1 The Photon

		$c = 2.998 \cdot 10^8 \left[\frac{\mathrm{m}}{\mathrm{s}} \right]$
		$h = 6.626 \cdot 10^{-34} \left[\frac{\text{m}^2 \text{ kg}}{\text{s}} \right]$
$c\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	speed of light	
$h\left[\frac{\mathrm{m}^2\mathrm{kg}}{\mathrm{s}}\right]$	planc's constant	$ hbar{h} = \frac{h}{2\pi} $
e [C]	electorn charge	$e = 1.602 \cdot 10^{-19} \text{ [C]}$
m_e [kg]	electron mass	$m_e = 9.109 \cdot 10^{-31} \text{ [kg]}$
$k_B \left[\frac{\mathrm{m}^2 \mathrm{kg}}{\mathrm{s}^2 \mathrm{K}} \right]$	bolzmann constant	$h = 1.291 \cdot 10^{-23} \text{m}^2 \text{kg}$
$\epsilon_0 \left[\frac{\mathrm{F}}{\mathrm{m}} \right]$	vacuum permittivity	$k_B = 1.381 \cdot 10^{-23} \left[\frac{\text{m}^2 \text{kg}}{\text{s}^2 \text{K}} \right]$
		$\epsilon_0 = 8.854 \cdot 10^{-12} \left[\frac{\mathrm{F}}{\mathrm{m}} \right]$
		$1 \text{ [eV]} = 1.602 \cdot 10^{-19} \text{ [J]}$

1.1 Photon & Electron

$$\lambda \text{ [m]}, \nu \text{ } \left[\frac{1}{s}\right] \text{ Wavelength, Freq.} \qquad \lambda = \frac{c}{\nu} \quad \nu = \frac{c}{\lambda} \quad \omega = 2\pi\nu$$

$$k \qquad \text{Wavenumber} \qquad k = \frac{2\pi\nu}{c}$$

$$E \text{ [J]} \qquad \text{Energy} \qquad E = h \cdot \nu = \hbar \cdot \omega$$

$$\vec{F_c} \text{ [N]} \qquad \text{Coulomb Force} \qquad \left| \vec{F_c} \right| = \frac{Q_1 \cdot Q_2}{4\pi\epsilon_0 r^2}$$

1.2 Photoelectric effect

$$\begin{array}{ll} V \ [\mathrm{V}] & \mathrm{Voltage} \\ \phi_0 \ [\mathrm{eV}] & \mathrm{Work \ function} \\ I \ [\mathrm{A}] & \mathrm{Photo-current} \\ n \ [\mathrm{m}^{-3}] & \mathrm{Volume \ density \ of \ electrons} \\ A \ [\mathrm{m}^2] & \mathrm{Area} \\ v \ [\frac{\mathrm{m}}{\mathrm{s}}] & \mathrm{velocity \ of \ electrons} \\ \end{array}$$

$$h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$V(\nu) = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

$$I = nAve$$

1.3 Blackbody Radiation

$$L$$
 [m] length of blackbody cube k_i wave constants E_x Electric field in x-direction $\langle E \rangle$ Average Energy N Number of states D Density of states U Blackbody radiation U Power radiated

$$E_x(x,y,z) = E_{0x}\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$k_x = n\frac{\pi}{L} \quad k_y = m\frac{\pi}{L} \quad k_z = l\frac{\pi}{L} \qquad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$N(k) = \frac{1}{3\pi^2}k^3L^3 \qquad D(k) = \frac{k^2}{\pi^2}$$

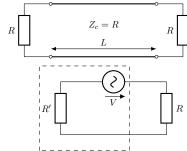
$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \cdot \frac{\hbar\omega}{\exp\left(\frac{-\hbar\omega}{kT}\right) - 1}d\omega \qquad u(\nu) = \frac{8\pi h\nu^3}{c^3\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)}d\nu$$

 $I(\omega) = c \cdot u(\omega)$

Equipartition-Theorem: Each degree of Freedom has an energy of kT

1.4 Johnson-Noise

This is the noise created in a one-dimensional circuit (like a coax-cable).



