JOURNAL OF APPLIED ECONOMETRICS
J. Appl. Econ. **13**, 185–202 (1998)

DOES MORE CALCULUS IMPROVE STUDENT LEARNING IN INTERMEDIATE MICRO- AND MACROECONOMIC THEORY?

J. S. BUTLER, T. ALDRICH FINEGAN AND JOHN J. SIEGFRIED*
Department of Economics, Vanderbilt University, Nashville, TN 37235, USA

SUMMARY

Using a selection bias correction model with ordered probit, we estimate how a second semester of calculus affects students' grades in intermediate economic theory. Selection bias correction is needed because similar aptitudes and interests often lead students to enroll and do well in both mathematics and economics. A sample of students enrolled in 49 classes of intermediate micro and 41 classes of intermediate macro is used to estimate the model. The results show a predicted payoff from a second semester of calculus of about one whole letter grade in intermediate micro, but no payoff in intermediate macro. © 1998 John Wiley & Sons, Ltd.

1. INTRODUCTION

Given that most undergraduate programmes in economics have a calculus prerequisite (Siegfried *et al*., 1991, n. 8) it must be widely believed that mathematical aptitude and skills are useful in learning undergraduate economics. The relationship between incremental and average values, for example, is pervasive in economics. Mathematics is useful in the decomposition of complex economic problems and in revealing fallacies in '*almost* always true' economic arguments. It encourages precision and helps train the mind to examine the side effects of a policy change, while clarifying relationships and improving student comprehension. The analogies between economics and other disciplines are conveniently made through mathematics (e.g. at an undergraduate engineering college where physics was required of all students, marginal cost was introduced to one of the authors of this essay as being 'just like' velocity).

The core of the typical undergraduate economics curriculum is principles of economics and intermediate microeconomics and intermediate macroeconomics. Principles is a service course, taken annually by upwards of a million American undergraduate students, many of whom specialize in disciplines in which mathematics plays no role. Mathematics beyond high school algebra is seldom used in principles courses, although there is some evidence that it may be useful (Brasfield, McCoy, and Milkman, 1992).

The intermediate theory courses, on the other hand, enroll but one-tenth of the students who take principles (Siegfried and Wilkinson, 1982). Most students in intermediate micro and intermediate macro concentrate on economics or related disciplines. Commonly adopted intermediate theory textbooks use some elementary calculus in at least the footnotes or appendices, and sometimes directly in the text. The pervasive role of constrained maximization (of utility, profits,

* Correspondence to: J. J. Siegfried, Department of Economics, Box 1819-B, Vanderbilt University, Nashville, TN 37235, USA. E-mail: siegfrjj@ctrvax.vanderbilt.edu

CCC 0883–7252/98/020185–18$17·50

Received 22 November 1993
Revised 15 September 1995

social welfare, etc.) in intermediate theory courses means that differential calculus is helpful; areas between cost and demand curves (e.g. deadweight welfare loss, consumer and producer surplus) are frequently measured and interpreted with integral calculus. Integral calculus can also help instructors to escape from the implausible assumptions of a linear world.

Most college and university calculus courses, including those at Vanderbilt University, follow a sequence with differential calculus covered in the first semester and integral calculus covered in the second.[1] Consequently, requiring integral calculus as a prerequisite for the intermediate theory courses would limit enrollment to students who have taken two semesters of calculus. As a result, the most common calculus prerequisite for the economics major in America is a one-semester requirement. Moreover, the popularity of a minimum one-semester calculus requirement has increased greatly in the last decade. While in 1980, only 26% of the economics major programmes of research universities and selective liberal arts colleges required a course in calculus (Siegfried and Wilkinson, 1982), by May 1990 81% required calculus either as a prerequisite or as a course in the curriculum (Siegfried *et al.*, 1991).

There appears to be widespread agreement that calculus is valuable, perhaps even essential, for understanding economics at the depth of an undergraduate major, but there is little information about *how much* calculus is useful. Because the economics major at Vanderbilt has required one semester of calculus for the last decade and the university's general education requirements encourage (but do not require) all students to take two semesters of calculus, almost two-thirds of Vanderbilt undergraduates who take intermediate-level theory have had two or more semesters of calculus, while the remainder have completed one semester. Using a sample of those students, we ask whether those who took two or more semesters of calculus performed measurably better in intermediate micro and intermediate macro than did students with only one semester of calculus. The answer has obvious relevance for how much calculus should be required.

Because relatively few of the intermediate theory instructors at Vanderbilt use integral calculus directly in their courses, one might be surprised if the second semester of calculus helps student performance in their courses. On the other hand, if training in mathematics primarily imparts general quantitative skills and increases students' confidence in the use of all kinds of quantitative methods, students who are exposed to an additional semester of calculus may perform better in intermediate micro and intermediate macro because their mastery of basic algebra and differential calculus is improved.

Students with high mathematical aptitude are more likely to do well in intermediate theory courses than their counterparts simply because this aptitude and ability to learn economics are positively related. These same students are also likely to take more calculus (or more intensive offerings in calculus). Hence, a simple comparison of grades in intermediate theory courses earned by students with different amounts of calculus might incur a serious selection bias. So far as we know, ours is the first study of grades in intermediate economic theory that attempts to control for this bias.[2]

---

[1] A one-semester survey of calculus course is frequently offered as an alternative to a two- or three-semester sequence. We use such a course as a benchmark for the analysis in this paper.

[2] In studying the effect of class size in introductory economics on subsequent performance in intermediate theory courses, Raimondo, Esposito, and Gershenberg (1990) included a control (*MATH*) for whether or not the student had fulfilled the calculus prerequisite (at least one semester of calculus). Their results showed a significant positive association between *MATH* and students' grades in both intermediate theory courses with a significantly larger correlation in micro than macro. While these findings are consistent with their expectations (and ours) about the beneficial effects of some calculus on the ability to learn intermediate economic theory, the lack of attention to selection bias casts some doubt on the

2. DESIGN OF THE STUDY

The Sample and Background Information

The data for this study come from the student records of nearly all Vanderbilt University undergraduates who entered the College of Arts and Science as freshmen between August 1983 and August 1986 and took Intermediate Microeconomic Theory (hereafter MICRO-2) or Intermediate Macroeconomic Theory (hereafter MACRO-2) or both. To obtain necessary control variables, we limit our sample to students who took at least one calculus course and both semesters of principles of economics at Vanderbilt before taking either intermediate theory course.[3] A total of 609 students satisfying these constraints in 49 different classes took MICRO-2; 490 students in 41 different classes took MACRO-2.

As in three-quarters of all US colleges and universities, students at Vanderbilt take two semesters of principles before intermediate-level theory. The first semester is macroeconomics (hereafter, MACRO-1), which is the prerequisite for second-semester microeconomics (MICRO-1). Most students who complete MICRO-1 and want more economics take intermediate micro before intermediate macro because of the course number sequence; but neither intermediate theory course is a prerequisite for the other. So the typical sequence of required courses (besides statistics) taken by economics majors is MACRO-1, MICRO-1, MICRO-2, then MACRO-2. As a result, there is usually no break or only a summer break between principles and intermediate micro, compared to a typical two- or three-semester gap between principles and intermediate macro.[4]

Unlike the two principles courses, which are taught in large classes, the two intermediate theory courses are offered in small sections (three to five per semester) with enrollments normally limited to 35 students; actual enrollments are usually smaller. Twenty-three different faculty members taught at least one section of MICRO-2 during the period studied; fifteen different instructors taught MACRO-2.[5] We use course grades to measure student understanding at the end of each

correct interpretation of these results. It is possible that the students in their sample who had fulfilled the calculus prerequisite would have done better in intermediate micro and macro even if they had not fulfilled the calculus prerequisite. In a study of principles of economics, Brasfield, McCoy, and Milkman (1992) found a similar positive effect of a first semester of calculus on student performance in both macro and micro. They too, however, made no attempt to control for selection bias. For a general discussion of selection bias in economics education research, see Melfi and Waldman (1987).

