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TRIZ-Fractality of mathematics

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Abstract

In the report one of the possible variants of solving contradictions “volume of knowledge – mastering” is considered. It is proposed the procedure of systematization knowledge on the base of its fractality. It is proved the assumption that knowledge is also fractal as everything in nature. There are analogues in the development of traditional nature objects: crystals, plants, animals, and knowledge. The procedure of systematization of mathematic knowledge is illustrated on the example of the development of Numerical methods. Also the algorithm of TRIZ-fractal maps forming is presented and the teaching procedure by TRIZ-fractal map is described.

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1. Introduction

At the previous “TRIZ Future 2006” conference in Kortreik one of the authors proposed a fractal knowledge model [1]. This model allows to impart knowledge in compact systematic form and therefore considerably to improve teaching efficiency.

Fractality of knowledge is used explicitly or implicitly by several TRIZ-experts (Nikolai Khomenko, Larry Smith, Nikolay Shpakovsky and others). Nikolay Shpakovsky (TRIZ-master since 2006) is the most consistent with use of fractal method. He published the book “Trees of evolution” [2]. Only evolution processes of technical systems are considered in this book. TRIZ-researchers practically do not attend to evolution of non-technical (in particular mathematical) systems.

Fractality of knowledge is not instrumental as a separate concept. In the previous article it was proposed to pay attention to three components: “the seeding grain”, “resources” and “rules of construction”. “The rules of construction” are considered as TRIZ tools (methods of technical and physical contradictions solution and Su-Field

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conversion). Acceptance of these assumptions allows introducing a new concept – TRIZ-fractality. TRIZ-fractality means self-similarity of transformations based on TRIZ tools.

“The rules of construction” of technical systems are closely analyzed in the Nikolay Shpakovsky book. It is interesting to extend knowledge fractality instrumental approach also to non-technical systems. In the article there is an attempt to apply TRIZ-fractal approach to mathematics as a system in particular to numerical methods.

2. About knowledge fractality

The development of knowledge can be compared with the development of natural objects: plants, crystals, animals. Mandelbrot showed in his works, that everything in nature is self-similar, that is fractal [3].

Simply put, the development is realized by self-recurrence, self-imitation of an initial specimen or a pattern. Let's illustrate this position on the example of crystal growing. The shape of a crystal is defined by the seeding grain (or the initial specimen). Three components are necessary for a crystal growing: a seeding grain, constructional material and the rules of construction. We try to obtain the structure fern, using Mandelbrot Fractal Geometry. We take geometrical object represented in figure 1a as “the seeding grain” (only three upper lines are considered as the “seeding grain”, the vertical line located below does not relate to the “seeding grain”). Let's formulate the rules of “transition” from the current into the higher state of system, i.e. the rules of growing of “fractal fern”. The proportionally reduced copy of entire model substitutes each element (line) of prototype; thus, one step of the iteration is realized (see figure 1 b, 1 c).

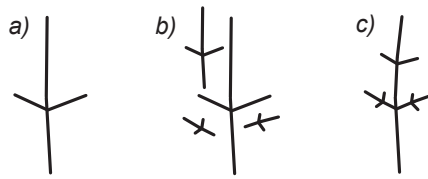


Figure 1: One-step realization of the iteration of fractal image construction.

In general, the quantity of iteration is unconfined and the more iteration is realized, the more the fractal model is adequate to real object and “the fractal fern” is nearer to the real. In figure 2 the sequence of iterations is represented: zero (prototype), the third, the fifth and the eighth.

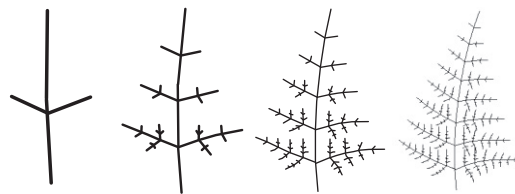


Figure 2: A fractal model of fern.

It should be noted that even in such simple plants as fern the prototypes and the rules of “transition” from the iteration to the iteration are more complex than in the example given above. However, there is a basic principle of self-similarity in all plants. Animals have more complex process of self-similarity. It is possible to assume that all the necessary information about the prototypes (patterns) in animals is placed in the genes [4], and the laws of nature determine the rules of “transition”. By the way, it is the knowledge of these laws that is the basic purpose of the education.