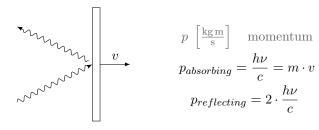
 $\langle V^2 \rangle$ Noise Voltage

 $\Delta \nu$ Bandwidth

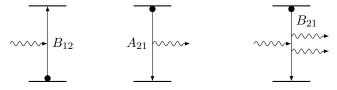
 $E = E_0 \cdot \sin(k_x \cdot x)$

 $\langle V^2 \rangle = 4R \cdot k_B T \cdot \Delta \nu$

1.5 Momentum of a photon



1.6 Absorption, spontaneous and stimulated emission



absorbtion spontaneous emission stimulated emission

 n_1 Number of electrons in the lower energy state

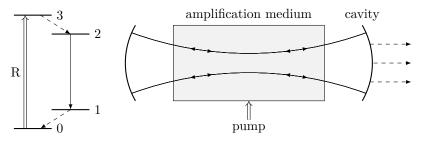
 n_2 Number of electrons in the higher energy state

$$\frac{dn_2}{dt} = \underbrace{n_1 \cdot u(\nu) \cdot B_{12}}_{\text{absorbtion}} - \underbrace{n_2 \cdot u(\nu) \cdot B_{21}}_{\text{stimulated emission}} - \underbrace{n_2 \cdot A_{21}}_{\text{spontaneous emission}}$$

$$\frac{n_2}{n_1} = e^{-\frac{h\nu}{k_B T}} = \frac{u(\nu)B_{12}}{u(\nu)B_{21} + A_{21}}$$

$$B_{21} = B_{12} = B \qquad A_{21} = \frac{8\pi h\nu^3}{c^3}$$

1.7 Laser-optical amplification



Electrons are excited from the ground state "0" to the level "3" by pumping through incoherent radiation. The electrons then fall onto a long-lived state n_2 (State "2") from level "3". The pumping can be done either optically by shining a strong incoherent light or by passing a current. It is also assumed that the lower state is quickly emptied by a fast process with lifetime τ_1 . As a result, the population in state "2" is:

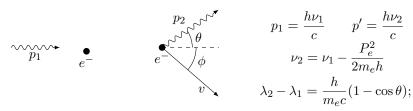
$$n_2 = \frac{R}{A_{21}}$$
 whereas $n_1 \approx 0$ because $A_{21} < \frac{1}{\tau_1}$

We have rherefore a population inversion between the two states. The likelihood of a stimulated emission process is larger than the one of absorbtion. If we enclose the system in an optical cavity, we can achieve self-sustained oscillation at the frequency ν .

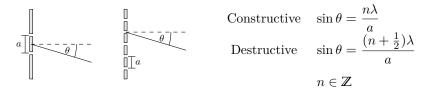
2 Wave mechanics

	frequency	wavelength	momentum	energy
Particle		$\lambda_b = \frac{h}{p}$	p = mv	$E = \frac{1}{2}mv^2$
Wave	ω	$\lambda = \frac{2\pi c}{\omega}$	$p = \frac{\hbar\omega}{c}$	$E=\hbar\omega$

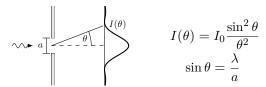
2.1 Compton Scattering



2.2 Double Slit and Bragg Diffraction



2.3 Single slit



2.4 Bohr-Sommerfeld equalization

Every single particle must satisfy the following equation. The quantized energy levels below relate to the hydrogen atom

$$p$$
 Momentum of particle E_n Energy of the nth state E_{ry} Rydberg Energy a_0 Bohr-radius E_{ry} Number of protons
$$\begin{aligned} &\int_{length} p \cdot ds = n \cdot h & n \in \mathbb{N} \\ &E_{length} & n \in \mathbb{N} \\ &E_{length} & n \in \mathbb{N} \\ &E_{length} & E_{length} & E_{len$$

3 Quantum Mechanics

3.1 Wave function

$$\psi(\vec{x},t) : \mathbb{R}^4 \to \mathbb{C} \qquad \iiint |\psi(\vec{x},t)|^2 d^3r = 1$$
$$\psi(\vec{x},t) = a\psi_1(\vec{x},t) + b\psi_2(\vec{x},t), \qquad |a|^2 + |b|^2 = 1$$