[3] When a student repeated a course, we counted the more recent grade in the calculus and principles courses, but only the first grade earned in the intermediate theory courses. Students who took the courses Pass/Fail or who withdrew were excluded. If these excluded students had taken considerably less calculus and had done (or would have done) less well in economic theory than the students remaining in our sample, the estimated regression coefficient for calculus attainment would be biased towards zero. There are two reasons, however, for believing that this particular selection bias is unimportant here. First, the attrition from excluding these students is very small, as they account for only 9% of the potential sample of MICRO-2 and just 6% of the MACRO-2 sample. Second, the mean math attainment of the students excluded from each sample was only about three tenths of one credit hour less than the mean of those who remained — a difference devoid of statistical significance (the relevant t-values for these differences are 0·70 for MICRO-2 and 0·50 for MACRO-2). For evidence that non-random data loss can sometimes lead to a serious selection bias problem, see Becker and Walstad (1990).

[4] Of the students in our sample who took MACRO-2, 19% completed it within one year after finishing MACRO-1; 69% did so some time during the second year; and 11% waited until the third year.

[5] Some instructors taught more sections than did others (this number varied from one to six), and the number of students in each section contained in our sample ranged widely (from one to 25). Even so, the four MICRO-2 instructors with the largest number of students in our sample together taught only 36% of the students; eight instructors together accounted for 59% of the total. Not surprisingly, there was more concentration of sample students in MACRO-2, which had four- and eight-instructor concentration ratios of 59% and 86%, respectively. Nonetheless, we think that the large number of instructors in our study enhances the extent to which our findings have relevance for intermediate theory courses in other

intermediate theory course. With no common exams and with each instructor free to choose his or her own text, some variation in course content and grading standards across sections clearly exists. We have minimized the resulting 'grade comparability' problem by including a control for differences in grading standards.[6]

Vanderbilt students differ not only in how many semesters of calculus that they take but also in the intensity and rigour of instruction that they receive in this subject (see Table I). There is a 'terminal' four-credit-hour one-semester course designed for those who seek only an introduction to the subject (Math 170); a two-semester introductory offering intended for most students (Math 171A-B); a more intensive two-semester sequence designed for students who plan to major in mathematics or math-related sciences (Math 172A-B); and two 200-level courses in intermediate calculus. Most students taking intermediate economic theory have completed either Math 170 or both semesters of Math 171; but enough can be found on each of the seven rungs of the calculus ladder to warrant treating each as a separate attainment group.

Estimation Methods and Expectations

To cope with the selection bias problem, we use a two-stage estimation procedure. The first stage employs an ordered probit model to predict the highest level of calculus attained by each student prior to taking each intermediate economic theory course.[7] Separate estimates are needed for each group of students because some took MICRO-2 but not MACRO-2 (or vice versa) and a few took more calculus in between these two courses. In the second stage, the student's grade in MICRO-2 or MACRO-2 (the 'outcome') is regressed on the actual level of calculus attained, the grade earned in that calculus course, the predicted residual in the grade equation that we would expect on the basis of the actual level of calculus attained, and a roster of control variables reflecting ability and motivation. Individuals are the unit of observation. Ordinary least squares estimation is used because there are twelve categories of grades which are commonly interpreted as cardinal measures of performance (as is implied by the calculation of 'grade point averages').

Selection bias applications have employed mainly bivariate models in the selection equation (Heckman, 1976), but Garen (1984) extended them to continuous selection variables and Jimenez and Kugler (1987) adapted the model to employ an ordered probit selection equation. Ordered probit (Greene, 1990, pp. 703–6) is appropriate for our application because various calculus courses can be ordered by the difficulty of material covered, but the differences among them cannot be measured cardinally.

universities and colleges. Among the instructors in our sample are economists currently teaching at Auburn University, Bar-Ihlan University, Baylor University, Berry College, Bloomsburg University, Duke University, Indiana University, North Carolina State University, Rhodes College, Seoul National University, Southern Illinois University, Syracuse University, Tulane University, the University of Loughborough, the University of Maryland, the University of Saskatchewan, the University of Tennessee-Chattanooga, Villanova University, and Wichita State University. A few still teach at Vanderbilt! Altogether faculty currently at 20 different institutions are represented in our sample. Only four of these intermediate theory instructors were advanced PhD students at Vanderbilt, and they were in their fifth or sixth year when they taught.

[6] We also included similar grade comparability controls for instructors of principles courses.

[7] The first use of ordered probit in economics education research was by Spector and Mazzeo (1980). Melfi and Waldman (1987) present a general discussion of this subject. See Becker and Kennedy (1992) for a graphical presentation of ordered probit.

Table I. Calculus courses taken by Vanderbilt undergraduates: descriptions and ranking in ordered probit

Course (Credit hours)	Description
Math 170 (4)	*Analytic Geometry and Calculus*. A basic course in the rudiments of analytic geometry and differential and integral calculus, with emphasis on applications. Designed for students who do not plan further study in calculus. No background in trigonometry is required or used in the course.
Math 171A-B (3-3)	*Analytic Geometry and Calculus*. A two-semester course in basic calculus recommended for most students. 171A: functions, limits, differentiation of algebraic functions, applications of differentiation, introduction to integration. 171B: differentiation and integration of transcendental functions, methods of integration.
Math 172A-B (4-4)	*Analytic Geometry and Calculus*. A more intensive two-semester course in basic calculus designed for students interested in majoring in math or math oriented science. 172A: functions, limits, differentiation of algebraic functions, integration, applications including extrema problems, areas, volumes, and centroids. 172B: differentiation and integration of transcendental functions, applications, methods of integration, coordinate geometry, polar coordinates, infinite series.
Math 221A-B (3-3)	*Intermediate Calculus*. A two-semester sequence earmarked for students who have completed 171A-B. Analytic geometry, polar coordinates, infinite series, vectors, parametric equations, vector analysis, partial differentiation, and multiple integrals. Prerequisite: 171B.
Math 222 (3)	*Intermediate Calculus*. A one semester course for students who have completed 172A-B; content equivalent to 221B. Indeterminate forms, solid analytical geometry, vectors in three space, partial derivatives, multiple integrals. Prerequisite: 172B.

Ordered probit ranking, bottom to top (cumulative credit hours shown in brackets)	Number of students who completed stated calculus course before taking: MICRO-2	MACRO-2
Math 170 [4] Calculus Survey	164	128
Math 171A [3] Calculus I	49	30
Math 172A [4] Calculus I for Math Majors	11	9
Math 171B [6] Calculus II	228	202
Math 172B [8] Calculus II for Math Majors	42	39
Math 221A [9] Calculus III	31	25
Math 221B or 222 [12 or 11][a] Calculus IV	84	57
Total	609	490

[a]Math 221B and 222 are considered equivalent courses. Students who take 221B complete 12 hours of calculus (171A-B followed by 221A-B), while those taking 222 complete 11 hours (172A-B followed by 222).