The evolution of the self-developing (“living”) organisms can be represented in the following form: crystals, algae, corals, ferns, fishes, highest plants, birds, mammals, men. That is to the present moment man possesses the highest level of complexity. It is possible to assume that in man the fractality should appear not only on the physical level, but also on the spiritual level, i.e. in the consciousness. A man experiences with the consciousness, i.e. the

system of knowledge is constructed. Then it is natural to assume that the knowledge is also fractal. In fact, knowledge is the reflection of the world picture, and if the world is fractal knowledge is fractal too.

Let's see the analogues of fractal concepts for knowledge: fundamental and applied (Table 1).

	Pattern	Resources	Iterative rules
Fractal fundamental knowledge	Axioms, starting positions	Observations, facts	Developing axioms according to the principles of integration (Dao bears one, one bears two, two bear three, three bear all (parable 42) [5])
Fractal applied knowledge	Fundamentals of correspondence problem domain.	Resources of problem domain	Developing axioms according to Su-Field conversion and ways of solving contradictions.

Table 1: Analogues of fractal and quasi-fractal object concept.

As it is shown in the table TRIZ tools are iterative rules for applied knowledge. In this case it is possible to use a new concept that is TRIZ-fractality. TRIZ-fractality is fractal transformations of systems (knowledge, technical and social objects, etc) in case when TRIZ tools are used. These tools, first of all, are methods of technical and physical contradictions solution and Su-Field conversion.

Now we should consider the way of using of TRIZ-fractality for solution of the main contradiction of the Higher education, that is contradiction between the volume of transferred information and time that is required for its learning. At the present time knowledge, skills and experience are transferred in non-systematic or weak-systematic form. At the same time every student creates his own system of ordered knowledge that is different for different sciences and does not correspond to teacher's systematization. It is proposed to teach student first definite concrete system, that is TRIZ. Then all Subjects will be trained according to this system. In this case systematization will be one and the same for all Subjects, will be created consciously and will correspond to teacher's systematization.

Let's consider for example the process of revision of the section of Mathematics "Numerical Methods" according to the TRIZ-fractality.

3. Evolution of numerical methods

Numerical methods are defined as methods of approximate solution of typical mathematical problems, which come to performing of finite quantity of elementary number operations. These methods are various [6-9]: linear algebraic equation systems solution, equations and nonlinear algebraic equation systems solution, numerical integration and differentiation, solution of Cauchy problem for ordinary differential equation, etc. In addition there is a list of specific methods for almost every above-mentioned field of application (Appendix A). Due to the area limits part of methods is shown with numbers. The description of these numbers is mentioned below.

1.1.1 – Cramer method; 1.1.2 – Gauss method with a complete choice of a conducting element; 1.1.3 – Gauss method without a choice of a conducting element; 1.1.4 – Gauss method with a partial choice of a conducting element; 1.1.5 – method of a square root; 1.1.6 – method of rotation of linear systems; 1.1.7 - method of optimum exception and edging; 1.1.8 – method of reflection.

1.2.1.1 – simple iteration method (Jacobi method); 1.2.1.2 – Seidel method; 1.2.1.3 – Iterative method with Chebyshev set of parameters; 1.2.1.4 – Richardson's method; 1.2.1.5 – top relaxation method.

1.2.2.1 – method of minimal discrepancies; 1.2.2.2 – method of minimal amendments; 1.2.2.3 – method of prompt release; 1.2.2.4 – method of conjugate gradients; 1.2.2.5 – Monte-Carlo method; 1.2.2.6 – Iterative methods with use spectral equivalence operators.

2.2.1.1 – simple iteration method; 2.2.1.2 – Newton's method; 2.2.1.3 – chord method; 2.2.1.4 – modified Newton's method; 2.2.1.5 – Steffenson method; 2.2.1.6 – interpolation method of the different orders; 2.2.1.7 – top relaxation method; 2.2.1.8 – Aitken method; 2.2.1.9 – the combined methods.

2.2.2.1 – method of tests (half-division method); 2.2.2.2 – method of secants; 2.2.2.3 – interpolation method of the different orders; 2.2.2.4 – inverse interpolation methods of the different orders.

