

09/02/2021 Plasma Beam Dump 10/11/4980
1st Meeting w/ Guoxing Xia

LHC → Proton-proton collisions involve quarks

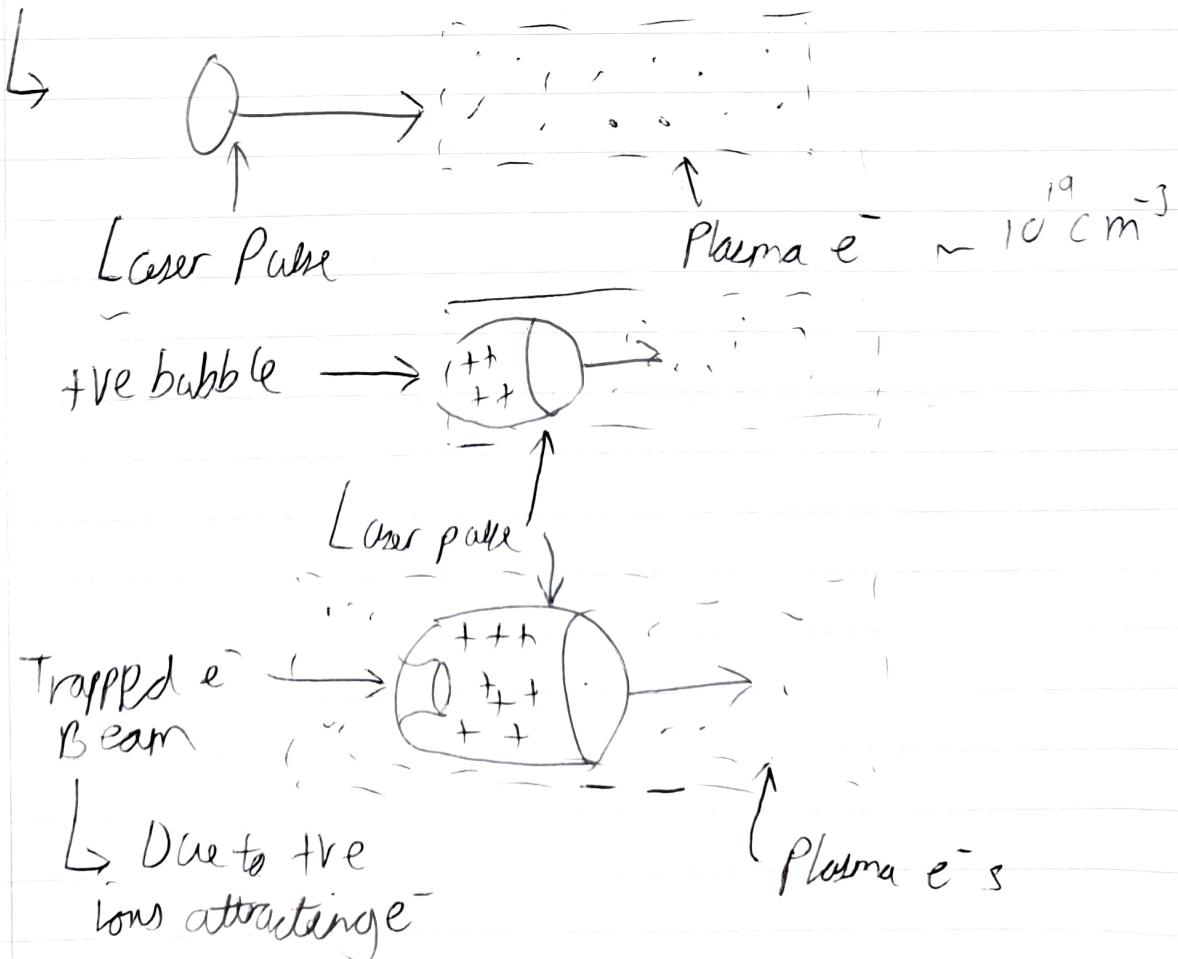
ILC → ILC is a linear collider over 10km, 30-50km
Position-electron collisions provide clearer picture
as they are fundamental

↳ But e^+e^- produces "sever" radiation which
is a problem!

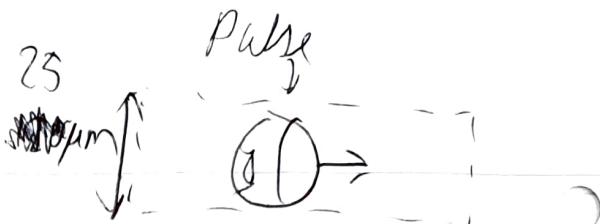
Plasma Accelerator

↳ Accelerating field in plasma is 3-4 orders of
magnitude higher than conventional accelerators

Laser Plasma Acceleration



Bubble $\sim 35 \mu\text{m}$
lasts 120 fs



Plasma accelerator

↳ can shrink to 9cm^3

↳ Save money and space building large particle accelerators

↳ Tabletop accelerator

Plasma Accelerator

↳ produces lots of high energy

↳ After collisions, still lots of high energy!

↳ Dump the plasma beam in a material to absorb energy

Conventional Beam dump design:

- Stainless steel pressure vessel
- 11m length
- 10 bar pressure, 100% safety margin
- 155°C Water temp.

What about using a plasma beam dump?

Plasma beam dump offers high decelerating gradients in a low density medium.

$$E_{Wb} = \frac{mc^2 w_p}{e} = 9.6 \sqrt{\frac{n_e}{10^{22}}} \text{ GV m}^{-1}$$

For $n_e = 10^{18} \text{ cm}^{-3}$

$$E_{Wb} = 96 \text{ GV m}^{-1}$$

- Compared with stopping power in solid material
e.g. ILC beam dump 250 GeV in 11m $\sim 23 \text{ CV m}^{-1}$
- Plasma density is orders of mag. lower than solid
e.g. graphite $\sim 10^{23} \text{ cm}^{-3}$
- Plasma dump can also be applied to e^+ , μ^- with high decelerating gradient

Motivation \Rightarrow Find optimum parameters to maximise beam energy loss in plasma

Next week: Discuss the background Physics

This week

Objectives: Read up papers sent by Guoxing,
literature 

11/02/2021

Literature

Bonatto et al. paper

↳ "Compact disposal of high energy electron beams using passive or laser driven plasma decelerating stage"

LPA = Laser-driven Plasma Accelerator

PIC = Particle-in-Cell

LPA's provide compact way to produce high-energy e^- beams \rightarrow particle accelerators!

↳ But disadvantage is the high energy beam needs to be disposed

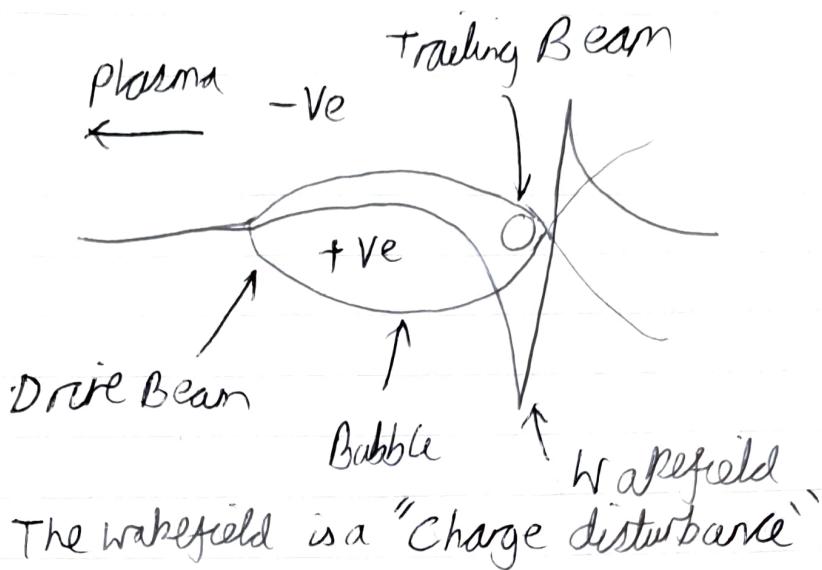
↳ conventional beam dumps inefficient

↳ plasma beam dump can decelerate high energy e^- beams to non-relativistic speeds in just a few cms!