3.2 The Schrödinger equation

$$V(x,t) \quad \text{potential} \quad m \quad \text{mass}$$

$$i\hbar \cdot \frac{\partial \psi}{\partial t}(\vec{x},t) = -\frac{\hbar^2}{2m} \cdot \nabla^2 \psi(\vec{x},t) + V(\vec{x},t) \psi(\vec{x},t)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\psi = A \cdot e^{i(\vec{k}\vec{x} - \omega t)} \qquad \vec{k} = \begin{bmatrix} k_x & k_y & k_z \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$E = \omega \hbar = \frac{\hbar^2 k^2}{2m}, \qquad k^2 = |k|^2$$

3.2.1 Phase and Group Velocity

The phase velocity v_{φ} describes how fast the phase of the wave moves forward. The group velocity v_g describes how fast the energy is moving forward.

$$v_{\varphi} = \frac{\omega}{k}$$
 $v_g = \frac{\partial \omega}{\partial k}$

For a particle wave, the phase velocity v_{φ} is half the group velocity v_{q}

$$v_{\varphi} \cdot 2 = v_g$$

Stationary (Time independent) States

In a stationary state, the wave function is a product of a function $\varphi(\vec{x})$ independent of time and a function $\chi(t)$ independent of space.

$$\psi_n(\vec{x},t) = \varphi_n(\vec{x}) \cdot \chi_n(t) = \varphi_n(\vec{x}) \cdot e^{-i\frac{E_n}{\hbar}t}$$

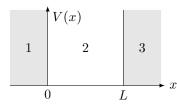
$$-\frac{\hbar^3}{2m} \nabla^2 \varphi_n(\vec{x}) + V(\vec{x}) \varphi_n(\vec{x}) = \varphi_n(\vec{x}) \cdot E_n$$

$$\iiint |\psi|^2 d^3 \vec{x} = \iiint |\varphi|^2 d^3 \vec{x} = 1$$

$$\psi(\vec{x},t) = \sum a_n \varphi_n(\vec{x}) \cdot e^{-i\frac{E_n}{\hbar}t} \sum |a_n|^2 = 1$$

Requirements: The wave function must be continous, as well as it's derivative

Example: 1D infinite potential well



$$V(x)$$

$$\psi_{1} = \psi_{3} = 0$$

$$-\frac{\hbar^{3}}{2m} \frac{\partial^{2}}{\partial x^{2}} \varphi_{2}(x, t) = E \varphi_{2}(x, t)$$

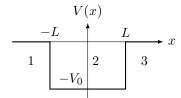
$$\varphi_{2} = A \sin(kx) + B \cos(kx)$$

Boundary cond.: $\varphi_2(0) = \varphi_2(L) = 0$

$$\varphi_{2n} = A \cdot \sin(k_n x) \quad \psi_{2n} = A \cdot \sin(k_n x) \cdot e^{-i\frac{E_n}{\hbar}x}, \quad \text{Normalize:} \quad A = \sqrt{\frac{2}{L}}$$

$$E_n = n^2 \cdot \frac{\hbar^2 \pi^2}{2mL} = n^2 \cdot E_0, \qquad k_n = \frac{n\pi}{L}$$

3.2.4Example: 1D finite potential well



The Energy E can be either bigger or smaller than 0. If E > 0, the wave function will decay exponentially in region 1 and 3. If E < 0, the wave will propagate away from the potential well.

Inside the well: The general solution to the rearranged Schrödinger's is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_2(x) = (E - V_0)\varphi_2(x)$$

$$\varphi_2(x) = A_2e^{ikx} + A_2'e^{-ikx} \qquad E = \frac{k^2\hbar^2}{2m} \quad k = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

Outside the well: There are two cases, which can apply:

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\varphi_1(x) = E\varphi_1(x)$$

1. E > 0:Unbound state

$$\varphi_1 = A_1 e^{ikx} + A_1' e^{-ikx} \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

The unbound state does not make sense to be investigated, because the particle is free to be anywhere. In the following, only the unbound state is considered.