Application of ordered probit

The ranking of calculus courses by attainment level used in the ordered probit and the actual number of intermediate theory students at each level are reported in Table I. Except for the two bottom rungs, the cumulative number of hours of calculus completed rises with rung level. The exception involves placing the terminal, less rigorous Math 170 below the first semester of mainstream Math 171 — a decision recommended by knowledgeable faculty in the Vanderbilt Department of Mathematics and supported by the higher value of the likelihood function using the test of Vuong (1989). On the top rung, the cumulative number of hours of calculus varies somewhat among students due to different tracks.[8]

The instruments in the ordered probit include a student's SAT Math score; the number of years of advanced math, physics, and chemistry (each field considered separately) taken in high school; his or her *expected* major at the time of application for admission; the student's sex; and whether the student had achieved a high enough score on the College Board Foreign Language Achievement Test to satisfy the Foreign Language Option of Vanderbilt's general education requirement. The rationale for the last variable is as follows: besides showing competence in basic math and a foreign language, each student must demonstrate a higher level of proficiency in one of these areas. The foreign language option can be satisfied either by completing an intermediate level course in a foreign language or by achieving a target score on the College Board exam. The mathematical reasoning option is most often satisfied by taking a full year of calculus. We surmised that students who satisfied the foreign language option by test score would be less likely to take a full year of calculus because they wouldn't need it to satisfy the requirement.

The effect of calculus on performance in MICRO-2 and MACRO-2

In the second-stage regressions, which explain grades earned in MICRO-2 and MACRO-2, the predicted residual from the grade equation serves as a control for the selection bias problem discussed earlier. A set of dummy variables for actual calculus attainment is also included, along with a companion set of variables for the grade (expressed on a 4-point scale) earned in the highest level of calculus completed before taking the intermediate theory course. We expected a positive sign for each of these variables and larger positive coefficients for higher levels of actual calculus attainment; that is, students with more calculus (predicted and actual) and higher grades in their last completed calculus course should have done better in intermediate theory, *ceteris paribus*. In this schema, the level of attainment measures exposure to and practice using a set of tools which have applications in economic theory (especially micro theory), while the grade earned measures proficiency.

The second-stage regressions also include control variables, most of which are common to both MICRO-2 and MACRO-2. These include a student's SAT Math and Verbal scores — general measures of aptitude for learning — and his or her overall grade point average at the end of the freshman year, a control for effort and discipline in academic work once aptitude is held constant. A student's achievement in intermediate level theory is also likely to be influenced by a set of factors related to the subject matter of each course: the amount of potential interest in the course subject, the economic intuition that the student brings to the course, and the strength of the analytical foundation acquired in the prerequisite principles courses. All three factors are likely related to how well the student performed in principles; hence we include the grade earned in each semester of principles as a predictor of each intermediate theory grade. Owing to the longer lag and the lesser content overlap between macro principles and intermediate macro than between the two micro offerings, we expect MICRO-1 to be more important in explaining MICRO-2 grades than MACRO-1 is in explaining MACRO-2 grades. We were unsure about the

[8] Students who complete 171B go on to 221A, whereas those completing the more intensive 172B go directly into 222, which is the equivalent of 221B.

strength of either crossover association (between grades in macro principles and intermediate micro, and vice versa).

As noted earlier, we included two 'deflators' in each regression to control for differences in instructors' grading standards. For the intermediate theory course itself, the deflator measured the difference between the average grade of the students in that particular section of MICRO-2 or MACRO-2 and the cumulative GPA of those same students in all earlier courses. Instructors with positive residuals presumably have easier grading standards than those with negative residuals; therefore we expect a positive association between this deflator and the student's grade in intermediate theory, *ceteris paribus*.[9]

For the two introductory courses, which are often taken by freshmen, a less satisfactory grading standard deflator had to be used — the difference between the average grade of the students in MACRO or MICRO principles and the average grade of the same students in all other courses taken during the *same* semester. Again, positive residuals indicate easier grading standards, although the variance in grading standards should be smaller across sections of principles than across sections of intermediate theory because there are fewer different instructors of principles. In this case we expect a negative sign in our second-stage regressions since an easier grading standard in principles implies a level of real achievement lower than the student's actual grade.

Two additional control variables used in both second-stage regressions are class size at the end of the semester and each student's sex. Normally, no more than 35 students may enroll in any section of intermediate theory, but actual enrollments vary considerably. Of the 49 sections of MICRO-2 containing at least one student from this study, nine have enrollments under 20, nineteen have 20 to 29 students, and twenty-one contain 30 to 39 students. The size distribution of the 41 sections of MACRO-2 is similar.[10] Other things being equal, one might expect students in smaller classes to do better because more attention can be given to individual students, attendance may be better, and pedagogical techniques emphasizing active learning may be used. The risk is that class size may be endogenous, driven by instructor characteristics, perceived grading standards, etc., which could lead to a positive association between grade and class size. Hence we have no expected sign for this variable.

The same uncertainty applies to the influence of the sex of student. While one generalizes here at some peril, female students at Vanderbilt are often perceived to complete assignments more diligently and attend class more consistently. Whether a student's sex will matter after controlling for freshman GPA and everything else is an open question.

Finally, we include in each regression two sets of variables related to the time at which the student took the subject theory course. The first set reports the class year in which each course was taken. We surmised that students who take either course — but especially MICRO-2, a

---

[9] There is another possible reason why an instructor might have a positive residual: superior teaching. Suppose Professors Drone and Engage have exactly the same grading standards, cover the same topics, and even ask the same questions on exams. If each instructor teaches in the manner befitting his name, Engage's students will do better in the course, relative to their performance elsewhere, than will the victims of Drone.

While this result is entirely possible, the important questions here are empirical (and, we suspect, unanswered): how large is the variance in real teaching effectiveness across instructors in multisection courses, relative to the variance in grading standards; and what is the correlation, if any, between these two attributes? Until we know the answers, it would be unwarranted to assume that much of the variance in our deflator variables can be explained by differences in how much students in different classes learn. The various sections of MICRO-2 and MACRO-2 do not use common exams.

[10] Enrollment is under 20 students in eight sections of MACRO-2, 20 to 29 in eleven sections, and over 30 in twenty-two sections.

prerequisite for many upper-level electives — in the sophomore year are highly motivated and on a fast track in the major; those who do not take either course until their senior year may be latecomers to economics who had abandoned majors in other fields. So we expect a positive coefficient for sophomore year and a negative coefficient for senior year (junior-year takers are the reference group).[11]

A second set of timing variables classifies the students who took intermediate micro into three groups: those who took MICRO-1 and MICRO-2 in the same academic year (no gap), those who took MICRO-1 in the spring and MICRO-2 in the subsequent fall (summer gap), and those who waited at least one semester after completing MICRO-1 before taking MICRO-2 (longer gap). Because intermediate micro builds directly on micro principles, we urge prospective majors to complete this sequence as early as possible. So we expect students with a longer gap between these courses to do less well in MICRO-2.