3.2.1.1 – relaxation method; 3.2.1.2 – Picard method; 3.2.1.3 – Newton's method; 3.2.1.4 – modified Newton's method; 3.2.1.5 – Newton method with a parameter.

3.2.2.1 – nonlinear Jacobi method; 3.2.2.2 – nonlinear Seidel method; 3.2.2.3 – hybrid methods.

4.1.1 – the formulas of rectangular; 4.1.2 – the formulas of a trapeze; 4.1.3 – Simpson's formulas; 4.1.4 – the component formulas; 4.1.5 – Runge method; 4.1.6 extrapolation Richardson's method; 4.1.7 – Romberg's method.

4.2.1 – Newton-Cotes formulas; 4.3.1 – Gauss formulas; 4.3.2 – Hermit formulas.

5.1.1.1 – method of Lagrange; 5.1.1.2 – Newton method; 5.1.1.3 – Aitken method of proportional parts; 5.1.1.4 – Bessel and Everett methods; 5.1.1.5 – inverse interpolation method.

5.1.2.1 – Hermit method; 5.1.2.2 – method of Chebyshev polynomials.

5.2.1 – linear spline-interpolation; 5.2.2 – parabolic spline-interpolation; 5.2.3 – cubic spline-interpolation; 5.2.4 – trigonometrically interpolation; 5.2.5 – interpolation by rational functions.

6.1.1.1 – Euler method; 6.1.1.2 – modified Euler method; 6.1.1.3 – Euler-Cauchy method; 6.1.1.4 – modified Euler- Cauchy method with the subsequent iterative processing; 6.1.1.5 – Adams method; 6.1.1.6 – purely-implicit methods (Gyre method etc.).

6.1.2.1 – methods of the different orders of accuracy; 6.1.2.2 – explicit methods; 6.1.2.3 – implicit methods.

6.2.1 – the solution with the help of the Laplac equations; 6.2.2 – the solution with the help of the thermal conductivity equations; 6.2.3 – the solution with the help of the wave equations.

7.1.1 – matrix screw die method; 7.1.2 – reducing method. 7.2.1 – simple iteration method (Jacobi method); 7.2.2 – Seidel method; 7.2.3 – top relaxation method; 7.2.4 – alternate-triangular method; 7.2.5 – modified alternate-triangular method;

7.2.6 – implicit method with Chebyshev set of parameters;

7.2.7 – method of variable orientations.

7.3.1 – hybrid methods; 8.1 – method of variable orientations.

Such a great number of poorly systematized knowledge (methods) does not allow studying it sufficiently in a short time.

Existing systematization of numerical methods is very manifold; this fact complicates teaching process. It is proposed to choose only two qualification criteria according to which evolution of numerical methods proceeds. These criteria associate directly with TRIZ concept of ideality. In compliance with the first criterion on evolution process of numerical methods is associated with increase of mathematical models adequacy to its real physical prototype. For example, behaviour of different nature macro models systems is described by linear and nonlinear algebraic equations systems, behaviour of micro models is described by differential equations systems and behaviour of micro models of distributed systems is described by differential equations in the form of partial derivatives. According to the second criterion, the evolution process of numerical methods is concerned with increase of ideality of existing models realization. For example, at first direct numerical methods were used for linear algebraic equations solution, then iterative one-step methods and iterative multi-step methods followed and etc. The ideality criteria consist of accuracy, convergence, number of arithmetic operations, etc.

Let's consider in detail the line of numerical methods development according the first criteria that is we will consider the development of mathematical models which describe objects of the real world more and more adequately.

The first models were linear equation systems. For example, equation system that describe the planets location. But scientist found out soon that if an acceptable region of variates that are included in the equation system is wide enough then test data and estimated data will be considerably different. This situation appears because the World is not linear in principle and linearization can be performed in small range. To approximate estimated data to test ones wide acceptable region was divided into some small parts and parameters of linear equation systems were defined separately. It led to the great volume of calculations. That is a contradiction appeared: increase of closeness of agreement of linear equation systems solution to test data led to unacceptable increase of volume of calculations. To solve this contradiction it was used a method "transition to another dimension" where "another dimension" means transition to the category of non-linear functions (equations).

