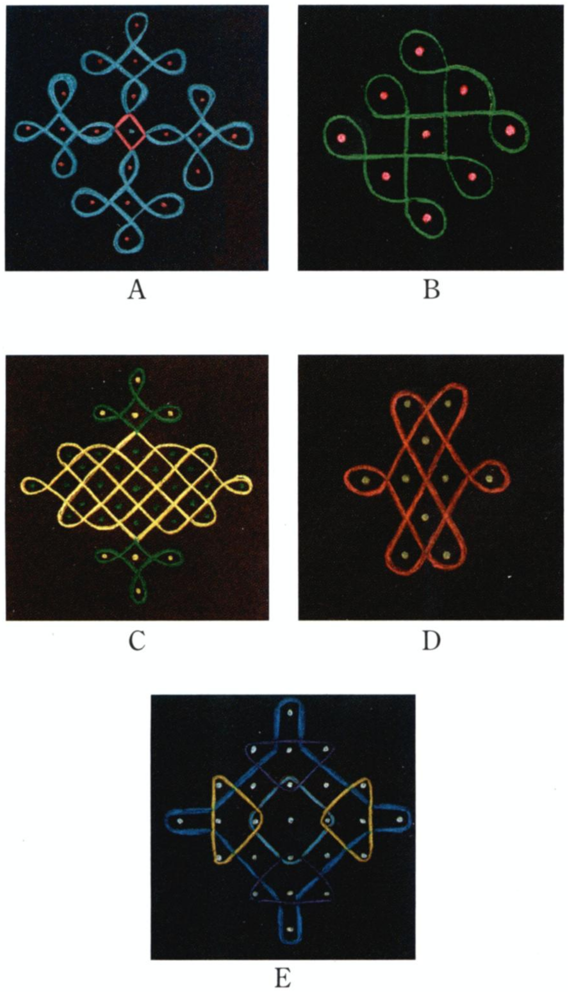
Do kolam contain Euler paths or Euler circuits? The students examined the kolam that they had made in art class to find the answer. They first tried to eliminate the possibility of Euler paths and Euler circuits by finding disconnected parts or more than two odd vertices in each of their designs. After this step, students were surprised at the number of choices they had to make when trying to trace over the designs without retracing any edge. In the end, they found that many of the kolam, despite appearing otherwise, are one continuous path and, therefore, are Euler paths and Euler circuits. Some of them are shown in **figure 5**. The black dot in each drawing shows the start and end of the tracing that the student used.

Additional Investigations

SMALL GROUPS OF STUDENTS THEN CONDUCTED different investigations and reported back to the rest of the class. Three of these investigations are described here.

Are kolam symmetric? Many kolam have reflectional symmetry, rotational symmetry, or both. Most other designs demonstrate what the students called "near symmetry"—when one detail in the design spoils the symmetry. Some examples of symmetrical kolam are shown in **figure 6**.

Are there mathematical relationships among the numbers of pulli, intersections, and edges in kolam? One student read online that, for a certain type of kolam, there are always the same relationships among the numbers of these features. In the kolam examined, there is exactly one pulli in every space enclosed by edges of the kolam. **Figure 7** shows examples of this type of design and the data that the students gathered. One group concluded that each kolam of this

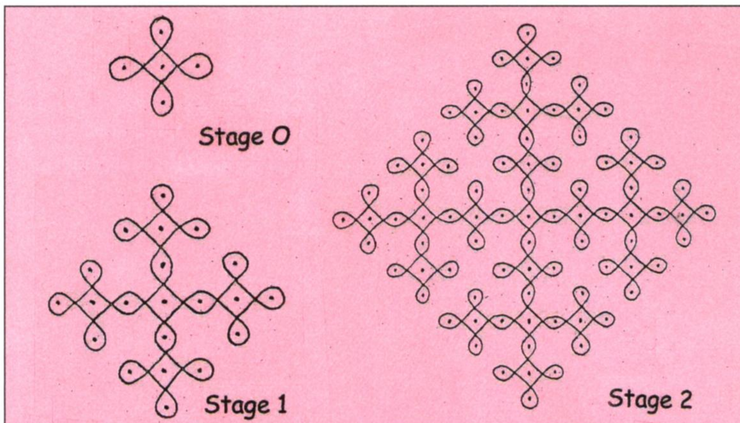


Kolam	Number of Pulli	Number of Intersections	Number of Edges
A	21	20	40
B	9	8	16
C	33	32	64
D	10	9	18
E	25	24	48

Fig. 7 Designs and data

type has the following characteristics:

- The number of edges is always even.
- The number of pulli is always one more than the number of intersections.
- The number of edges is always twice the number of intersections.
- If you are given two of the three numbers, you can always find the missing number, since the number of pulli plus the number of intersections equals the number of edges plus 1.