The last set of timing variables applies to students who took intermediate macroeconomics. This set distinguishes whether the student took MACRO-2 after MICRO-2 (what 80% of our students do), both concurrently, or without having had MICRO-2 (the reference group). Given that much of contemporary macroeconomics is built on microeconomic foundations, one might expect students who have had MICRO-2 to do better in MACRO-2, *ceteris paribus*. Students who take the intermediate courses concurrently (only 8% of our sample) may have greater aptitude — or at least tolerance — for economic theory than their peers, but the academic demands of two theory courses may limit the time available for each. Consequently, the sign for this interval is uncertain.

3. EMPIRICAL RESULTS

Instrumental Regressions

The results of the ordered probits predicting the level of calculus attained by students who took each intermediate theory course contain few surprises; see Table II. As expected, students who had higher SAT Math scores, two years of advanced mathematics in high school, and expected to major in a natural science took more calculus than their counterparts who had lower scores, less math in high school, and expected to major in humanities or a social science other than economics. Prospective economics majors took slightly less calculus than the reference group, those undecided about their major, but the difference was not statistically significant.[12] Having taken at least one year of high school physics was also significantly associated with more calculus but one year of high school chemistry was not. The coefficients for more than one year of physics and chemistry have the expected positive sign, but are not statistically significant.

Contrary to our expectations, a high score on a foreign language achievement test appears to have no influence on the amount of calculus taken by these students, other things being equal.

[11] The reader may wonder why the student's actual major was not included as a control variable; surely concentrators in a discipline have a greater incentive to do well in a required course, especially one that lays a foundation for further work. We contemplated this addition until we realized that a student's grade in intermediate micro theory often helps him or her to decide whether or not to major in economics! Hence, to include this variable would create an identification problem. Even so, certain anomalous results reported later may be related to variations across subsets in the fraction of students majoring in economics.

[12] This result is not surprising, since student decisions to enroll in calculus are less likely to be based on whether calculus will help them learn intermediate economic theory in the future, but rather more likely to be based on students' interest or aptitude, class time, instructor's personality, etc.

Table II. Ordered probit estimates of level of calculus attained[a]

Variable[b]	Expected sign	Students taking MICRO-2 Mean (SD)	Students taking MICRO-2 Coefficient (t-value)	Students taking MACRO-2 Mean (SD)	Students taking MACRO-2 Coefficient (t-value)
Constant		—	−3·09 (5·48)	—	−2·62 (3·95)
SAT-math $\times 10^{-2}$	+	6·25 (0·60)	0·50[d] (6·12)	6·25 (0·60)	0·48[d] (5·23)
Foreign lang. proficiency [1,0]	−	0·11 (0·32)	0·02 (0·14)	0·09 (0·29)	0·23 (1·22)
Sex (female = 1; male = 0)	?	0·39 (0·49)	0·25[d] (2·59)	0·36 (0·48)	0·22[e] (1·96)
Expected major:					
Economics	?	0·34 (0·48)	−0·11 (0·86)	0·36 (0·48)	−0·18 (1·31)
Other social science	?	0·17 (0·38)	−0·29[e] (1·99)	0·15 (0·36)	−0·27 (1·59)
Natural science	+	0·21 (0·41)	0·43[d] (3·10)	0·20 (0·40)	0·32[e] (2·05)
Humanities	−	0·07 (0·25)	−0·37[e] (1·78)	0·07 (0·26)	−0·39[e] (1·80)
Years of HS Advanced Math (Y_m)					
$1 \leqslant Y_m < 2$	+	0·49 (0·50)	0·24 (1·07)	0·49 (0·50)	−0·00 (0·02)
$Y_m = 2$	+	0·45 (0·50)	0·93[d] (4·04)	0·45 (0·50)	0·67[d] (2·83)
$Y_m > 2$	+	0·01 (0·11)	0·77[e] (1·70)	0·01 (0·11)	0·28 (0·55)
Years of HS physics (Y_p)					
$1 \leqslant Y_p < 2$	+	0·67 (0·47)	0·26[d] (2·71)	0·67 (0·47)	0·27[d] (2·50)
$Y_p \geqslant 2$	+	0·02 (0·14)	0·38 (1·07)	0·01 (0·11)	−0·11 (0·20)
Years of HS chemistry (Y_c)					
$1 \leqslant Y_c < 2$	+	0·82 (0·39)	−0·12 (0·69)	0·82 (0·39)	−0·18 (0·75)
$Y_c \geqslant 2$	+	0·12 (0·32)	0·17 (0·75)	0·13 (0·34)	0·20 (0·75)
TRUNCATION POINTS[c]					
(1)	+		0·27[d] (7·29)		0·21[d] (5·59)
(2)	+		0·33[d] (8·16)		0·27[d] (6·46)
(3)	+		1·52[d] (20·32)		1·55[d] (18·26)
(4)	+		1·79[d] (23·07)		1·88[d] (20·73)
(5)	+		2·04[d] (23·72)		2·15[d] (20·58)
OVERALL RESULTS					
Log likelihood			−886·67		−698·09
Outcomes predicted correctly			37·9%		41·2%
Number of Observations			609		490

[a]The dependent variable is the level of calculus attained, as shown by the ordered probit ranking in the lower panel of Table I.
[b]Omitted reference groups: other or unstated expected major; less than one year advanced math, physics, and chemistry in high school.
[c]In an ordered probit, an underlying, normally distributed, latent variable has a mean which is a function of observable variables. The latent variable gives rise to a set of observed dummy variables for ordered categories based on ranges between unobserved but estimable truncation points which correspond to levels of effort, ability, or other factors reflected in the explanatory variables. If L categories are observed, there are $L-1$ truncation points, of which the first is normalized to be zero, so that $L-2$ truncation points are estimated and reported in the table. The values correspond to standard deviations of the latent normally distributed variable.
[d]Significant at 0·01 level, one- or two-tailed test, as appropriate.
[e]Significant at 0·05 level, one- or two-tailed test, as appropriate.

Perhaps the students with this proficiency, who were not required to complete the math reasoning option, chose to do so out of superior motivation. (They comprise only about 10% of each sample.) An additional selection factor is that 70% of the students who took MICRO-2 and 83% of those who took MACRO-2 were economics majors.

Female students in our samples took significantly more calculus than the male students, *ceteris paribus*. This association holds both before and after controlling for SAT Math score and other predictors. The explanation may be that women students at Vanderbilt find mathematics a more congenial discipline than economics. Half of recent Vanderbilt graduates with a concentration in math have been women, compared to one-third of those majoring in economics.

Altogether, the explanatory variables predict 38% of the calculus attainment outcomes among MICRO-2 students and 41% among MACRO-2 students. These fractions are respectable in view of the fact that most rungs on our calculus ladder differ by only one or two semester hours of cumulative work, not by a whole semester. By a more generous test, 55% of the predicted outcomes in the MICRO-2 sample and 59% in MACRO-2 matched their actual calculus attainment, plus or minus one rung.

Regressions Explaining Intermediate Theory Grades

The results of the second stage regressions are reported in Table III. We first discuss our findings on calculus attainment and grades; then we summarize the results for control variables. Both raise interesting policy issues.