Passive Damping: Highly relativistic e^- beam propagates in an initially quiescent plasma, exchanging energy with self-excited wakefield

Active Damping: Beam propagates in the wake of a laser pulse, experiencing a net E field that is a superposition of the self-excited and laser driven wakefields

Wakefield Excitation



Consider 1D

↳ If amplitude of wakefield is small compared to cold non-rel. wave breaking E field

$$E_0 = \frac{c m_e w_p}{e}$$

Where $w_p = K_p c = \sqrt{\frac{4\pi n_0 e^2}{m_e}}$

e^-
Plasma

Frequency

n_0 = Background e^- Plasma density

Active damping → Better at reducing beam energy than passive damping

Guoxing Xia et al.

↳ "Plasma Beam Damps for EuPRAXIA Facility"

Active Plasma Beam Damp = APBD

Passive Plasma Beam Damp = PPBD

PPBD

- ↳ • Head of bunch experiences no decelerating field due to finite response time of plasma
 - Particles at bunch tail experience a decelerating field
- ↳ BUT eventually becomes non-rel. and fall behind rest of the bunch & reaches an accelerating phase of wakefield
- ↳ Beam re-acceleration \Rightarrow BAD! We need to deaccelerate \square

Counter Beam Re-acceleration:

- ↳ • inset foils in plasma to absorb re-accelerated particles
- Tailor plasma density along beam propagation direction to change relative phases of wakefield along beam driver
- Use ~~APBD~~ APBD!
 - ↳ An excited wakefield is employed in plasma prior to beam propagation such laser driver & beam driver wakefields flatten decelerating field along bunch.
 - ↳ Prevents formation of reacceleration fields

BUT APBD much more complex to implement.

LWFA - Laser Wakefield Accelerator

PWFA - Plasma Wakefield Accelerator

Wu et al.

"Collective deceleration: toward a compact beam dump"

→ Conventional beam dump based on ionization/radiation loss mechanisms are cumbersome, costly and result in radiological hazards

With plasma beam dump

↳ required plasma density is low so hazards of radioactivation due to individual nuclear collisions significantly reduced

↳ More important for particle Energies > GeV where heavy secondary particles e.g. μ^- are produced and need more stopping power

→ Uniform plasma deceleration ineffective at certain distance

↳ Need a structured tailored plasma to further electron deceleration

Bethe - Bloch formula for stopping Power:

$$-\left(\frac{dE}{dx}\right)_T = \left(\frac{F}{\beta^2}\right) \left[\ln\left(\frac{2m_e \gamma^2 v^2}{I}\right) - \beta^2 \right]$$

Change in Electron kinetic energy w.r.t. distance into medium

$$F = \frac{4\pi e^4 n_{em}}{m_e c^2} = e^2 K_{pe,m}^2$$

(oc)

$\gamma = \text{Relativistic Factor}$
 n_{em} is e^- density in stopping material

$K_{pe,m} = \omega_{pe,m}/c$ is plasma wavenumber

$\beta = \frac{v}{c}$ normalised e^- velocity

I represents a specific average of excitation and ionisation potentials in atom

→ Electron energy absorbed by excitation/ionisation of bound e^- in atoms of stopping material

Collective Deceleration

Collective Stopping Power for wakefield deceleration of e^- bunch

$$\hookrightarrow - \left(\frac{dE}{dx} \right)_{\text{coll-wave break}} = m_e c \omega_{pe} \left(\frac{n_b}{n_e} \right)$$

n_b = e^- bunch density

n_e = plasma density

⇒ In principle, energies from decelerated b can be deposited in form of organised plasma wakefields
 Unlike heat energy in conventional dump, may be recycled into electricity?

↪ Save energy by reusing it to accelerator

↪ ALSO mitigates cooling plasma problem

16/02/2021

2nd Meeting w/ Gaoming Xia
+ Niki Vitorica
(Part - doc)

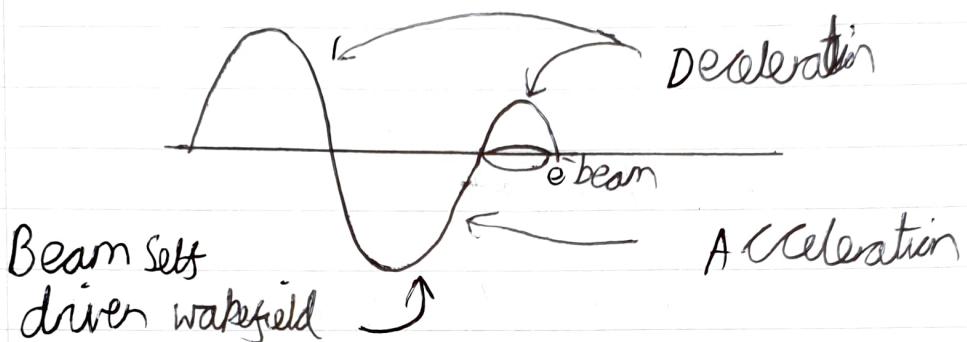
Main Goal: Find a way to deposit more energy into a plasma dump

Transverse - Focus/defocus

Longitudinal - Accelerates/decelerates

Passive Plasma Beam Dump

↳ Beam self driven wakefield

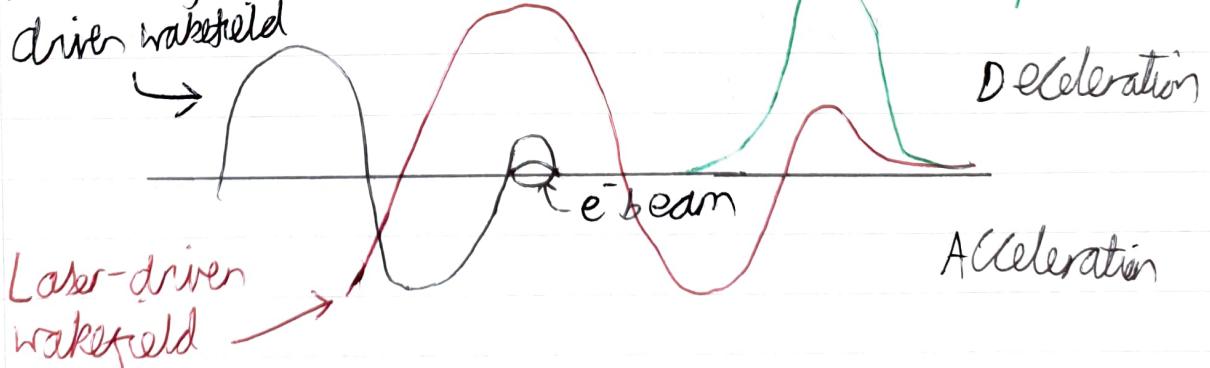


If e^- beam falls behind bunch, it experiences a reacceleration! \rightarrow Energy loss saturation!

Active Plasma Beam Dump

↳ Beam + laser driven wakefield

Beam self driven wakefield



If e^- beam falls behind bunch, the laser driven wakefield flattens reacceleration \rightarrow No reacceleration!

Motivation: Plasma/Laser wakefield accelerators have vastly increased in specs, energy, and tech

- ↳ Resulted in higher energy beams which means conventional beam dumps become far more costly
- ↳ Therefore need an alternative approach (plasma beam dump) to keep up with the advancing PWFA technology

How a Wakefield Accelerator Works

Using Electron Beam:

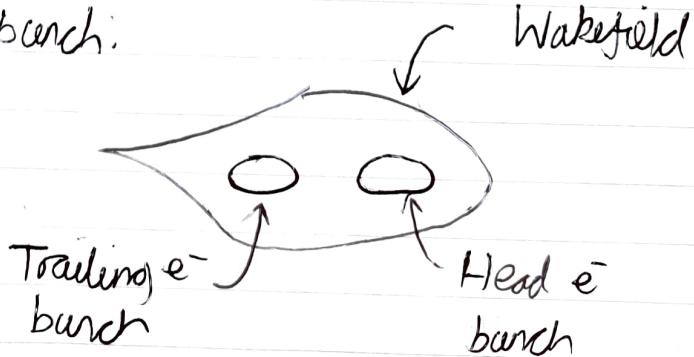
The head of the bunch of e^- propagates through a plasma and drives a "wake" of charge

The wake is driven out of the path of incoming e^- hence creating a charge imbalance that pulls it casting back in behind the passing e^- bunch

This effect creates a strong longitudinal field that accelerates particles in the back of the bunch.