Stage #	Stage 0	Stage 1	Stage 2	Stage 3
Total # of Parts	1·4+1 5	5·4+1 21	21·4+1 85	85·4+1 341

The rule for any stage is total of previous stage ·4+1

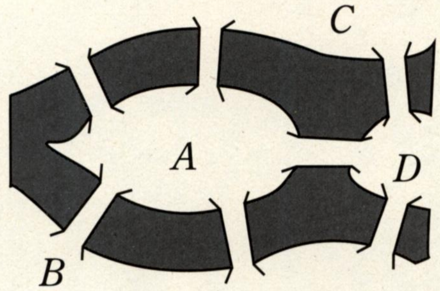
Fig. 8 Anklets of Krishna kolam

Are there "growing families" of kolam that follow certain rules? The students discovered several groups of kolam in which there seemed to be a growing pattern. Much like some of the geometric patterns they had studied earlier in the year (usually made of polygons), each group seemed to have stages that formed a growing pattern. The students investigated to see whether they could find the rule for any of these patterns. **Figure 8** shows one such group and the iterative rule that the students found. "Anklets of Krishna" is the name of a specific design of kolam. (It would be exciting to follow up this work with a unit on fractals.)

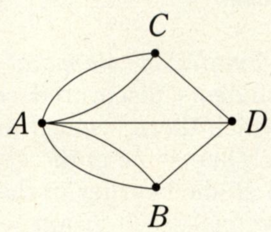
Concluding Activities

AT THE END OF THE TWO WEEKS AND AFTER ALL the students had shared their findings with classmates, a day of concluding activities wrapped up the unit. For homework the night before, the students were asked to think about or research other practical uses or applications of Euler paths and Euler circuits. They compiled the list that follows and discussed the uses in class.

- Work or travel routes (street sweepers, snow plows, mail carriers, etc.)
- Plumbing or wiring plans
- Routes in electronic games
- Molecular structure studies
- Computer languages
- Floor plans
- Digital circuits



A very famous problem involves the bridges of a town called Königsberg. The river divided the town of into four separate land masses, *A*, *B*, *C*, and *D*. Seven bridges connected various parts of town. On Sundays, it is said that the people of Königsberg would stroll through the town and try to take a walk, crossing each bridge exactly once, and return home. The famous mathematician, Leonard Euler, made a graph to represent the town and its bridges, as shown below.



Using his graph, Euler solved the problem. Can you?

Fig. 9 The Königsberg Bridge problem

As another concluding activity, the students were given the famous Königsberg Bridge problem (see **fig. 9**). Maddie's solution is typical of those turned in by the students (**fig. 10**). Finally, each student was asked to pick one aspect of the study and write a paragraph, using any notes or drawings. They generated the following list of topics:

- Kolam as Art
- Some Basics of Graph Theory
- The Mathematics of Kolam
- How Mathematics Can be Learned through Art
- Real-World Applications of Euler Circuits and Paths
- Finding Patterns in Kolam
- Ephemeral Art
- Connecting Numbers of Pulli, Intersections, and Edges in Kolam
- Euler and the Königsberg Bridge Problem
- Connecting Art and Mathematics

The following paragraphs were written by the students. They encourage me, as I hope they will the reader, to design more of these units for middle school mathematics classes.

You cannot go over each bridge and end up in the same place because every vertex is odd so there cannot be a path or circuit.

Fig. 10 Maddie's explanation represented that of many students.

Connecting Art and Mathematics

"We've been studying kolam in art and math classes. There's a lot of math in these designs—their symmetry, the paths and circuits they have in them, and all the number patterns and rules you can find. But there's also immense beauty. The shapes, the designs, the colors, and the symbols and traditions they show. They are considered ephemeral or temporary art, but I think the traditions and the connections between generations last a long time, just like the mathematics they contain."—*Evie*

Real-World Applications of Euler Circuits and Paths

"You might be wondering where in the real world we find uses for Euler paths and circuits. I did too. But suppose I am New York City's Subway Inspector. I need an optimal route to inspect the tracks and, hopefully, return to my starting point. Or I'm the DC Tour Guide, or the parking meter person, mailman, or in charge of garbage removal. I want to go everywhere I need to just once and return to where I began. Now there's one more way I never knew. The women of India actually have Euler circuits and paths in the kolam designs they create as a part of their everyday life. Euler might not have thought of them, but I'm glad someone did. They're beautiful, I felt like an artist myself, and I learned some fun math!"—*James*

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To read another article on teaching Euler's graph theory, see "Discovering Euler Circuits and Paths through a Culturally Relevant Lesson" by Rebecca R. Robichaux and Paulette R. Rodrigue in the March 2006 issue of MTMS.—*Ed.* □