While the sheer number of coefficients in Table III suggests a proliferation of explanatory variables, many of these are alternative categories of characteristics. For example, each student falls into only one level of calculus interval, one calculus grade interval, and so on. Counted this way, there are 15 explanatory variables in each intermediate theory regression.

The dependent variable is the letter grade in MICRO-2 or MACRO-2, converted to a 4-point numerical scale (A = 4·0, A − = 3·7, B + = 3·3, etc.). The mean grade received in each course was B − (2·65 in MICRO-2, 2·71 in MACRO-2).

Attainment and grades in calculus

Does a second semester of calculus lead to a better grade in intermediate economic theory? Based on the results in Table III, the answer is an emphatic 'yes' in the case of micro theory and a less certain 'no' for macro theory.

The term correcting for selection bias in calculus attainment from the first-stage probit has the expected positive sign in each second-stage estimate, but the coefficient is not significant at the 5% level. The correction term here serves only as a control for selection bias; its low t-value suggests only that selection bias is less important in this analysis than we had anticipated.

The effects of more calculus itself are revealed by the coefficients of the actual attainment levels. For MICRO-2, the story they suggest has two themes. First, it makes little difference *which* one semester exposure to calculus (reference group 170, or 171A or 172A) a student has had; the coefficients for 171A and 172A are not significantly different from the omitted benchmark, 170. Second, completing *either* two-semester sequence leads to a large and highly significant improvement in one's grade — one whole letter grade for students who completed the six-hour mainstream 171A-B, and one and one-half letter grades for students who finished the eight-hour intensive 172A-B. Both coefficients are easily significant at the 1% level. This much of the pattern fits our expectations. The size of the payoff is quite large.

Table III. OLS regressions explaining grades in intermediate theory courses

Variable[a]	Expected sign	MICRO-2		MACRO-2	
		Mean (SD)	Coefficient (t-value)	Mean (SD)	Coefficient (t-value)
Intercept	—	—	−1·64	—	−0·03
			(3·48)		(0·06)
Selection bias correction	+	−0·00	0·10	−0·00	0·10
(Predicted residual)		(0·92)	(1·29)	(0·92)	(1·20)
Level of calculus attained:					
Math 171A	+	0·08	0·39	0·06	−0·61
		(0·27)	(1·04)	(0·24)	(1·36)
Math 172A	+	0·02	−0·18	0·02	−1·72
		(0·13)	(0·21)	(0·13)	(1·16)
Math 171B	+	0·37	1·02[b]	0·41	−0·31
		(0·48)	(3·49)	(0·49)	(0·91)
Math 172B	+	0·07	1·52[b]	0·08	−0·42
		(0·25)	(3·53)	(0·27)	(0·91)
Math 221A	+	0·05	1·33[c]	0·05	−0·46
		(0·22)	(2·27)	(0·22)	(0·76)
Math 221B or 222	+	0·14	0·75[c]	0·12	−0·64
		(0·35)	(1·67)	(0·32)	(1·21)
Grade in last calculus course:					
Math 170	+	3·06	0·36[b]	3·10	−0·09
		(0·70)	(4·36)	(0·66)	(0·98)
Math 171A	+	2·22	0·26[c]	2·31	0·09
		(0·86)	(2·21)	(0·92)	(0·63)
Math 172A	+	2·94	0·42	3·14	0·40
		(0·80)	(1·54)	(0·53)	(0·87)
Math 171B	+	2·62	0·10[c]	2·62	−0·03
		(0·93)	(1·85)	(0·91)	(0·58)
Math 172B	+	2·63	−0·01	2·56	−0·04
		(0·90)	(0·10)	(0·93)	(0·30)
Math 221A	+	3·10	−0·09	2·96	−0·04
		(0·77)	(0·55)	(0·86)	(0·24)
Math 221B or 222	+	3·15	0·11	3·14	0·02
		(0·76)	(1·04)	(0·82)	(0·16)
Grade deflator of instructor in intermediate	+	−0·16	0·88[b]	−0·14	0·98[b]
theory course		(0·27)	(8·28)	(0·20)	(6·06)
Taken in Sophomore year	?	0·32	0·07	0·09	−0·14
		(0·47)	(0·94)	(0·28)	(1·21)
Taken in Senior year	−	0·06	−0·02	0·14	−0·14
		(0·24)	(0·13)	(0·35)	(1·56)
MICRO-1 and MICRO-2 in same academic year	+	0·35	0·04	—	—
		(0·48)	(0·46)		
At leat one semester between MICRO-1 and MICRO-2	−	0·27	0·13	—	—
		(0·44)	(1·85)		
MACRO-2 after MICRO-2	+	—	—	0·79	−0·05
				(0·41)	(0·51)
MACRO-2 and MICRO-2 taken together	?	—	—	0·08	−0·22
				(0·27)	(1·54)
Grade in MACRO-1	+	2·73	0·20[b]	2·76	0·23[b]
		(0·73)	(3·93)	(0·72)	(3·85)
Grade in MICRO-1	+	2·67	0·29[b]	2·72	0·32[b]
		(0·74)	(5·93)	(0·72)	(5·93)
Instructor's grade deflator:					
MACRO-1	−	−0·32	−0·33[c]	−0·32	−0·24
		(0·20)	(2·20)	(0·21)	(1·55)
MICRO-1	−	−0·29	−0·11	−0·29	−0·33
		(0·16)	(0·53)	(0·16)	(1·57)
Class size (intermediate theory course)	?	28·2	−0·002	28·0	−0·00
		(5·5)	(0·45)	(5·8)	(0·05)
Freshman Grade Point Average	+	2·79	0·29[b]	2·79	0·58[b]
		(0·46)	(3·04)	(0·45)	(5·58)
Sex (female = 1; male = 0)	?	0·39	0·13[c]	0·36	0·06
		(0·49)	(2·09)	(0·48)	(0·80)
SAT-Math score $\times 10^{-2}$	+	6·25	0·12[c]	6·25	0·05
		(0·60)	(1·75)	(0·60)	(0·63)
SAT-Verbal score $\times 10^{-2}$	+	5·56	0·04	5·57	−0·04
		(0·67)	(0·78)	(0·66)	(0·77)
OVERALL RESULTS					
Mean (SD) of dependent variable		2·65		2·71	
		(0·91)		(0·86)	
Adjusted R^2		0·44		0·39	
Number of observations		609		490	

[a]Omitted reference groups in MICRO-2 regression: attained Math 170; took MICRO-2 in Junior year; took MICRO-1 in spring, MICRO-2 next fall. Omitted reference groups explaining MACRO-2: attained Math 170; took MACRO-2 in Junior year; did not take MICRO-2 before or during MACRO-2.
[b]Significant at 0·01 level, one- or two-tailed test as appropriate.
[c]Significant at 0·05 level, one- or two-tailed test as appropriate.

The grade earned in calculus also matters for students with only one semester's work, where one full letter grade better is associated with an improvement of about one-third of a grade in MICRO-2, or one 'notch' in a +/− grading scheme. For students who complete at least two semesters of calculus, the grade in the last semester matters much less — perhaps because the new topics are not directly relevant for students of MICRO-2 but still reinforce command over basic quantitative relationships. A useful policy lesson can be drawn from the fact that the second semester grade coefficients are much smaller than the associated attainment coefficients. Many students are reluctant to undertake a full year of calculus because the second semester is harder than the first and they fear earning a low grade in the second. The coefficients in Table II suggest, however, that a student who completes 171B with a C − could expect to do 0·9 of a letter grade better in MICRO-2 than he or she would have done by taking the terminal Math 170 and getting the average grade of B in it.[13] Of course, taking two semesters of calculus may still reduce the student's overall GPA, depending on the grades actually earned in each semester of calculus and in Econ 231.