↳ Energy from the particles in the head bunch is transferred to those in the back, through the plasma wake

2 electron bunch:



Energy is transferred from the head e^- bunch to the trailing e^- bunch
↳ The trailing e^- bunch is also focussed

Jakobsson o. et al.

↳ "Tailored plasma-density profiles for enhanced energy extraction in passive plasma beam dumps"

Plasma wakefield accelerators provide accelerating gradients up to hundreds of GeV m^{-1} $\rightarrow \sim 10^3$ larger than those in conventional RF cavities

Recent Results for Laser/Plasma Wakefield Accelerator

- Guiding of 850 TW laser pulse through 20 cm long pre-formed plasma channel at BELLA centre, yielding 5 pC e^- bunch at 7.8 GeV
- Final Focus Test Beam facility at SLAC demonstrated a fraction of a 42 GeV e^- beam had its energy doubled by the self driven wakefield producing accelerating gradient of $\sim 52 \text{ GeV m}^{-1}$ in 85 cm long plasma
- 400 GeV proton bunch as a driver to demonstrate trapping and acceleration of e^- in AWAKE experiment at CERN

Beam Dump

Conventional dumps are metre-long water cooled metal/graphite blocks. \rightarrow Stopping power via Bethe-Bloch formula

↳ Lots of nuclear radioisotopic hazards produced

Plasma Beam Dump

Consists of relatively low density plasma
↳ radionuclide hazard reduced

Damps with uniform plasma density eventually reach an energy loss saturation due to particle reacceleration

↳ Instead can tailor a non uniform plasma density against longitudinal beam profile

Active damping \rightarrow ~95% Total beam energy loss

Passive damping \rightarrow ~60-70% Total beam energy loss

\rightarrow Particles at head of bunch preserve their initial energy

\rightarrow Particles trailing behind undergo reacceleration

Tailored plasma Density *

\Rightarrow In linear to quasilinear regime, transverse & longitudinal wakefield components out of phase by $\sqrt{\pi/2}$

↳ max accelerating field behind bunch occurs at the point where transverse field changes from focusing to defocusing

\Rightarrow reacceleration of non-rel. particles falling behind bunch result in them attaining rel. energies just before defocusing region

↳ ~~increase plasma density resulting in shortening of plasma wavelength, moving defocusing region towards head of bunch~~

- ⇒ As bunch continues to lose particles, it forms a secondary bunch of re-accelerating particles behind head of bunch, just in front of defocusing region
- ↳ Hence increasing plasma density results in shortening of plasma wavelength, moving defocusing region towards head of bunch and over reaccelerating particles
- ↳ These particles rapidly ejected in transverse direction, hence avoiding beam energy loss saturation

NOTE: In uniform plasma density, saturation can also eventually be overcome by defocusing of particles in Secondary Bunch (Reacceleration peak), when its charge is large enough

↳ But does include ejection of particles with energy above initial energy with no major beam energy reduction

S_{chirp} = The distance at which the first beam particles i.e. those experiencing higher amplitude decelerating wakefield have reached non rel. velocities and start falling behind

↳ Here reach an acceleration phase and we reach an energy loss saturation

Different Plasma Density Profiles:

- Uniform
- Multi-staged uniform
- Linearly increasing
- Constant rate of wavelength change

Investigated

\Rightarrow Defocusing causes transverse ejection?

Bonatto et al.

\hookrightarrow "Advances in plasma-based beam dump modelling"

Important measurement is $\frac{\gamma(s)}{\gamma_0}$

\hookrightarrow Ratio of relativistic factor at distance s into plasma
to relativistic factor at $s=0$

Saturation Distance s^*

$$s^* = \frac{\gamma_0}{K_p E_{z,\max}/E_0}$$

$K_p = w_p/c$ is plasma wavenumber

$w_p = \left(\frac{n_0 e^2}{m_e \epsilon_0} \right)^{1/2}$ plasma frequency

$E_0 = \frac{c m_e w_p}{e}$ cold non-relativistic wavebreaking
electric field

s^* = The distance at which particles under max
wakefield $E_{z,\max}$ reach condition $\frac{\gamma(s)}{\gamma_0} \ll 1$
(non-relativistic velocity) and suffer phase
slippage

Reacceleration Problem

- ⇒ Wu et al. proposed use of vacuum regions & periodic thin foils to eliminate reacceleration
- ⇒ Hanahoe et al. explored linear/quadratically increasing plasma-density profiles to defocus particles from acceleration peaks (causing ejection)
- ↳ vessel shaped foil wrapped around plasma cell to stop ejected particles
- ⇒ Jakobsson et al. explored role played by rate of plasma wavelength change with respect to propagation distance
- ↳ changing plasma density n controls plasma wavelength $\lambda \propto \frac{1}{\sqrt{n}}$
- ↳ Hence more location of acceleration/deseleration and focusing/defocusing phases of wakefield

Hanahoe et al.

- ↳ "Simulation study of a passive plasma beam dump using varying Plasma density"

Conventional Dump

- ↳ Large Electron-Position Collider (LEP) uses 2m aluminium dump to stop 100 GeV e^- beam
- ↳ ILC proposed water dump stops 500 GeV beam in 11m
- ↳ decomposition generates H_2 or O_2 gas that needs to be removed

Stopping Power - Average loss of energy with distance

↳ At high energies, losses dominated by bremsstrahlung

T_c = Critical Energy

↳ Where Energy loss due to Bremsstrahlung equal to losses due to other factors

$$T_c = \frac{800 \text{ MeV}}{Z + 1.2}$$

Atomic Number of Stopping Material

Stopping power due to radiation

$$-\frac{dT}{dx} = Z \alpha \frac{4e^4 n_e}{mc^2} (\gamma - 1) \ln(183 Z^{-\gamma_3})$$

n_e = e⁻ density of stopping material

⇒ Passive dump is unable to decelerate head of bunch due to finite plasma response time

18/02/2021

To mitigate reacceleration peaks in passive dump

↳ Change plasma density $n_0 \rightarrow n_1 \rightarrow n_2$

∴ Reacceleration peak is

decelerated if $\left(\frac{3}{2}\right)^2 \leq \frac{n_1}{n_0} \leq \left(\frac{5}{2}\right)^2$

defocused if $1 \leq \frac{n_1}{n_0} \leq 4$

We want to decrease peak so magnitude of acceleration is reduced.

We want to defocus peak so accelerated particles can be transversely ejected.

→ This is with EuPRAXIA beam parameters

$$\sigma_f = \sigma_r = 5 \mu\text{m}$$

$$\text{charge} = 30 \text{ pC}$$

$$\text{Energy} = 1 \text{ GeV}$$

$$\text{RMS Energy spread } \frac{\sigma_E}{E} = 1\%$$

$$\text{Angular Divergence} = 10^{-5} \text{ rad}$$

$$+ \text{ Plasma Density} = n_0 = n_{\text{beam}} = 10^{17} \text{ cm}^{-3}$$

23/02/2021

Meeting with Gaoxing Xia

- Sanjeev Kumar (Post doc)
- Niki Vitoratos (post doc)

Write down ~~FLASH~~ forward beam parameters

$$[n_0, E_c, \sigma_r, \sigma_z]$$

$$\boxed{n_b = \frac{n_0}{(2\pi)^{3/2} \sigma_r^2 \sigma_z}}$$

Beam
Density

Calculate beam density
from parameters

have roughly
~~we want roughly~~ $n_p \approx n_0$

$$n_p = n_b \quad \left\{ \begin{array}{l} n_p = 10n_b \\ n_p = n_b \\ n_p = 0.1n_b \end{array} \right.$$

σ_r = transverse width

σ_z = longitudinal width

Plasma Density = n_p

$$E_{acc} = 244 \frac{N_0}{4 \times 10^{10}} \left(\frac{600 \mu m}{\sigma_z} \right)^2 MeV m^{-1}$$

$$E_{acc} \sim E_{dec}$$

$$L \sim \frac{E_0}{E_{dec}}$$

*Put these parameters into the beam code and run the simulation!

- * Request access into computer with email
- * Download application for FBPIC
- Mention Guoxing Xia + Cockcroft institute

Beam parameters constant

↳ But plasma density is a variable

* Installed FBPIC into anaconda spyder

Via

Conda Install numba scipy numpy mkl

Conda Install -c conda-forge mpi4py

Pip Install fbpic

* Requested access to CSF by emailing
ds-ri-team@manchester.ac.uk

25/02/2021

Literature - W.Lu et al.

