1 The Photon

$c\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	speed of light
$h\ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}}\right]$	planc's constant
$e\ [\mathrm{C}]$	electorn charge
$m_e\ [\mathrm{kg}]$	electron mass
$k_B\ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}^2\,\mathrm{K}}\right]$	bolzmann constant
$\epsilon_0\ \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$	vacuum permittivity

$$c = 2.998 \cdot 10^{8}\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

$$h = 6.626 \cdot 10^{-34}\ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}}\right]$$

$$\hbar = \frac{h}{2\pi}$$

$$e = 1.602 \cdot 10^{-19}\ [\mathrm{C}]$$

$$m_e = 9.109 \cdot 10^{-31}\ [\mathrm{kg}]$$

$$k_B = 1.381 \cdot 10^{-23}\ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}^2\,\mathrm{K}}\right]$$

$$\epsilon_0 = 8.854 \cdot 10^{-12}\ \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$$

$$1\ [\mathrm{eV}] = 1.602 \cdot 10^{-19}\ [\mathrm{J}]$$

1.1 Photon & Electron

$\lambda\ [\mathrm{m}]\,,\ \nu\ \left[\frac{1}{\mathrm{s}}\right]$	Wavelength, Freq.
$E\ [\mathrm{J}]$	Energy
$\vec{F_c}\ [\mathrm{N}]$	Coulomb Force

$$\lambda = \frac{c}{\nu} \quad \nu = \frac{c}{\lambda} \quad \omega = 2\pi\nu$$

$$E = h \cdot \nu = \hbar \cdot \omega$$

$$\left|\vec{F_c}\right| = \frac{Q_1 \cdot Q_2}{4\pi\epsilon_0 r^2}$$

1.2 Photoelectric effect

$V\ [\mathrm{V}]$	Voltage
$\phi_0\ [\mathrm{eV}]$	Work function
$I\ [\mathrm{A}]$	Photo-current
$n\ \left[\mathrm{m}^{-3}\right]$	Volume density of electrons
$A\ \left[\mathrm{m}^2\right]$	Area
$v\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	velocity of electrons

$$h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$V(\nu) = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

$$I = nAve$$

1.3 Blackbody Radiation

$L\ [\mathrm{m}]$	length of blackbody cube	k_i	wave constants
E_x	Electric field in x-direction	$< E >$	Average Energy
N	Number of states	D	Density of states
u	Blackbody radiation	I	Power radiated

$$E_x(x,y,z) = E_{0x}\cos(k_x x) sin(k_y y) sin(k_z z)$$

$$k_x = n\frac{\pi}{L} \quad k_y = m\frac{\pi}{L} \quad k_z = l\frac{\pi}{L} \qquad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$N(k) = \frac{1}{3\pi^2}k^3 L^3 \qquad D(k) = \frac{k^2}{\pi^2}$$

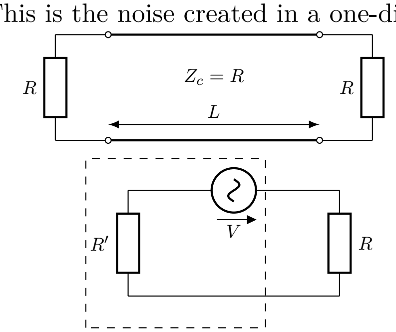
$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \cdot \frac{\hbar\omega}{\exp\left(\frac{-\hbar\omega}{kT}\right) - 1} d\omega \qquad u(\nu) = \frac{8\pi h\nu^3}{c^3\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)} d\nu$$

$$I(\omega) = c \cdot u(\omega)$$

Equipartition-Theorem: Each degree of Freedom has an energy of kT

1.4 Johnson-Noise

This is the noise created in a one-dimensional circuit (like a coax-cable).

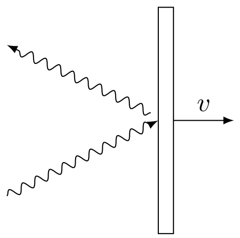


$\langle V^2 \rangle$	Noise Voltage
$\Delta\nu$	Bandwidth

$$E = E_0 \cdot \sin(k_x \cdot x)$$

$$\langle V^2 \rangle = 4R \cdot k_B T \cdot \Delta\nu$$

1.5 Momentum of a photon

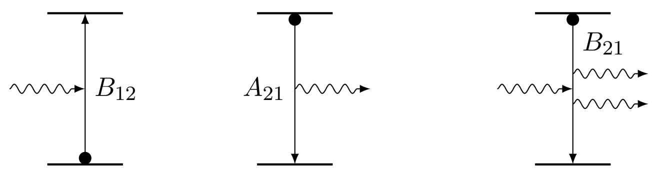


p $\left[\frac{\text{kg m}}{\text{s}}\right]$ momentum

$$p_{absorbing} = \frac{h\nu}{c} = m \cdot v$$

$$p_{reflecting} = 2 \cdot \frac{h\nu}{c}$$

1.6 Absorption, spontaneous and stimulated emission



absorbtion spontaneous emission stimulated emission

n_1 Number of electrons in the lower energy state

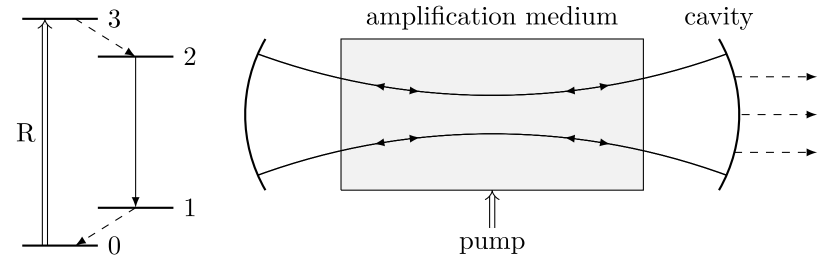
n_2 Number of electrons in the higher energy state