Is the return in intermediate micro understanding from taking a second semester of calculus large enough to warrant requiring all concentrators to do so? The important role of micro theory in many economics electives invites an affirmative answer. Critics would point out, however, that large benefits are not universal (the standard error for the coefficient of Math 171B is 0·29), that the amount of calculus actually used in most sections of intermediate micro is relatively small, that some excellent economics students take only one semester of calculus, and that informing prospective majors of the probable benefits from a second semester is preferable to requiring it.[14]

For takers of MICRO-2, there is a third conclusion to report: contrary to our expectations, the gain from taking three or four semesters of calculus appears to be smaller than the gain from stopping with two. In part, this appearance is deceptive: students who complete 171B and want more calculus go on to 221A, and the MICRO-2 coefficient for 221A (1·33) is larger than that for 171B (1·02), although not significantly so. But there is also a real paradox. Students continuing on from 172B go straight into 222, the equivalent of 221B, and the coefficient for this much calculus (11 hours or 12) is smaller than those for 171B and 172B.[15]

There are several possible explanations for this puzzle. One relies on the fact that the top rung of the calculus ladder contains relatively few economics majors. Students taking MICRO-2 or MACRO-2 because it is required by their major program — and a prerequisite for upper-level electives — have a greater incentive to excel than do students taking the same course for other reasons.[16] A second possibility is that more of these top-rung students in calculus may be majors in mathematics or natural sciences with heavier course loads that leave less time for work in economics than economics majors usually have. A third is that students with eleven or more hours of calculus may have less of the intuition needed to excel in economics, especially in policy

[13] This inference follows from the fact that the attainment coefficient for Math 172B (1·02) is ten times as large as the coefficient for a one-letter grade difference in that course (0·10). The average grade in 172B is a B−. So a one-letter grade reduction in 172B (from B− to C−) reduces the net gain from completing 171B by only one tenth of a letter grade (from 1·02 to 0·92).

[14] The size of the calculus coefficients is sensitive to the specification of the model. In an alternative estimate that did not include the grade in *macro* principles as a predictor of grade in *micro* theory, the coefficients for levels 171B, 172B, 221A, and 221B or 222 were all about 0·2 of a grade smaller.

[15] The regression coefficients for Math 172B, 221A, and 221B or 222 are not significantly different from each other; but the difference between that for 172B (1·52) and the one for 221B or 222 (0·75) just misses significance at the 10% level with a two-tailed test; the t-value is 1·63.

[16] Among the MICRO-2 takers in our study, economics majors accounted for 70% of the students who had completed 171B and 172B, but only 45% of those taking 221B or 222.

applications. In other words, beyond some point in undergraduate studies, proficiency in mathematics and economics may be gross substitutes rather than complements for producing learning *in intermediate theory courses*.

To test the first of these hypotheses, we dropped the 218 non-majors from our MICRO-2 sample and re-estimated the MICRO-2 regressions. If less motivation by non-majors accounts for the puzzling pattern, then it should disappear once these students are dropped. But as Table IV shows, a similar profile of coefficients (but with generally smaller t-values) appears for economics majors. While some difference in effort between majors and non-majors cannot be ruled out, the main reasons for this pattern would seem to lie elsewhere.

How much do the beneficial effects of more calculus depend on the amount of math students encounter in intermediate micro theory? We were unable to classify MICRO-2 instructors by how much calculus they used in class, but we did identify the textbooks used in 39 course sections. A subset of 284 students in our sample read Browning and Browning (1986), a popular text whose first two editions contained no calculus and relied heavily on graphs. A smaller subset of 168 students used five other textbooks that made some use of calculus, at least in footnotes or chapter appendices, and more use of algebra, including expressions with delta notation.[17] On re-estimating the second-stage regressions for each subset, we found no significant association between grade in MICRO-2 and amount of calculus taken by those students in sections that used Browning and Browning, but a strong and highly significant association in the subset which used more mathematical texts. There, the estimated payoff from a second semester of math was roughly two and one-half letter grades higher than for the benchmark group of students who had completed only Math 170, other things being equal. Students who had taken three or more semesters of calculus reaped a gain of about three letter grades. Even the students who had taken only one semester of the more rigorous Math 171 did significantly better than those who took 170, although there was also a payoff to earning a higher grade in 170. Thus it appears that the return to taking more calculus and to assimilating more of it in a beginning survey course is strongly related to the direct use of mathematics in intermediate micro theory.[18]

In marked contrast to our findings for intermediate micro, the results in Table III suggest that *neither* the amount of calculus that a student has had, at least beyond the most rudimentary introduction, nor how well he or she did in it has any effect on performance in intermediate macro. In fact, the coefficient for every attainment level (with respect to Math 170, the reference rung) has a negative sign! Given the lesser use of calculus in the teaching of MACRO-2 than in MICRO-2, we were prepared to find that more calculus helped less in learning macro theory than micro. But we still thought that the revisiting of algebra, the stimulus to orderly thinking, the training in precision of expression, and the emphasis on relationships within systems of equations that are a part of learning more calculus would have some payoff in understanding macroeconomic theory. Evidently, that is not so.

To find out whether the consistently negative signs for all six calculus levels in Table III had some collective statistical significance, we re-estimated the MACRO-2 equation after dropping all the attainment variables. The F-ratio used to test the resulting decline in the sum of squared

[17] The five textbooks in this subset were Eaton and Eaton (1988), Gould and Lazear (1989), Hirschleifer (1988), Thompson (1981), and Varian (1987). (Some sections may have used different editions of these texts.)

[18] Students who enrolled in sections of MICRO-2 using more mathematical texts had completed only a little more calculus, on average, than students in sections using Browning and Browning; the percentage of each subset that had completed at least two semesters of calculus was 69 and 61, respectively. Thus, the differential return to more calculus in these subsets is not attributable to differences in math attainment.

Table IV. Regression coefficients for calculus attainment explaining grade in MICRO-2: all students and economics majors alone

Level of calculus attained	Expected sign	All students coefficient (t-value)	Econ majors coefficient (t-value)
Math 171A	+	0·39	0·77[b]
		(1·04)	(1·69)
Math 172A	+	−0·18	0·02
		(0·21)	(0·02)
Math 171B	+	1·02[a]	1·20[a]
		(3·49)	(3·40)
Math 172B	+	1·52[a]	1·31[a]
		(3·53)	(2·49)
Math 221A	+	1·33[a]	1·18[b]
		(2·27)	(1·81)
Math 221B or 222	+	0·75[b]	0·55
		(1·67)	(0·97)
Mean (SD) of dependent variable		2·65	2·71
		(0·91)	(0·85)
Adjusted R^2 from complete regression		0·44	0·46
Number of observations		609	391

[a]Significant at the 0·01 level, one- or two-tailed test as appropriate.
[b]Significant at the 0·05 level, one- or two-tailed test as appropriate.

residuals is non-significant (0·60). The same conclusion holds when both the attainment and grade variables are dropped.[19]

Instructors' grade deflators in intermediate theory courses

This variable, which we interpret as an (inverse) indicator of grading standards, turned out to play a large role in both regressions, with t-values of 8·28 in MICRO-2 and 6·06 in MACRO-2. Since the grade deflator measures the difference between the average grade that the instructor awarded in intermediate theory and the average grade that his or her students had received in all previous semesters' work at Vanderbilt, it is not surprising that the regression coefficient in each theory course is almost unity. That is, a student who took a section of theory from an instructor with an *ex-post* deflator of plus one could expect to receive a course grade higher by almost one letter than would have been received from an 'average' instructor with a deflator of near zero.[20] The estimated coefficients for the grade deflator underscore the importance of controlling for this factor.