~~"~~ "Limits of linear plasma wakefield theory for electron or positron beams"

Linear Theory

→ Green's function solution for the plasma response to an arbitrary relativistic charge bunch

$$\hookrightarrow \text{Form } \rho_b = e_{\perp}(r) e_{\parallel}(\xi)$$

Where

$$\boxed{\xi = z - ct}$$

$$E_z(r, \xi) = Z(\xi) R(r)$$

$$Z'(\xi) = -4\pi \int_{\xi}^{\infty} d\xi' e_{\parallel}(\xi') \cos(k_p(\xi - \xi'))$$

$$R(r) = \frac{k_p^2}{2\pi} \int_0^{2\pi} d\theta \int_0^{\infty} r' dr' \rho_{\perp}(r') k_0(k_p |r - r'|)$$

Where K_0 is the zero-order modified Bessel function

→ We evaluate these expressions for a Gaussian longitudinal profile

$$\hookrightarrow \rho_{11}(\xi) = q n_b e^{-\xi^2/2\sigma_z^2}$$

at a position along the axis where $\xi \ll \sigma_z$

Result is

$$E_z(0, \xi) = \left(\sqrt{2\pi} \frac{q}{e} \frac{mcw_p}{e} \frac{n_b}{n_p} \right) \left(K_0 \sigma_z e^{-K_0^2 \sigma_z^2/2} \right) R(0) \cos(K_0 \xi)$$

where

$$R(0) = k_p^2 \int_0^\infty r dr' \rho_\perp(r') K_0(k_p r')$$

For wide beams, i.e. $\rho_\perp = 1$ for $r=0$ and remains close to unity for r' much larger than k_p^{-1} , $R(0) \approx 1$

For fixed bunch length σ_z and normalised beam

density n_b/n_p , expression for E_z

$$\text{maximised at } \epsilon = \frac{eE}{mcw_p} = \frac{2\sqrt{\pi}}{e}$$

$$\frac{n_b}{n_p} \approx 1.3 \frac{n_b}{n_p} \text{ for } K_0 \sigma_z = \sqrt{2}$$

↳ well known 1D result

\Rightarrow Summary

Proper expression for the wake amplitude from narrow bunches

$$\hookrightarrow k_p \sigma_r \ll 1$$

$$E_{zM} = (236 \text{ MeV m}^{-1}) \left(\frac{q}{e} \right) \left(\frac{N}{4 \times 10^{10}} \right) \times \left(\frac{0.06 \text{ cm}}{\sigma_z} \right)^2 \ln \left(\sqrt{\frac{10^{16} \text{ cm}^{-3}}{n}} \frac{50 \text{ km}}{\mu_r} \right)$$

σ_r = Transverse Beam width

σ_z = Longitudinal beam width

FLASH Forward Beam Parameters

e- Beam

Macro. rep. rate 1-10 Hz

Micro rep. rate 0.040-3 MHz

Bunch Charge 50-800 pC

Energy 400-1250 MeV

Bunch Length 50-6000 fs rms

Chirp -0.5 - +0.5 % rms

Transverse emittance 1-3 mm mrad rms proj.

Focal Spot Size 5x5 (spec.), 30x30 (meas.) $\mu\text{m} \times \mu\text{m}$

Energy Resolution 0.02 %

Laser Beam

Nominal energy	550 MJ
Peak Power	25 TW
Pulse Length	>30 fs
Wavelength Range	770 - 830 nm
Focal length (lens & trans.)	18, >0.5 m
Time jitter	~100 (~10) fs
Polarisation	linear/circular

Plasma Sources

Gas flow	< 20 mbar l/s
Gas species	variable & mixed
density range	< 5×10^{17} , $< 1 \times 10^0 \text{ cm}^{-3}$
(windowless, w/end caps)	
Cell length & diameter	< 500, < 10 mm
Transverse ionis. injection length	~10 μm
Ignition methods	laser, high voltage discharge
Discharge	25 KV , 400 ns, 4.1 nF

02/03/2021

MPhys Meeting

With Guoxing Xia
Niki Vitoratou post doc

~~breaking limit~~

Plasma wakefield comes from electron acceleration

Can use either MV m^{-1} \rightarrow E field strength
 or MeV m^{-1} \rightarrow Accelerating gradient

BOTH CORRECT!

1 GeV 10 GeV m^{-1}
beam gradient

$$\hookrightarrow \frac{1 \text{ GeV}}{10 \text{ GeV m}^{-1}} = 10 \text{ cm long plasma}$$

Chat \Rightarrow FBPIC link to GitHub

Download scripts and check them out

Hanohoe Thesis Notes Chapter 6

Plasma Beam Dumps work via collective forces

\hookrightarrow as opposed to random interactions in conventional beam dumps

- \hookrightarrow Particles in plasma oscillate coherently and thus achieve high decelerating gradients even at low plasma density
- \hookrightarrow Since plasma response is collective, it is possible to recover electricity from plasma
 - \hookrightarrow Conventional dump, energy recovery limited to low grade heat
- \hookrightarrow Recovery of electricity also lowers cooling requirements for the plasma dump

Passive Dump:

Saturation Length = L_{sat}

↳ Rate of energy loss of bunch dramatically decreases

$$L_{sat} \approx \frac{T_0}{eE_{dec}}$$

Beam Initial Energy: T_0
Max decelerating Gradient : E_{dec}

Approx. the propagation length at which max decelerating gradient E_{dec} decelerates a portion of the beam to non-rel velocity

(NOTE - ACTIVE DUMP - Wakefield created by spreading laser, whole of the bunch is thus decelerated)

LIMITS

Active - • Available laser pulse puts limit on max energy absorption

Passive - • Length of dump
• Continuing ability of the decelerating bunch to drive plasma wakefield

Stopping power of a plasma beam depends on bunch parameters

↳ but limited by wakebreaking field

$$E_{wb} = \frac{m_e c w_p}{e}$$

W_b field depends on e⁻ plasma freq. w_p

→ Essentially, E_{wb} is the maximum decelerating gradient of the plasma

Note: Decelerating gradient of plasma beam dump is independent of mass of particle being decelerated

↳ as opposed to conventional beam dump where stopping power is inversely proportional to particle mass

⇒ Bremsstrahlung is a method of energy loss in plasma beam dump

↳ but ^{reduced} factor \propto of ratio of densities of plasma & conventional dump

↳ Pair production of muons may occur due to bremsstrahlung but stopping power of muons same as e⁻ in plasma dump (no dependence on mass)

Energy loss due to Bremsstrahlung can be neglected

↳ Stopping power $\approx 30 \text{ keV m}^{-1}$

↳ Very small relative to wakefields

Reacceleration \rightarrow Wu et al. suggested foil after Lsat

- ↳ However foil experiences extreme conditions that leads to damage over time which dump is expected to operate
- \Rightarrow vary plasma density to defocus low energy particles & prevent reacceleration
- ↳ Much greater durability than foil

Low plasma density \rightarrow Focusing region

High plasma density \rightarrow defocusing region

High plasma density defocuses witness particles as transverse E field overcomes bunch B field

↳ Defocusing region of wakefield therefore lies behind accelerating region

↳ Force low energy portion of bunch through defocusing region to expel them

↳ If accelerating gradient is low enough, particles can pass accelerating region into defocusing region. Stop them from slipping to wakefield

↳ Shift plasma wakefield relative to bunch to push low energy particles into defocus region

↳ Achieved by increasing plasma density to shorten plasma wavelength

Stepped Plasma Density Increase

Once a significant portion of accelerated particles reach defocusing, density is increased to shift particles into defocusing region

↳ Expected particles must be sufficiently low so they are easily absorbed by walls / end of beam dump.