Transition to non-linear equations allowed describing functioning of technical object more or less adequately but only in one field sub-system. For example, welding process had been described only in electrical sub-system. But in any real technical object processes referring to different sub-systems take place. In the welding process, electrical

sub-system is only initial one then heat sub-system appears, then deformation, hydraulic and other sub-systems follow. While solving non-linear equations of different sub-systems not connected with each other significant mistakes appear. That is a contradiction appears again which can be solved by the principle of combination. Technical object starts to be described with the system of non-linear equations.

Solution of non-linear equations systems is a long process, which requires a big amount of calculations. For more accurate description of a real object there are attempts to get solutions with the least step of parameters increment and it will result more quantity of calculations.

The method of “partial or surplus actions” allows proceeding to interpolating of functions between accurate solutions received in the non-linear equation system. In this case it is possible to solve non-linear equation system less times.

All above-mentioned models described a technical object as a stationary system that depleted significantly description of the object. Introduction of one more variate was realized in the form of addition of one more variate. But only usage of the Standard “Coordination of rhythm” allowed passing on to differentials and integral.

Calculating of derivatives and integrals allows obtaining only local behavior of test object. To pass to description of behavior in wide range it is used a combination method and then transition to differential equations.

Deeper description of test object requires transition from linear differential equations to non-linear partial differential equations (the method of transition to another dimension, see analogue above). To receive necessary solutions for definite time it is used the principle of split and then pass to finite-difference steady equations.

It is again required to involve transition with time addition that is non-steady finite-difference equations start considering.

Evolution of models of test object description and indication of solving contradictions and methods of their resolution are shown in picture 3.

Now we will define TRIZ-fractality of separate groups of Numerical Methods specifying shortage and TRIZ tools which realize transitions from one method to another.

Let's start from the first model, methods of solution of linear algebraic equations.

4. Development of methods of linear algebraic equation systems and equations solution

At first it is required to eliminate redundancy while considering of the development of solution methods of linear equation systems. The point is that a number of methods have practically equal ideality and in this case only one method was used for considering. So first we need to range the methods of solution linear equation systems solution as per ideality ratio (TRIZ ideality meant).

In close view the line of development of methods of linear equation systems solution may be shown in the following way: direct methods → iterative one-step methods → iterative multi-step methods → iterative variation type methods.

Each of these groups also developed in the directions of ideality increase.

Most of accepted direct methods of linear equation systems solution may be considered as variants of Gaussian method and some of their details differ. The basic method is the method of simple iteration.

Most of iteration methods of variation type are also variants of each other depending on system matrix.

Thus the general development line has the following view: Gaussian method → simple iteration method → Iterative method with Chebyshev set of parameters → top relaxation method → method of the minimal discrepancies → method of the conjugate gradients → iterative methods with use of spectral equivalence operators.