[19] When we showed these results to one of our colleagues who teaches intermediate macro, he pointed out how little calculus is typically used in this course, especially in comparison to MICRO-2, and questioned whether our original expectation of a positive payoff from a second semester was well founded. A professor of mathematics went further, wondering if students with a full year or more of calculus might perhaps feel frustrated by this constraint, especially after finding many more applications of calculus in MICRO-2. Still another colleague suggested that calculus was more useful in some models of modern macro theory than in others, and that if we could identify the instructors who emphasized the 'Minnesota–Rochester' branch of macro, we might observe some payoff to more calculus for those students who had taken MACRO-2 from them.

[20] In fact, the mean value of this deflator was slightly negative in both theory courses (−0·16 in micro and −0·14 in macro), as one might expect for required courses of above-average difficulty.

Grades in economic principles

Students who have done better in micro principles do better in MICRO-2, and likewise for macro principles and MACRO-2. One letter grade better in each principles course was associated with about one quarter of a grade improvement in the companion intermediate course, *ceteris paribus*. The stronger (t-value) linkage between the two micro courses presumably reflects a larger overlap in content and a shorter time lapse between them.

There is also a strong relationship between the converse pairs of courses. One letter grade higher in MACRO-1 is associated with an improvement of 0·2 of a grade in MICRO-2, while the coefficient for the nexus between grades in MICRO-1 and MACRO-2 is even larger, 0·3. It might seem odd that a student's performance in intermediate *macro* depends more on how well he or she did in *micro* principles than in macro principles. The explanation, we think, lies in the fact that our students take MICRO-1 after MACRO-1, that MICRO-1 is more difficult, and that grades in introductory courses measure aptitude and motivation for learning economics more than what is learned.

Our grade deflators for the principles courses have the expected negative signs, but the coefficients are smaller than their counterparts for intermediate theory, and only one is statistically significant.

Freshman GPA

This general measure of student effort, discipline, and ability is another highly significant predictor of grades in both theory courses. A letter grade advantage in freshman GPA is associated with an improvement of about 0·3 of a grade in MICRO-2 and 0·6 of a grade in MACRO-2, which suggests that the skills needed to do well in macro theory may be less specialized to economics than those required for success in micro theory.

SAT scores

Of the pair of SAT scores included in each regression, only the quantitative score predicting the student's grade in MICRO-2 achieves statistical significance at the 5% level (one-tailed test). This positive coefficient is small: an additional 100 points on the Math SAT is associated with only about one-eighth of a grade improvement in MICRO-2. Most studies find verbal SAT scores to be positively and significantly associated with test performance in principles courses; mathematical aptitude seems to be more important for scoring well on locally constructed principles tests such as those used here (Siegfried and Fels, 1979, p. 937). With grades in principles and freshman GPA included as predictors, it is hardly surprising that SAT scores play only a minor role in explaining performance in intermediate theory.

Class size

In neither intermediate theory course is there any significant association between class size and grade. Two possible reasons come to mind. First, while there is a substantial range of class sizes in these sections (16 to 39 in MICRO-2, 6 to 34 in MACRO-2), all or most of them may be small enough to capture most of the beneficial learning effects of a 'small class'.[21] A second reason is

[21] Alternatively, if instructors of intermediate theory teach classes of 15 the same way they would teach classes of 35 — a plausible hypothesis if they *expected* to have a class of 35 — then *none* of these sections may have captured the potential gains in learning from a 'small class'!

Raimondo, Esposito, and Gershenberg (1990) found that students who took macro *principles* in large lecture sections of 200 to 300 students did significantly less well in *intermediate* macro than students who had taken macro principles in small sections of 25 to 35. Curiously, the same inverse relationship was not observed for the micro sequence.

the possible feedback from instructor popularity to class size, which might offset any negative influence of class size on learning. But the instructor's grade deflator may already account for student withdrawals.[22]

Timing variables

None of our hypotheses concerning how a student's grade in each theory course might be influenced by when the course was taken receives much support from the data. First, the class year in which the course was taken seems to make no difference. Second, students who take both intermediate theory classes together seem to do a little less well in macro than those with no exposure to intermediate micro (this finding just misses significance at 10% on a two-tailed test). More surprisingly, macro theory takers who have completed micro theory do no better than those with no prior exposure to micro theory. Perhaps the spillover benefits from intermediate micro to macro have been overrated.

Worse still, the results related to the timing of beginning and intermediate micro contain a puzzle. Not only do students who take this sequence in the same academic year do no better than those with a summer gap (our reference group), but students with longer intervals actually do a little better than their peers (the regression coefficient is 0·13 and significant at the 10% level on a two-tailed test). Perhaps those who return to micro theory after a semester or longer do so with renewed interest, or perhaps they have taken in the interim another course or two in economics that proves of value. Even so, the results are somewhat disquieting. While our advice to majors to take MICRO-2 immediately after MICRO-1 may be well founded, it rests more on intuition than on evidence.[23]

Sex

Other things equal, women do a little better than men in micro theory, but about the same as men in macro. The regression coefficient for sex = female is 0·13 and significant at the 5% level for explaining grades in MICRO-2, but only 0·06 and non-significant in MACRO-2. Here is one possible explanation for the micro results. Previous research has found that women tend to do less well on multiple-choice exams than men, while the reverse is true on essay questions (Lumsden and Scott, 1987).[24] Many sections of principles courses at Vanderbilt, particularly micro principles, rely heavily (but not exclusively) on multiple-choice questions. Intermediate theory courses use mainly essay questions. Hence, controlling for the grade earned in principles is

[22] We were surprised to find that there was virtually no correlation in either sample between class size and the instructor's grade deflator: the simple correlation was zero for MICRO-2 and 0·05 (non-significant at 20%) for MACRO-2. This result does not show that class size is unaffected by the popularity of the instructor, but it does suggest that grading standards are not the main engine of this popularity, at least in small sections of intermediate theory.

[23] Raimondo *et al.* (1990) do find that the length of the period between completing the principles sequence and taking intermediate micro (counting the summer hiatus as a semester) has a significant negative influence on the student's grade in the latter course.