↳ Unclear how to do this in practice \otimes

Gradual Plasma Density Increase

↳ Density increase must be sufficiently fast so accelerated particles pass reacceleration and into defocusing region before they regain excessive energy

↳ Functional form should give roughly constant rate of plasma wavelength change with distance propagated by bunch
↳ Linear / quadratic forms

Linear Plasma

Rate of change of plasma wavelength w.r.t. position

$$\frac{d\lambda}{dz} = \frac{\pi c e^2 n_f}{E_0 l m_e} \left[\frac{e^2 n_i (1 + \frac{n_f z}{n_i l})}{E_0 m_e} \right]^{-3/2}$$

- Initial plasma density n_i

- Final plasma density n_f

→ Length l

- ⇒ In linear defocusing region $\frac{d\lambda}{4}$ behind max deceleration region
- ⇒ Prop. distance Δz over which λ_p changes by $1/4$ can be estimated assuming rate of change of λ_p is constant, energy gain ΔT is average $E_{acc} \times \Delta z$

$$\boxed{\Delta T = E_{acc} \Delta z = E_{acc} \frac{\lambda_0}{4} \left(\frac{d\lambda}{dz} \right)^{-1}}$$

λ_0 is initial plasma wavelength ($z=0$)

- ⇒ More rapid change in plasma density, less energy gained by decelerated particles
 - ↳ But density must remain low enough to be achievable & generate high decelerating field

- ⇒ Practicalities of achieving very plasma density not investigated in death
 - ↳ But stepped plasma densities achieved in beam & laser driven PWFA experiments using gas jet source
 - ↳ But only for $\sim mm$ plasma
Does not work for plasma $\sim m$

09/03/2021

Zoom Meeting with

Guoxing Xia
Niki Vitoratou

→ FB PIC

↳ tried running `lwf-a-Script.py`

↳ completed in 16:43 but no graphics?

Niki sent a GitHub link about openPMD

↳ Need to explain about FB PIC in report!

⇒ only work in 400-600 MeV e^-b case to shorten dump
↪ up to 1.2 GeV

↳ Guoxing will give us parameters, plot the Energy change

↳ $\frac{E}{E_0}$ on y axis, Propagation distance on x axis

⇒ Small tail at end is the saturation energy due to acceleration!

⇒ Contact Linbo about FB PIC simulations

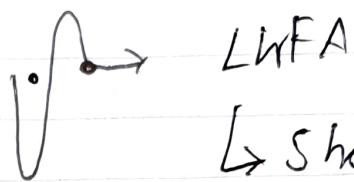
Guoxing sent link in chat about FD PIC

↳ Read & understand literature!
↳ It is a reliable Code

Nled access to machines by week 6

↳ Resend email crating upon inquiry \rightarrow Mention Guokong

\rightarrow FLASH forward is a more realistic beam

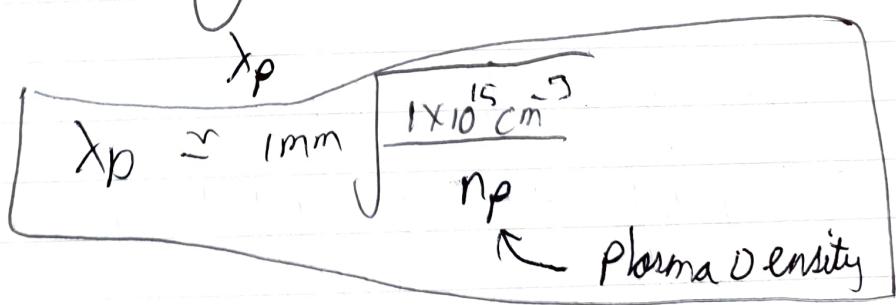
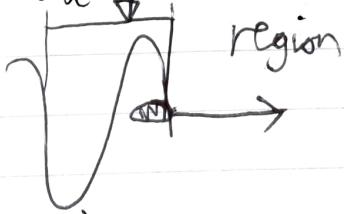


↳ Sharper than PWFA



Most lose energy & decelerate
Some particles gain energy

We want \rightarrow Where particles stay in deceleration



We change the region of particles by altering x_p
which is dependent on n_p .

⇒ GitHub openPMD has tutorials to understand it

16/03/2021

Zoom Meeting with

Gaoxing Xia
Niki Vitoratou

Still don't have access to CSF - need to consider using own laptop

Gaoxing sent an example of previous student report
↓ ↳ read it!

FLASH forward beam DESY website.

Checkout code from laptop → Contact Linbo for help

Worst case scenario → use MATLAB write your own code

Niki has set up team group →

Registered on
TEAMS.

↳ start on introduction?

→ 1 section on report about FLASH Forward

How does FBPIC work?

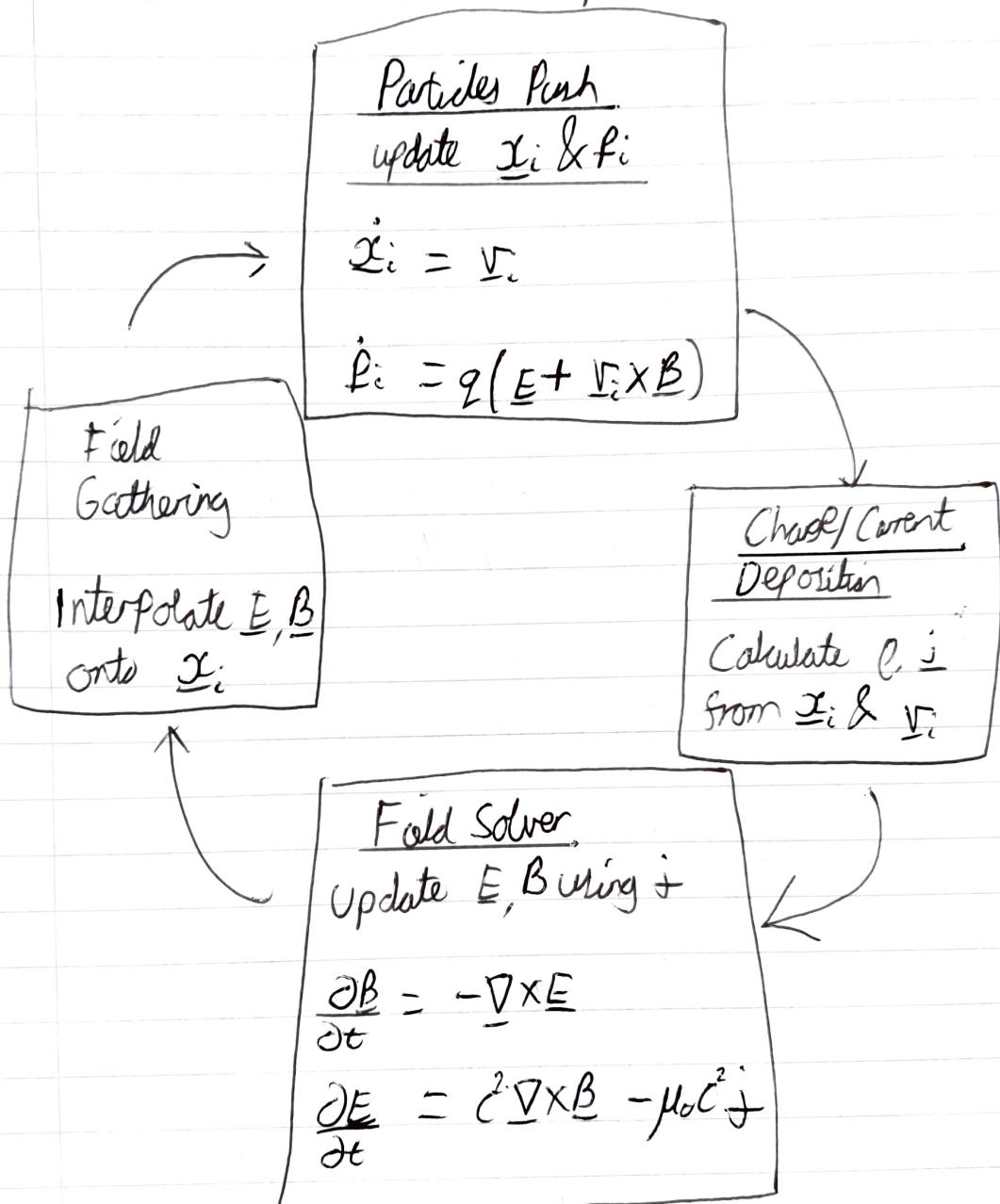
↳ Simulates the self-consistent interaction of charged particles & EM fields

Charged particles represented by macroparticles
↳ this lumps together several physical particles

FIELDS represented on a grid.