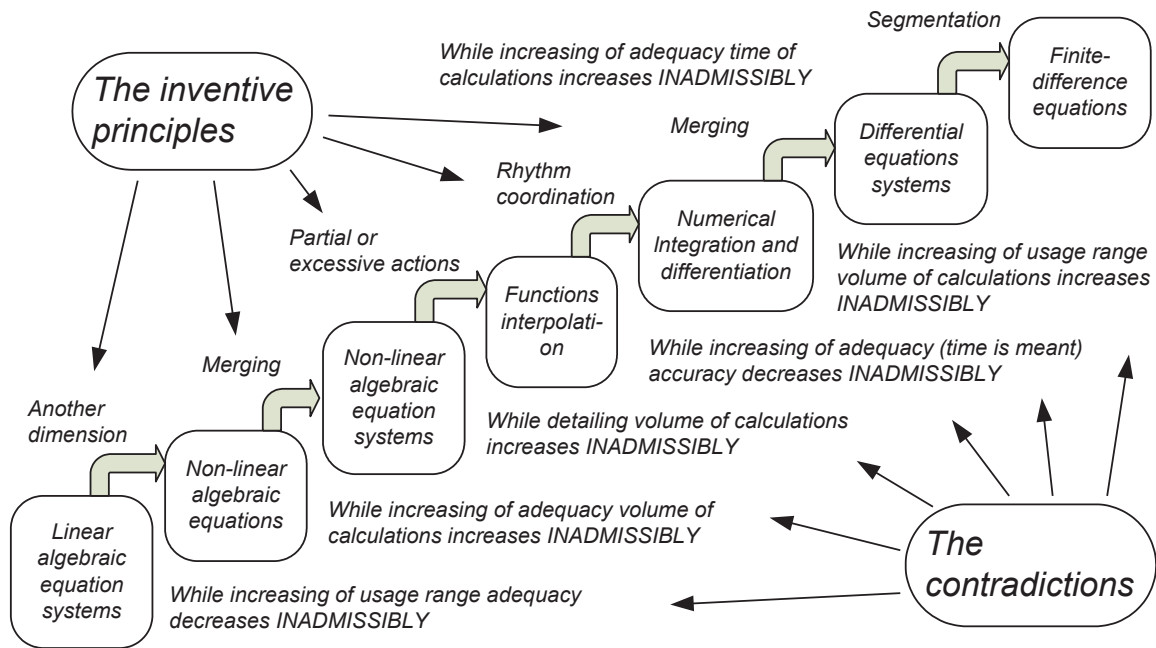


Figure 3: Evolution of objects description

4.1. About Gaussian method

4.1.1. Description of the method

We start description of the method according to the classical TRIZ approach, which is from reveal of system characteristics, main useful function, working object and elements as per the Law of system completeness.

System characteristic is to solve approximately a linear algebraic equation system (LAES).

Main useful function is to solve approximately a linear algebraic equation system of $Ax = b$ (1) type, where A – real square matrix of m degree, b – prescribed vector, x – desired vector; matrix determinant A is non zero.

Working object is problems of algebra and analysis.

Energy source is macro level models of the real world.

Engine is system of the type (1):

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m = b_2, \\ \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mm}x_m = b_m. \end{cases} \quad (1)$$

Transmission is calculation formulas of direct and counter motion.

Direct motion is a transition to a system $Cx = y$, which is equal to $Ax = b$. Counter motion consists in estimation of the unknowns x_1, x_2, \dots, x_m from the system $Cx = y$ (2), where C – a top triangular matrix with identities at the main diagonal.

$$\begin{cases} x_1 + c_{12}x_2 + \dots + c_{1m}x_m = y_1, \\ x_2 + \dots + c_{2m}x_m = y_2, \\ \dots \\ x_{m-1} + \dots + c_{m-1,m}x_m = y_{m-1}, \\ x_m = y_m. \end{cases} \quad (2)$$

Paying attention to the fact that matrix of the system is of the form of triangular, it is used a general formula of counter motion (3):

$$\begin{aligned} x_i &= y_i - \sum_{j=i+1}^m c_{ij}x_j, \\ i &= m-1, \dots, 1 \\ x_m &= y_m \end{aligned} \quad (3)$$

Actuator is the method equations modified for solving of concrete problems (choice of algorithm is possible: without choice of pivot entry, with partial choice of pivot entry, with full choice of pivot entry; it depends on specification of the problem).

The main features of a method:

- the method doesn't assume the solution of a special kind matrixes;
- restriction of a method consists distinction of conducting elements from zero (elements of the matrix main diagonal: $a_{11}, a_{22}, \dots, a_{mm}$);
- the main time of calculation is used up realization of a direct course. On average $m^2/3$ -actions are used up calculation one unknown quantity. This method is suitable for the solution of the equations general view systems with m-unknown quantities 100 order (medium order) on expenses of time;
- the resultant error of calculations the is more, than the order of a matrix is higher;
- the solution almost never is reached with absence of an error on the computer;
- the exact decision depends on a kind of a matrix. So the error is sharply increased at the decision of systems with the "zero" determinant.

4.1.2. Parameters of the method

Control parameter – the number of operations, accuracy of calculations (for example, $\varepsilon = 0,001$) – it is import resultant error to be within limits of fixed accuracy.