[24] Lumsden and Scott's evidence of the relative advantage of men and women on essay and multiple-choice exam questions is based on a large-scale study of introductory economics involving many institutions in Britain. Their evidence supports the psychological assertions that females have higher verbal skills than do males, at least at the typical age of university students taking their first economics course. On multiple-choice questions, female students scored 3% lower on micro questions and 4% lower on macro questions than did male students. On essay questions over the same content, female students scored 7% higher than males on both micro and macro questions (Lumsden and Scott, 1987, p. 370). An exam format that roughly balances these relative advantages would be one-third essay and two-thirds multiple-choice. That, of course, presumes that there are no real differences between performance of men and women in economics except those based on exam format. Moreover, the relative advantage college-aged women have on essay exams is not beyond question (Williams, Waldauer and Duggal, 1992a,b).

not really sex neutral: on balance, a given grade earned by a female tends to underpredict her performance in intermediate theory relative to what we would predict for a male who receives the same grade. The problem here is that this argument should apply to both intermediate theory courses, whereas we observe that women do significantly better only in intermediate micro.[25]

4. CONCLUSION

Using a large sample of students who took intermediate micro and intermediate macro from many different instructors, we find that a second semester of calculus is associated with better performance of students in intermediate micro, but not in intermediate macro. A second semester of calculus is associated with a full letter grade improvement in intermediate micro performance, *ceteris paribus*. The benefits are larger in sections that use textbooks containing more mathematics. How well students do in calculus seems related to their performance in intermediate micro if they take only one semester of calculus; but if they take more calculus, their performance in mathematics courses is unrelated to their success in intermediate micro theory. Perhaps they are beyond the level of calculus understanding that can help them in intermediate micro theory. How well students do in calculus at any level is unrelated to their success in intermediate macro.

The greater success in intermediate micro theory associated with a second semester of calculus is independent of the selection bias caused by students with greater quantitative aptitude simultaneously enrolling in more calculus courses and doing better in intermediate economic theory. Although we control for it, the evidence from our sample indicates that this bias is not serious.

Before these findings can be used as a basis to require two semesters of calculus as a prerequisite for interemdiate micro theory, opportunity costs need to be considered. In the larger context of a liberal arts education, the learning represented by an additional letter grade in intermediate micro theory plus the independent benefits of calculus II may be less valuable than the liberal arts elective that the second calculus semester supplants.[26] On the other hand, it is widely believed that success in intermediate micro theory creates additional benefits by improving student learning in economics electives with microeconomic foundations. So the stakes may be greater than one letter grade in intermediate micro theory alone.

ACKNOWLEDGEMENTS

We wish to thank Dean John H. Venable for the data, Hao Zhang for skilful computer programming, and a large number of colleagues, especially Kathryn Anderson, William Becker, William Damon, Rendigs Fels, William Greene, Cliff Huang, Peter Kennedy, Richard Larsen, Jane Lillydahl, Robert Margo, Robert Rich, Thomas Tietenberg, and Fred Westfield, for helpful advice.

[25] A possible reconciliation of the different effects in micro and macro is the fact that multiple-choice questions are used more frequently in principles of micro courses than in principles of macro courses at Vanderbilt.

[26] It would be difficult to estimate the contours of this opportunity cost, as the particular courses forgone to make room for an additional semester of calculus vary widely across students. One might compare the number of hours of coursework taken in broadly defined fields by MICRO-2 students with two semesters of calculus with data for students with only one semester. But that comparison at best would illuminate only what students who voluntarily took more calculus chose to give up. One cannot assume that future concentrators in economics who take more calculus because of a programme requirement would make the same sacrifices. Indeed, some might choose a different major.

REFERENCES

Becker, W. E., Jr and P. E. Kennedy (1992), 'A graphical exposition of the ordered probit', *Econometric Theory*, **8**, No. 1, March, 127–131.

Becker, W. E. and W. B. Walstad (1990), 'Data loss from pretest to posttest as a sample selection problem', *Review of Economics and Statistics*, **72**, No. 1, February, 184–188.

Brasfield, D., J. McCoy and M. Milkman (1992), 'The effect of university math on student performance in principles of economics', *Journal of Research and Development in Education*, **25**, No. 4, Summer, 240–247.

Browning, E. K. and J. M. Browning (1986), *Microeconomic Theory and Applications*, 2nd edition, Little, Brown, Boston and Toronto.

Eaton, B. C. and D. F. Eaton (1988), *Microeconomics*, W. H. Freeman, New York.

Garen, J. (1984), 'The returns to schooling: a selectivity bias approach with a continuous choice variable', *Econometrica*, **52**, No. 5, September, 1199–1218.

Gould, J. P. and E. P. Lazear (1989), *Microeconomic Theory*, 6th edition, Richard D. Irwin, Homewood and Boston.

Greene, W. H. (1990), *Econometric Analysis*, Macmillan, New York.

Heckman, J. J. (1976), 'The common structure of statistical models of truncation, sample selection, and limited dependent variables and a simple estimator for such models', *Annals of Economic and Social Measurement*, **5**, No. 4, Fall, 475–492.

Hirschleifer, J. (1988), *Price Theory and Applications*, 4th edition, Prentice Hall, Englewood Cliffs, NJ.

Jimenez, E. and B. Kugler (1987), 'The earnings impact of training duration in a developing country: an ordered probit selection model of Colombia's *Servicio Nacional de Aprendizaje* (SENA)', *Journal of Human Resources*, **22**, No. 2, Spring, 228–247.

Lumsden, K. G. and A. Scott (1987), 'The economics student reexamined: male–female differences in comprehension', *Journal of Economic Education*, **18**, Fall, 365–375.

Melfi, C. A. and D. M. Waldman (1987), 'Limited and discrete dependent variable models: use, applications, and comparisons', in W. E. Becker and W. B. Walstad (eds), *Econometric Modeling in Education Research*, Kluwer-Nijhoff, Boston, pp. 185–205.

Raimondo, H. J., L. Esposito and I. Gershenberg (1990), 'Introductory class size and student performance in intermediate theory courses', *Journal of Economic Education*, **21**, No. 4, Fall, 369–381.

Siegfried, J. J. and R. Fels (1979), 'Research on teaching college economics: a survey', *Journal of Economic Literature*, **17**, September, 923–969.

Siegfried, J. J. and J. T. Wilkinson (1982), 'The economics curriculum in the United States: 1980', *American Economic Review*, **72**, No. 2, May, 125–138.

Siegfried, J. J., R. L. Bartlett, W. L. Hansen, A. C. Kelley, D. N. McCloskey and T. H. Tietenberg (1991), 'The status and prospects of the economics major', *Journal of Economic Education*, **22**, No. 3, Summer, 197–224.

Spector, L. C. and M. Mazzeo (1980), 'Probit analysis and economic education', *Journal of Economic Education*, **11**, Spring, 37–44.

Thompson, A. A. (1981), *Economics of the Firm*, 3rd edition, Prentice Hall, Englewood Cliffs, NJ.

Varian, H. R. (1987), *Intermediate Microeconomics*, 1st edition, W. W. Norton, New York.

Vuong, Q. H. (1989), 'Likelihood ratio tests for model selection and non-nested hypotheses', *Econometrica*, **57**, March, 307–333.

Williams, M. L., C. Waldauer and V. G. Duggal (1992a), 'Gender differences in economic knowledge: an extension of the analysis', *Journal of Economic Education*, **23**, Summer, 219–231.

Williams, M. L., C. Waldauer and V. G. Duggal (1992b), 'Gender differences in economic knowledge: a further extension of the analysis', *Quarterly Review of Economics and Finance*, **32**, Winter, 138–143.