↳ Time evolution of system is discrete time steps

How it works: at each timestep



Normal PIC uses cartesian grid (xyz)

↳ FBPIC uses cylindrical coordinates (r, θ, z)
because it is more efficient

This is represented by a set of 20 radial grids,
each grid represents an azimuthal mode (m)

$m=0$ represents fields independent of θ

$m=1$ represents fields varying proportionally to $\cos\theta/\sin\theta$

$m \geq 2$ represents fields varying proportionally to
 $\cos(m\theta)/\sin(m\theta)$

↳ Takes into account further departures from
cylindrical symmetry

=> Usually only need 2 modes, 3 modes may be
needed for nonlinear effects

↳ Using a few 20 grids as opposed to full
3D grids uses a lot less memory

↳ Computational cost increases with m

Macroparticles deposit their charge and current
on each 20 radial grid and gathers the sum of
the fields from each grid.

NOTE: due to cylindrical coordinates, macroparticles
initialised far from the axis have a larger
weight (represent more particles)

↳ Weight is denoted in output \rightarrow Account for
this

Analytical Integration

- Standard PIC, Maxwell equations discretized using finite-differences
 - ↳ But can lead to spurious numerical dispersion in EM waves propagation and numerical growth of emittance for relativistic beams
- FBPIC Fourier transforms the fields and solves the fields in spectral space
 - ↳ derivatives more precisely calculated
 - ↳ algorithm becomes dispersion free in all directions
- ⇒ Due to the FT, field solver has no Courant limit, Δt can be freely chosen
 - ↳ Courant Limit: Time step in numerical analysis of explicit time integration schemes must be less than a certain time otherwise results will be incorrect
- ⇒ Common to choose $\Delta t = \frac{\Delta z}{c}$, Δz is resolution along z
- ⇒ At each timestep, FBPIC transforms fields into spectral space, advances in time and transforms back to real space

Due to cylindrical coords, spectral transform is a FT along z , Hankel transform along r ?

Hankel Transform:

→ Express a function $f(r)$ as the weighted sum of an infinite number of Bessel functions of the first kind $J_0(kr)$

Computational Cost

FT cost scales as $N_z \log(N_z)$

HT scales as $N_r^2 \rightarrow$ HT may dominate Computational time of large number of gridpoints used along r .

→ FBPLIC also removes problem of spurious Cerenkov radiation which appears in finite difference

17/03/2021

OpenPMO is a visualiser for our FBPLIC code

→ discrete vector fields on a mesh $E(r)$
→ discrete scalar fields on a mesh $T(r)$
→ particle property position Γ :

→ physical quantities are records

=> Actual components stored in multi dimensional arrays inside those records

→ Position x is a component not a record

Each record must fulfill these attributes:

- UnitSI: Conversion factor to SI units
- Unit Dimension: Passable dimensionality
- Time/TimeUnitSI: Iteration != time

Unit Dimension:

- Automatic unit description
- as Powers of 7 base SI units
 - ↳ e.g. $V\text{m}^1$ is $L^{1.0} M^{1.0} T^{-3.0}$

Base SI units:

- Electrical Current \Rightarrow Ampere [A]
- Thermodynamic Temp. \Rightarrow Kelvin [K]
- Amount of Substance \Rightarrow Mole [mol]
- Luminous Intensity \Rightarrow Candela [cd]
- Length \Rightarrow Metre [m]
- Mass \Rightarrow Kilogram [kg]
- Time \Rightarrow Second [s]

To handle Petabytes of data

- ↳ Use parallel community file formats
 - e.g., PHDF5.h5 (parallel/stripped, compressed)
 - ↳ (stripped some elements are present to save space)

- ↳ Asked Linbo for help about certain things in Microsoft Teams

23/03/2021

Meeting with

Guoxing Xia
Niki Vitoratou
Sanjeev Kumar

- ↳ Get visual results in the 2 weeks of Easter break
 - ↳ After, change parameters, maybe tailor plasma density to reduce
- Ziqian Xiang Paper

Plasma is ionised gas \rightarrow non uniform due to spatial distribution of generating source and under govern of diffusion

- E & B fields produced by plasma but this will influence behaviour of plasma ions in it
 - ↳ Fields induce ion displacement which can then also change the field
 - ↳ Response is always ahead of our calculation
- \Rightarrow Use fluid model in which individual particle is neglected and we treat plasma as a fluid of e^- s & fluid of +ve ions
 - ↳ Lorentz & Coulomb force interactions only, no collisions

e^- Bunch Assumption

→ Assume e^- speed $v \approx c$ as FLASH Forward beam of 400-1250 MeV

→ Initial bunch has constant density

↳ Assume effective electron bunch is invariant

$$\boxed{\begin{aligned} \nabla \cdot \underline{E} &= -4\pi p_{\text{perturb}} + 4\pi p_0 \\ \nabla \times \underline{E} &= -\frac{1}{c} \dot{\underline{B}} \\ \nabla \times \underline{B} &= \frac{4\pi}{c} \dot{\underline{j}} + \frac{1}{c} \dot{\underline{E}} \end{aligned}}$$

? not same units
a) Maxwell equations

$$\rightarrow \nabla \times \underline{E} = -\dot{\underline{B}} \quad \nabla \cdot \underline{E} = \frac{p}{\epsilon_0} ?$$

↳ implies $\frac{1}{\epsilon_0} = 4\pi ?$

↳ Email Gaoxing \Rightarrow CGS units ?

→ Assumption that unperturbed plasma has no contribution to E field, only perturbed generates E & B

↳ $n_{\text{plasma}} = n_{\text{perturb}} + n_{\text{unperturb}}$

Density of individual e^- propagating

↳ $\rho_e = q \delta(x - v_b t) = q \delta(r) \delta(z - v_b t)$

↳ Dirac delta, so charge only exists in certain points

χ = Longitudinal Displacement of e^-

r = Transverse displacement in cylindrical

z = longitudinal displacement

↳ equation of motion for electron

$$F = m \frac{dV_b}{dt} \quad F = -eE$$

$$\hookrightarrow \boxed{\frac{dV_b}{dt} = -\frac{eE}{m}} \quad \textcircled{1}$$

\Rightarrow Change of plasma density in local area
equal to divergence of the flux into, minus
flux out of

↳ Continuity equation for plasma

$$n_{\text{plasma}} - n_{\text{perturbed}} \left(\frac{\partial n_{\text{plasma}}}{\partial t} + \nabla \cdot j = 0 \right)$$

$$\boxed{\frac{\partial n_{\text{perturbed}}}{\partial t} + \nabla \cdot j = 0} \quad \textcircled{2}$$

Sub ① into differential of ② over time,
subbing gradient of E with z .

$$\hookrightarrow n_{pe} = \frac{\omega^2 Q}{ev_b} \sin\left(\omega(t - \frac{z}{v_b})\right) \Theta\left(t - \frac{z}{v_b}\right)$$

Take curl of $\nabla \times E = -\frac{1}{c} \dot{B}$ & combine with

$\nabla \times B = \frac{4\pi}{c} + \frac{1}{c} \dot{E}$, with continuity equation

$$\hookrightarrow \boxed{\frac{\partial J_1}{\partial t} = -e n_{npe} \frac{\partial V}{\partial t} = \frac{e^2 E n_{npe}}{m}}$$

J_1 = current density from that e^-

$$\hookrightarrow \frac{\partial^2 E}{\partial t^2} - c^2 \nabla^2 E = -wE - c^2 \nabla \left(-4\pi n_{npe} + 4\pi \rho_a \right)$$

\hookrightarrow cylindrical ∇^2 written as $\nabla_r^2 + \frac{\partial^2}{\partial z^2}$, $dz = c dt$

$$\hookrightarrow (\nabla_r^2 - k_p^2) E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \nabla \left(-4\pi n_{npe} + 4\pi \rho_a \right)$$

$$= 4\pi \omega^2 \frac{e}{c} \cdot \nabla \left(\delta(r) \Theta(t - \frac{z}{c}) \right)$$

$$\sin \left[\omega \left(t - \frac{z}{c} \right) \right]$$

If $k_p^2 = \frac{\omega^2}{c^2}$, and only consider longitudinal fraction

$$\boxed{(\nabla_r^2 - k_p^2) E_z = \frac{4\pi \omega^2 e}{c} \cdot \delta(r) \Theta(t - \frac{z}{c}) \sin \left[\omega \left(t - \frac{z}{c} \right) \right]}$$

\hookrightarrow Kelvin-Helmholtz equation using Green's function & continuity method for E_z

$$E_z = 2e k_p^2 K_0(k_p r) \Theta(t - \frac{z}{c}) \cos(\omega(t - \frac{z}{c}))$$

K_0 is zeroth order Bessel function of 2nd kind

Now e^- beam with density $\rho(r, \theta, z - ct)$

$$E_z(r, \theta, \xi) = - \int_{-\infty}^{\xi} d\xi' \int_0^{\infty} dr' \int_0^{2\pi} d\theta' 2k_p^2 \rho_b(r', \theta', \xi')$$

$$K_0(k_p |r - r'|) \cos(k_p (\xi - \xi'))$$

in Comoving frame

30/03/2021.