Number of arithmetic operations required for solving the system as a rule may be calculated exactly. For example, to make a direct motion of the Gaussian method it is necessary to work out $\frac{(m^2-1)m}{3} + \frac{m(m+1)}{2} = \frac{m(m+1)(2m+1)}{6}$ operations, the main number of these operations ($m^3/3$ degree) should be use to

calculate elements of matrix C. To perform counter motion it is required $\sum_{i=1}^{m-1} (m-i) = \frac{m(m-1)}{2}$ of multiplications.

And the conclusion is that to realize the Gaussian method it is required $\frac{m(m+1)(2m+1)}{6} + \frac{m(m-1)}{2} = \frac{m(m^2+3m-1)}{3}$ operations of multiplication and division.

4.1.3. Formulation of contradictions

Let it is required to increase method's rate of convergence (to decrease number of final arithmetic calculations).

The rate of convergence is limited by the number of arithmetic calculations for direct and counter motion (time of performing direct and counter motions).

Time of motions performing depends directly on a type (a structure) and a degree of a matrix.

The solution will be found faster if the degree is less. The case with structure of a matrix is similar, because the solution will be found faster if the structure is simpler.

Therefore, to decrease the number of final arithmetic calculations it is necessary to make a mathematical model of the real world that contains a matrix of a medium degree (<100), or, and that is better, of the small degree which angle minors shall be non-zero. But such mathematical model does not describe fully the behavior of systems macro models.

There is a contradiction: while increasing of the rate of convergence of the Gauss Method the number of the real world objects macro models decreases intolerably.

To solve this contradiction it is proposed to use the method of transition to another dimension, that is to find solution without change of a mathematical model (for example, there should be matrixes of any type of high degree to solve LAES with iteration method).

4.2. About simple iteration method

4.2.1. Description of the method

System characteristic is to solve approximately linear algebraic equations systems.

The main useful function is to solve approximately the linear algebraic equations systems of $Ax = b$ (1) type, where A is a real square matrix of the m -degree, b is a prescribed vector, x is a desired vector, and the determinant of matrix is non-zero.

Working object is Algebra and Analysis problems.

Energy source is macro level models of the real world.

Engine is a system of the type (1).

Transmission is calculation formulas of a method.

The initial system, according to the principle of “preliminary action”, will be transformed to a kind $x = A_2x + B_1$, i.e. the j -equation is solved obviously relating to j -unknown quantity, and every thing else is transferred in the right part (4).

$$\begin{cases} x_1 = a_{211}x_1 + a_{212}x_2 + \dots + a_{21m}x_m + b_{11}, \\ x_2 = a_{221}x_1 + a_{222}x_2 + \dots + a_{22m}x_m + b_{12}, \\ \dots \\ x_m = a_{2m1}x_1 + a_{2m2}x_2 + \dots + a_{2mm}x_m + b_{1m}. \end{cases} \quad (4)$$

The solution is as a limit of a sequence (5):

$$x^{n+1} = A_2x^n + B_1 \quad (5)$$

Actuator is the equations of a method, which are transformed for the solution of specific problems.

The initial values $x_i^{(0)} (i = 0, 1, \dots, m)$ are set at random.

Iterative process completion is defined either by specifying of the maximal number of iterations $n_{0,}$, or by the following condition (6):

$$\begin{aligned} \max |x_i^{(n+1)} - x_i^{(n)}| &\leq \varepsilon \\ 1 \leq i \leq m, \varepsilon &> 0 \end{aligned} \quad (6)$$

4.2.2. Parameters of the method

Control parameters are the number of operations, accuracy of calculations (for example, $\varepsilon = 0,001$): it is import resultant error to be within limits of fixed accuracy.

- If $\|A_2\| < 1$, the system of the equations (4) has the unique solution and the iterative process (5) converges to the solution with rate of a geometrical progression (according to the theorem of a sufficient condition of convergence of a simple iteration method).
- The iterative process (5) converges to the solution of system (4) if and only if all own values matrix A_2 on the module are less than unit (according to the theorem of a necessary and sufficient condition of convergence of a method).

To decrease the number of operations while transition to the system (4) it is necessary to try to find system with the smallest value $\|A_2\|$.

The numerical parameters, which depend on the number of iteration, are often worked in iteration methods to speed up convergence. The way of choice of iterative parameters is found with a test of convergence. Values of parameters, which convergence is the fastest, are called optimal.