2. E < 0: Bound state

$$\varphi_1 = B_1 e^{\delta x} + B_1' e^{-\delta x} \qquad \delta = \sqrt{-\frac{2mE}{\hbar^2}}$$

We see that as $x \to -\infty$, the Term B'_1 , as well as B_3 approaches ∞ . Since the wave function cannot approach ∞ , $B'_1 = B_3 = 0$ is a condition.

$$\varphi = \begin{cases} \varphi_1 = B_1 e^{\delta x} & x < -L \\ \varphi_2 = A_2 e^{ikx} + A_2' e^{-ikx} & -L < x < L \\ \varphi_3 = B_3' e^{-\delta x} & L < x \end{cases}$$

Boundary conditions: We require, that the wave function is continuous, as well as it's spacial derivative. Therefore, we have:

$$\varphi_1(-L) = \varphi_2(-L) \qquad \varphi_2(L) = \varphi_3(L)$$
$$\frac{\partial}{\partial x}\varphi_1(-L) = \frac{\partial}{\partial x}\varphi_2(-L) \qquad \frac{\partial}{\partial x}\varphi_2(L) = \frac{\partial}{\partial x}\varphi_3(L)$$

$$|\cos(kL)| = \frac{k}{k_o}, \quad \tan(kL) > 0$$

$$k_0 = \sqrt{\frac{2mV_0}{\hbar^2}}$$

$$|\cos(kL)|$$

$$k_0 = \sqrt{\frac{k_o}{\hbar^2}}$$

 $\begin{array}{c} \textbf{Odd solutions: only odd (sine)} \\ \textbf{components} \end{array}$

$$\left|\sin\left(kL\right)\right| = \frac{k}{k_o}, \quad \tan(kL) > 0$$

$$k_0 = \sqrt{\frac{2mV_0}{\hbar^2}}$$

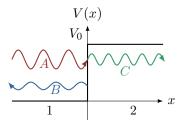
$$\left|\sin(kL)\right|$$

$$1$$

$$k_0$$

$$k_0$$

3.3 Example: 1D potential step function



An incoming plane wave from the left hits a potential step at x = 0. In region 1, two waves are added together, one is traveling to the right and one to the left. If $E > V_0$, the wave is transmitted to region 2. if $E < V_0$, the wave decays exponentially in region 2.

In **Region 1**, the general solution to the Schrödinger equation is:

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_1(x) = E\varphi_1(x), \quad \varphi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}, \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

In $\bf Region~2,$ there are two cases, which can apply:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_2 = (E - V_0)\varphi_2(x)$$

1. $E > V_0$: Transmission

$$\varphi_2 = Ce^{ik_2x}, \qquad k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

2. $E < V_0$: Complete reflection

$$\varphi_2 = Ce^{\delta_2 x}, \qquad \delta_2 = \sqrt{\frac{2m(V_0 - 2)}{\hbar^2}}$$

Applying the **initial conditions**, which require the wave function and it's derivative to be continuous at x = 0, we get the following expression for A, B, C:

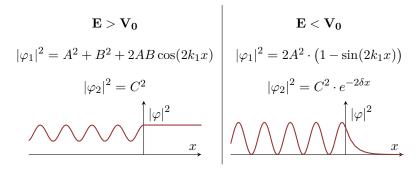
$$\varphi_1(x=0) = \varphi_2(x=0) \qquad \frac{\partial}{\partial x} \varphi_1(x=0) = \frac{\partial}{\partial x} \varphi_2(x=0)$$

$$\mathbf{E} > \mathbf{V_0} \qquad \mathbf{E} < \mathbf{V_0}$$

$$A + B = C \qquad A + B = C$$

$$k_1(A-B) = k_2C \qquad A = B$$

The **probability density function** $|\psi(x,t)|^2 = |\varphi(x)|^2 = \varphi \cdot \varphi^*$ can then be computed and sketched:

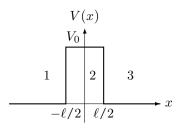


To find the **transmission coefficient** T and the **reflection coefficient** R, we normalize A=1. Then, we can define $B=\sqrt{R}$ and $C=\sqrt{T}$. Then, we can solve for R and T:

$$T = \frac{4k_1k_2}{(k_1 + k_2)^2} \qquad R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

If $E < V_0$, nothing is transmitted and therefore T = 0 and R = 1.