→ Energy Loss

$$\xi = z - ct \quad \text{Co-moving Co-ordinates}$$

r = Radial Distance

$s = ct$ Propagation distance

$\gamma = e^-$ energy

↳ Energy Loss

$$\frac{d\gamma}{ds} \approx -k_p \frac{E_z}{E_0} \quad \text{as } v \approx c$$

W here $k_p^2 = \frac{\omega^2}{c^2}$ plasma wavenumber

E_z = Longitudinal component of beam
driven wakefield

$E_0 = C M_e W_p / e$ Wave-breaking E field

↳ Max E field in cold & non-relativistic plasma
in linear region

NOTE : $n_{beam} \ll n_{plasma}$ Linear Region

$n_{beam} \approx n_{plasma}$ Quasilinear Region

$n_{beam} \gg n_{plasma}$ Non linear Region

Linear theory works well in quasilinear &
linear region

→ In this project we use $n_{beam} \approx n_{plasma}$

↳ So linear theory is accurate enough

To get γ

$$\hookrightarrow \int_{\gamma_0}^{\gamma} d\gamma = \int_0^s -k_p \frac{E_z}{E_0} ds$$

$$\gamma - \gamma_0 = -k_p s \frac{E_z}{E_0}$$

$$\therefore \boxed{\gamma(s) = \gamma_0 - k_p s \frac{E_z}{E_0}}$$

$$\text{Also } \int_{U_0}^U dU = \int_V dV \gamma(\xi, r) \frac{n_b}{n_0}$$

$$U - U_0 = \int_V dV \gamma(\xi, r) \frac{n_b}{n_0} \quad \gamma = k_p s \frac{E_z}{E_0}$$

$$U = U_0 - k_p s \int_V dV \left(\frac{E_z}{E_0} \right) \left(\frac{n_b}{n_0} \right)$$

$$U_0 = \int_V dV \gamma_0 \frac{n_b}{n_0} = \gamma_0 \int_V dV \frac{n_b}{n_0}$$

$$\frac{U}{U_0} = \frac{1 - k_p s \int_V dV \left(\frac{E_z}{E_0} \right) \left(\frac{n_b}{n_0} \right)}{\gamma_0 \int_V dV \frac{n_b}{n_0}}$$

$$\Rightarrow U = \int_V dV \gamma(\xi, r) \frac{n_b}{n_0} \quad \text{sub in } \gamma(\xi, r)$$

$$U = \int_V dV \left(\gamma_0 - k_p s \frac{E_z}{E_0} \right) \frac{n_b}{n_0}$$

$$U = \int_V dV \gamma_0 \frac{n_b}{n_0} - \int_V dV k_p s \frac{E_z}{E_0} \frac{n_b}{n_0}$$

$$U = U_0 - \int_V dV k_p s \frac{E_z}{E_0} \frac{n_b}{n_0}$$

$$\frac{U}{U_0} = 1 - \frac{\int_V dV k_p s \frac{E_z}{E_0} \frac{n_b}{n_0}}{\int_V dV \gamma_0 \frac{n_b}{n_0}}$$

$$\Rightarrow \frac{U}{U_0} = 1 - k_p s \frac{\int_V \frac{dV E_Z}{E_0} \frac{n_b}{n_0}}{n_0 \int_V dV \frac{n_b}{n_0}}$$

↳ Linear decreasing function of distance

06/04/2021

Collective Stopping Power for Wakefield
deceleration of e- bunch

$$\Rightarrow -\left(\frac{dE}{dx}\right)_{\text{coll-wavebreak}} = m_e c \omega_{pe} (n_b/n_e)$$

ω_{pe} = plasma frequency

* Is exact resonant excitation of a wakefield with
bunch length $\sigma_L/\lambda_{pe} \approx 0.5$ density ratio

Transverse size $\sigma_T/\lambda_{pe} \geq 0.3$, $\frac{n_b}{n_e} < 10$

→ Ajman trying to run sim on his Laptop, (work out
deviations)

↳ LinBo very helpful!

↳ Unable to run sim, Ajman's laptop does not have
enough memory.

⇒ Use $n_b \approx n_e$

Ratio of collective deceleration in plasma &
Bethe - block stopping power

$$\hookrightarrow R = \frac{(dE/dx)_{\text{coll-wave break}}}{(dE/dx)_{\text{ind}}} \\ \approx \frac{m_e c w_p e \beta^2}{F \Lambda} \\ = \frac{m_e n_e \lambda_{pe}}{n_{e,m} r_0} \frac{\beta^2}{2\pi\Lambda}$$

Λ is log term of
 $(dE/dx)_{\text{ind}}$

\hookrightarrow With typical numbers

$$\hookrightarrow R \approx 1000$$

\hookrightarrow plasma dump is 10^3 larger in stopping power

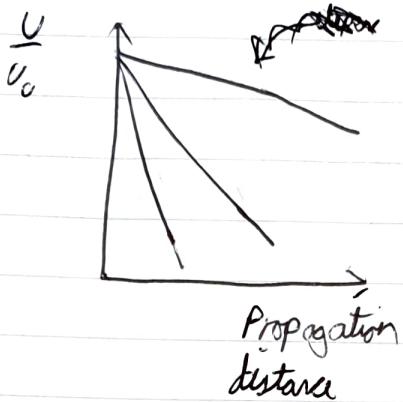
Also : Radioactivating hazard reduced by $(\frac{n_e}{n_{e,m}})$
due to gas tenacity

$\hookrightarrow R$ increases as we use energies $> 6\text{GeV}$ and
muons account

13/04/2021

Meeting with Guoxing Xia
Sanjeev Kumar

- ↳ charge particle per cell number, $\frac{\text{total}}{\text{partic}}$
- ↳ Analytical solution only accounts for linear, does not model simulation for after saturation L_{sat}



larger gradient \Rightarrow larger deceleration in smaller distance

\Rightarrow Parameter Scanning

Thesis

- ↳ introduction
 - ↳ conventional beam dump
 - ↳ plasma beam dump
- ↳ plasma beam dump
 - ↳ basic theory
 - ↳ higher deceleration field, equation to compare
- ↳ FLASH forward beam
 - ↳ Based on this parameters
 - ↳ Energy loss derivations / Curve

↳ Assume $n_b = n_p \Rightarrow$ quasilinear regime

↳ Section 3/4

↳ Derivation of positive code

↳ Future considerations

↳ Conclusion

16/04/2021

↳ Scaling down simulation so can run on Aoran's laptop

↳ Take every lower limit of FLASH forward beam parameters

$$\hookrightarrow K_{p_0} = \frac{w_{p_0}}{c}$$

$$w_{p_0} = \sqrt{\frac{n_0 e^2}{E_0 m_e}}$$

→ Changed $N = 1,000,000$

to $N = 1,000$

↳ Linbo said too small, lots of numerical noise period

↳ changed back to 1,000,000

⇒ Can reduce drag period to see only a few periods, reduce sim time

↳ sim Time $\leq 18t$ hours !!