The formula containing iterative parameter (for simple iteration method) is following (7):

$$\frac{x^{(n+1)} - x^{(n)}}{\tau} + Ax = B \quad (7)$$

In the formula (7) τ is constant iterative parameter (it doesn't depend on numbers of iteration).

Optimal iterative parameter is set by the formula (8):

$$\tau_0 = \frac{2}{\lambda_{\min}(A) + \lambda_{\max}(A)} \quad (8)$$

In the formula (8) $\lambda_{\min}, \lambda_{\max}$ are the minimal and maximal own numbers of matrix A_2 .

Parameter (8) is called optimal, because it minimizes error (9) value:

$$\rho_0 = \frac{1 - \xi}{1 + \xi}, \quad (9)$$

$$\xi = \frac{\lambda_{\min}}{\lambda_{\max}}.$$

The error estimate of a method is (10):

$$\|x_n - x\| \leq \rho_0^n \|x_0 - x\| \quad (10)$$

In the formula (10): $n=0,1,\dots$; x_n is n-approximation to the solution, x_0 is initial approximation, x is the solution of system.

4.2.3. The resolution of contradictions

In the method of simple iteration it is possible to pick up optimum value of iterative parameter, wherein the prescribed accuracy will be reached for the minimal number of iterations. The mathematical model will not change here.

For this purpose iterative parameter should depend on numbers of iteration, so as to decrease total number of iterations (11):

$$\tau = \tau_n \quad (11)$$

The Chebyshev formulas are used for an optimum choice of positive numbers $\tau_1, \tau_2, \dots, \tau_n$, for which the norm of an error $\|x_n - x\|$ of the n-iteration is minimal.

Thus, the formulated earlier contradiction is resolved, as anyone macro level models of the real world can be used.

5. TRIZ-Fractal map

While analysis (similar analysis, which presented in item 4) for all numerical methods included in increase of ideality line it is possible to find TRIZ-fractality of development of the methods of the linear algebraic equations system solution line. This line of development will look in the following way. Gauss method \rightarrow "Another dimension", "feedback", "preliminary action" \rightarrow Simple iteration method \rightarrow "Intermediary" \rightarrow Iterative method with Chebyshev set of parameters \rightarrow "Preliminary action", "intermediary" \rightarrow Top relaxation method \rightarrow "Intermediary" \rightarrow Method of the minimal discrepancies \rightarrow "Intermediary", "segmentation" \rightarrow Method of the conjugate gradients \rightarrow "Preliminary action" \rightarrow Iterative methods with use of spectral equivalence operators.

After study of all basic development of numerical methods lines (see figure 3) it is also possible to make TRIZ-fractal map of numerical methods (Appendix B).

6. Teaching procedure by TRIZ-Fractal map

While use of TRIZ-fractal map the following procedure of transfer of applied knowledge is realized, in particular it is a procedure of teaching the section of Mathematics "numerical methods ". Students study all TRIZ tools before. If for any reasons there is no opportunity to study all tools, students study only principles of resolution of a contradiction. Then the elements for base horizontal coordinate of a map are trained. It is evolution of mathematical models of a prototype (technical object) system for numerical methods description. Training starts with the simplest first model. It is shown with examples how to describe object with the help of system of the linear equations and what it is necessary for. Then the task becomes complicated, and the students are offered to describe prototype system in detail. At the same time they see, that there are problems connected to increase the volume of calculations. The students are offered to formulate the contradiction and to resolve it by TRIZ-tools, i.e. to offer next (more ideal) mathematical model of the object description. The teacher helps students if it is necessary. The first step finishes here. Then students "open" similarly all subsequent models. Then they pass to training of vertical lines of development. The steps are carried out almost similarly, except that ideology of a method and its mathematical realization review in detail on each step. It is necessary to note, that the students can "receive" equivalent on ideal method, as some methods have izo-identical ideality, and in figure (Appendix B) one representative of ideal methods is represented only. There is a movement throughout the map from the simplest method up to the most difficult one with the help of the above described way.