3.3.1 Example: 1D finite potential barrier



An incoming plane wave from the left hits a potential barrier with length l. The Transmission coefficient tells, how much of the wave can continue at the other side of the barrier (quantum tunneling).

In **Region 1** and 3, the general expression for the wave equation is the following:

$$\varphi_j(x) = A_j e^{ik_j x} + A'_j e^{-ik_j x}, \qquad k_j = \sqrt{\frac{2mE}{\hbar^2}}, \quad j \in \{1, 3\}$$

In **Region 2**, the expression is depending on V_0 . There are two cases:

1.
$$\mathbf{E} < \mathbf{V_0}$$
: $\varphi_2 = B_2 e^{\delta_2 x} + B_2' e^{-\delta_2 x}, \qquad \delta_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$

2.
$$\mathbf{E} > \mathbf{V_0}$$
: $\varphi_2 = A_2 e^{ik_2 x} + A_2' e^{-ik_2 x}, \qquad k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

Apply boundary conditions at $x = -\ell/2$ and $x = \ell/2$ in order to determine all constants. If the wave is only traveling from left to right, then $A_3' = 0$.

$$\varphi_1(-\ell/2) = \varphi_2(-\ell/2), \quad \varphi_2(\ell/2) = \varphi_3(\ell/2)$$
$$\frac{\partial}{\partial x}\varphi_1(-\ell/2) = \frac{\partial}{\partial x}\varphi_2(-\ell/2), \quad \frac{\partial}{\partial x}\varphi_2(\ell/2) = \frac{\partial}{\partial x}\varphi_3(\ell/2)$$

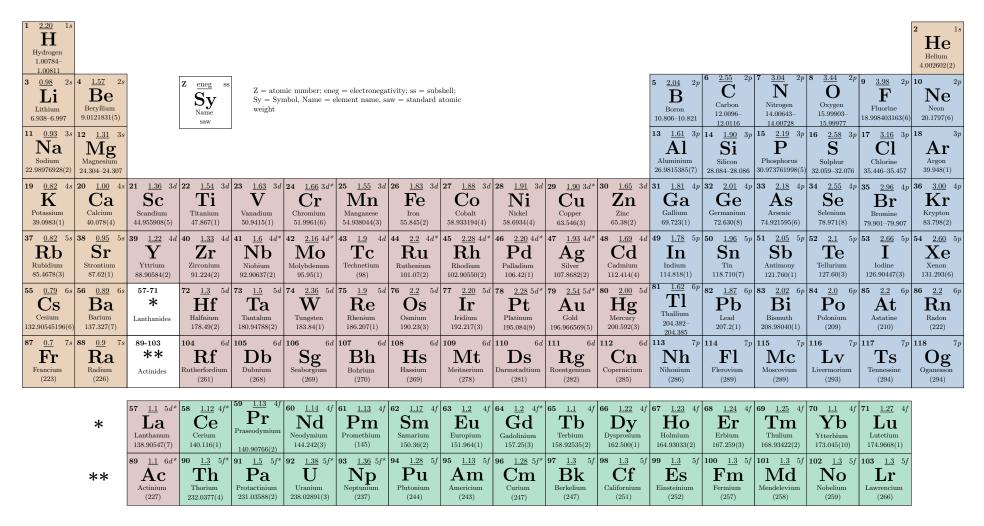
Then, the transmission coefficient T and the reflection coefficient R can be calculated as following:

$$R = \left(\frac{A_1}{A_1'}\right)^2, \qquad T = \left(\frac{A_3}{A_1}\right)^2$$

$$T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sinh^2(\delta_2 \ell)} \qquad T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sin^2(k_2 \ell)}$$

If $\mathbf{E} > \mathbf{V_0}$, the transmission coefficient has a maximum. If $k_2 \ell = n\pi \Rightarrow T = 1$ (resonance). The minimum of T is at: $k_2 \ell = \pi/2 + n\pi$.

4 Periodic Table of the Elements



Standard atomic weights taken from the Commission on Isotopic Abundances and Atomic Weights (ciaaw.org/atomic-weights.htm). Adapted from Ivan Griffin's IsTeX Periodic Table. © 2017 Paul Danese

An asterisk (*) next to a subshell indicates an anomalous (Aufbau rule-breaking) ground state electron configuration.