\Rightarrow Start to write code on plotting $\frac{V(s)}{V_0}$ against Propagation distance

$$\boxed{\frac{V(s)}{V_0} = 1 - \frac{\pi^3 k_p^2 L \left(\frac{n_b}{n_0}\right) \cos^2\left(\frac{k_p L}{2}\right)}{\gamma_0 (\pi^2 - k_p^2 L^2)^2}}$$

\Rightarrow From compact dispensable Bonatto

$$\gamma_0 = \gamma(s=0)$$

\downarrow
L = Beam length

S: Propagation Distance

Plasma
Kp: Wavenumber $k_p = \frac{w_p}{c}$, $w_p = \sqrt{\frac{n_0 e^2}{\epsilon_0 m_e}}$

L: Saturation Length $L_{sat} = \frac{V_0}{e E_{dec}}$

$$E_{acc} \approx E_{dec}$$

$$E_{acc} = 236 \text{ MV/m} \left(\frac{q}{e} \right) \left(\frac{N}{4 \times 10^{10}} \right) \left(\frac{0.06 \text{ cm}}{d} \right)^2 \ln \left(\frac{10^{16} \text{ cm}^{-3}}{n_0} \frac{50 \mu\text{m}}{or} \right)$$

n_b : Beam density $n_b \approx n_0$

n_0 : Plasma Density $n_b \approx n_0$

γ_0 : Lorentz factor at $s=0 \Rightarrow \text{me} \gamma_0 = 500$

$$\boxed{\lambda_p = 1 \text{ mm} \sqrt{\frac{1 \times 10^{15} \text{ cm}^{-3}}{n_0}}}$$

plasma wavelength

$$L_{\text{sat}} = \frac{U_0}{eE_{\text{dec}}} \leftarrow 6 \text{ eV}$$

↑

$$\propto V \text{ m}^{-1} \quad C \cdot V \text{ m}^{-1} = eV \text{ m}^{-1}$$

$$\begin{aligned}
 E_{\text{dec}} &= \frac{mc^2 w_p}{[C]} \\
 &= \frac{[kg][m s^{-1}][s^{-1}]}{[C]} \\
 &= kg m c^{-1} s^{-2} \\
 &= 1 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2} \\
 &\quad \cancel{\text{kg m s}^{-1} \text{ A}^{-2}} \\
 &= 1 \text{ V} \cancel{s \text{ A}^{-1}}
 \end{aligned}$$

NOTE L is Length of Beam!

22/04/2021

→ Coding $\frac{C}{U_0}$ against Prop. distance

$$\Rightarrow n_b = \frac{N_0}{(2\pi)^n \sigma_r \sigma_z}$$

4PM meeting with : Guoxing Xia
Sanjeev Kumar

↳ email Dr. Conor Fitzpatrick
to confirm interview date

↳ Do quick estimate of calculations before doing
Code

$$n_b = \frac{N_{\text{tot}}}{(2\pi)^{3/2} \sigma_r^2 \sigma_z}$$

σ_z = Bunch length

Use smaller value for σ_b cm !

↳ lower parameters

↳ prepare for next meeting with a presentation / 2/3
slides

24/04/2021

Parameter Ranges :

e^- charge : 50 - 800 pC

e^- Energy E_0 : 400 - 1250 MeV

e^- Bunch Length : 50 - 6000 fs

Plasma density n_{-0} : $5e17 - 1e20 \text{ cm}^{-3}$
 $5e23 - 1e26 \text{ m}^{-3}$

\Rightarrow Everything on lowest parameter except plasma density

\hookrightarrow at $5e^{23} \rightarrow$ energy ratio goes negative

27/04/2021

MPhys meeting with

Gaoxing Xia
Sanjeev Kumar

No. of particles per bunch \square

Not Plasma Density
or No. of macroparticles

$$N_0 = \frac{q}{e}$$

$q =$ Bunch charge

Need ~ 8 pages by next week!
 \rightarrow Coding!

\rightarrow Graph looks great \square

\rightarrow derivations

Introduction - FLASH Forward Beam
- Derivations

\rightarrow Future Considerations

$$\frac{U}{U_0} = 1 - k_p s \left(\frac{n_b}{n_0} \right) \left(1 - \cos(k_p L) \right)$$

$$\frac{U}{U_0} = 1 - \frac{\gamma_0 L}{\gamma_0 \int d\zeta n_b / n_0} \left[k_p \int d\zeta \left(\frac{E_2}{E_0} \right) \frac{n_b}{n_0} \right]$$

$$\frac{E_2}{E_0} = -k_p \int_{-\infty}^{\zeta} d\zeta' \cos k_p (\zeta - \zeta') \left[\frac{h_0}{n_0} + \frac{a^2}{2} \right]$$

$$a = C w_p$$

~~F = Eq~~

$$F = \frac{F}{e} = \frac{ma}{e} = \frac{m_e a}{e}$$

$$\int_0^L d\zeta \frac{n_b}{n_0} \sin(\pi \zeta / k_p L)$$

$$\left[-\frac{n_b}{n_0} \cos(\pi \zeta / k_p L) \frac{k_p L}{\pi \zeta} \right]_0^L$$

$$= -\frac{n_b}{n_0} \frac{k_p L}{\pi} \left[\cos\left(\frac{\pi}{k_p}\right) - 1 \right]$$

~~k_p L ≈ 0~~

$$= -\frac{n_b}{n_0} \frac{k_p L}{\pi} \left[\cos\left(\frac{\pi}{k_p}\right) - 1 \right]$$

$$\int_{-\infty}^{\zeta} \cos[k_p(\zeta - \zeta')] d\zeta'$$

$$= \left[\frac{1}{k_p} \sin[k_p(\zeta - \zeta')] \right]$$

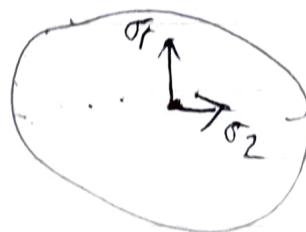
$$= -\frac{1}{k_p} \sin[k_p(\zeta - \zeta')] \quad g = k_p(\zeta - \zeta')$$

$$\frac{dg}{d\zeta'} = -k_p$$

$$x = \sin g \quad \frac{dx}{dy} = \cos g$$

$$\frac{dx}{d\zeta'} = \cancel{\frac{dx}{dy}} \frac{dx}{dy} \frac{dy}{d\zeta'} = -k_p$$

$$= +\cos g \cdot -k_p$$



$$\rightarrow -\frac{1}{k_p} \sin[k_p(\zeta - \zeta')]$$

04/05/2021

Meeting with Guoxing Xia
Sanjeev Kumar

→ send email to Bonatto to ask about definite integral

$$\rightarrow \int_{\infty}^{\infty} \cos [k_p (\zeta - \zeta')] d\zeta'$$

→ change beam energy, fix plasma density

→ change bunch charge, fix other parameters

→ ^{Vary} ~~fix~~ plasma density, fix other parameters

abonatto@gmail.com

q, U_0, L, n_a, n_b

Alexandre Bonatto

After L_{sat} → bunch extends length → acceleration

07/05/2021

Variable U_0 - Fix All other parameters

↳ γ_0 varies

↳ $\frac{U_0}{U_0}$ varies

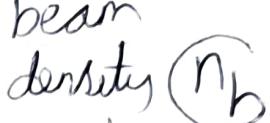
q, L, n_a, n_b

Variable $q \rightarrow$ Fix all other parameters

↳ Variable N



↳ Variable beam density n_b



↳ Variable E_{dec}



U_0, L, n_0

↳ Variable $\frac{U}{U_0}$



Vary plasma density n_0
beam length L

↳ Variable $\frac{U}{U_0}$

Q, U_0, n_0, n_b

Vary plasma density n_0

↳ Variable w_p, λ_p

Q, U_0, L, n_b

↳ Variable $K_p \rightarrow \frac{U}{U_0}$

↳ Variable $\frac{U}{U_0}$

E_{wb}

$K_p \rightarrow \frac{U}{U_0}$

L_{sat}

↳ L_{sat} does not change!

↳ cancels out.

$$5 \times 10^{23} - 10^{26}$$

Ideal bunch length = $30 \mu\text{m}$

Ideal bunch charge = 100 pC

Ideal $U_0 = 400 \text{ MeV}$

Ideal $n_0 = 5 \times 10^{23} \text{ m}^{-3}$

$$3e8 \times = 30 e^{-6}$$

$$\alpha = 10^{-13}$$

~~$$3e8 \times$$~~

$$3e8 \times = 30e^{-6}$$

$$\gamma = 10^6 e^{15}$$

11/05/22 1.

MPhys Zoom Meeting with Guoxing Xia
Sanjeev Kumar

↳ showed Final graphs of varying parameters

↳ Final Parameters:

$q = 100 \text{ pC}$
$L = 30 \mu\text{m}$
$U_0 = 400 \text{ MeV}$
$n_0 = 5 \times 10^{23} \text{ m}^{-3}$

Max Deceleration Field = 3.956 V/m^2