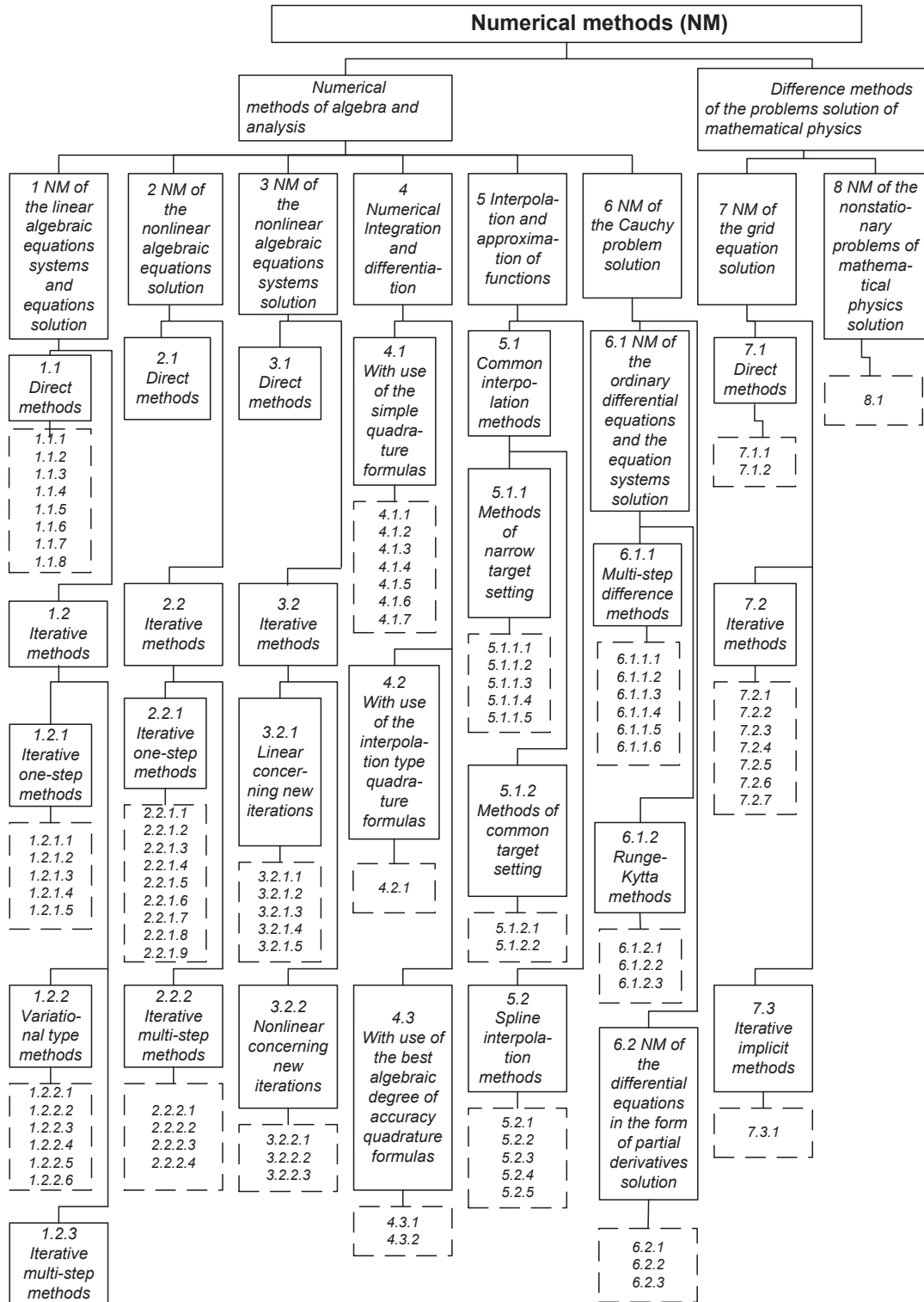
7. Conclusion

The analysis of numerical methods TRIZ-fractality made in the report allows to mark out standard ways of numerical methods development and to construct its TRIZ-fractal map. TRIZ-fractal map is a list of TRIZ-instruments used for transition on development lines. Such map allows to increase essentially the efficiency of master in numerical methods learning and to retain training hours at the same time.

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Appendix A: Numerical methods structure



Appendix B: TRIZ-fractal map