$$\frac{dn_2}{dt} = \underbrace{n_1 \cdot u(\nu) \cdot B_{12}}_{\text{absorbtion}} - \underbrace{n_2 \cdot u(\nu) \cdot B_{21}}_{\text{stimulated emission}} - \underbrace{n_2 \cdot A_{21}}_{\text{spontaneous emission}}$$

$$\frac{n_2}{n_1} = e^{-\frac{h\nu}{k_B T}} = \frac{u(\nu)B_{12}}{u(\nu)B_{21} + A_{21}}$$

$$B_{21} = B_{12} = B \qquad A_{21} = \frac{8\pi h \nu^3}{c^3}$$

1.7 Laser-optical amplification



Electrons are excited from the ground state “0” to the level “3” by pumping through incoherent radiation. The electrons then fall onto a long-lived state n_2 (State “2”) from level “3”. The pumping can be done either optically by shining a strong incoherent light or by passing a current. It is also assumed that the lower state is quickly emptied by a fast process with lifetime τ_1. As a result, the population in state “2” is:

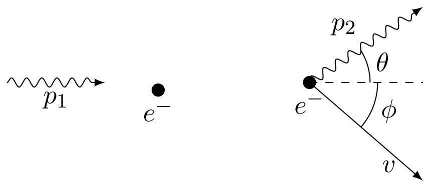
$$n_2 = \frac{R}{A_{21}} \quad \text{whereas} \quad n_1 \approx 0 \quad \text{because} \quad A_{21} < \frac{1}{\tau_1}$$

We have rherefore a population inversion between the two states. The likelihood of a stimulated emission process is larger than the one of absorbtion. If we enclose the system in an optical cavity, we can achieve self-sustained oscillation at the frequency ν.

2 Wave mechanics

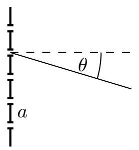
	frequency	wavelength	momentum	energy
Particle		$\lambda_b = \frac{h}{p}$	$p = mv$	$E = \frac{1}{2}mv^2$
Wave	ω	$\lambda = \frac{2\pi c}{\omega}$	$p = \frac{\hbar\omega}{c}$	$E = \hbar\omega$

2.1 Compton Scattering



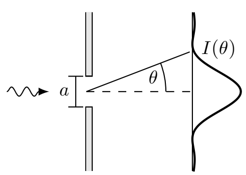
$$p_1 = \frac{h\nu_1}{c} \qquad p' = \frac{h\nu_2}{c}$$
$$\nu_2 = \nu_1 - \frac{P_e^2}{2m_e h}$$
$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta);$$

2.2 Bragg diffraction



$$\sin\theta = \frac{n\lambda}{a}$$

2.3 Single slit



$$I(\theta) = I_0 \frac{\sin^2\theta}{\theta^2}$$
$$\sin\theta = \frac{\lambda}{a}$$

2.4 Bohr-Sommerfeld equalization

Every single particle must satisfy the following equation. The quantized energy levels below relate to the hydrogen atom

p	Momentum of particle
E_n	Energy of the nth state
E_{ry}	Rydberg Energy
a_0	Bohr-radius
Z	Number of protons

$$\int_{length} p \cdot ds = n \cdot h \qquad n \in \mathbb{N}$$
$$E_n = -\frac{Z}{n^2} \cdot \frac{m_e e^4}{8\epsilon_0^2 h^2} = -\frac{Z}{n^2} \cdot E_{ry}$$
$$r_n = \frac{n^2}{Z} \cdot \frac{2\epsilon_0 h}{m_e e^2} = \frac{n^2}{Z} \cdot a_0$$
$$E_{ry} = 13.6 \ [\mathrm{eV}]$$
$$a_0 = 5.292 \cdot 10^{-11} \ [\mathrm{m}]$$

3 Quantum Mechanics

3.1 Wave function

$$\psi(\vec{r}, t) : \mathbb{R}^4 \to \mathbb{C} \qquad \iiint \left|\psi(\vec{r}, t)\right|^2 d^3 r = 1$$
$$\psi(\vec{r}, t) = a\psi_1(\vec{r}, t) + b\psi_2(\vec{r}, t), \qquad |a|^2 + |b|^2 = 1$$

3.2 The Schrödinger equation

$V(x,t)$	potential	m	mass

$$i\hbar \cdot \frac{\partial\psi}{\partial t}(\vec{x}, t) = -\frac{\hbar^2}{2m} \cdot \nabla^2\psi(\vec{x}, t) + V(\vec{x}, t)\phi(\vec{x}, t)$$
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$
$$\psi = A \cdot e^{i(\vec{k}\vec{x} - \omega t)} \qquad \vec{k} = \begin{bmatrix} k_x & k_y & k_z \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
$$E = \omega\hbar = \frac{\hbar^2 k^2}{2m}, \qquad k^2 = |k|^2$$

3.2.1 Phase and Group Velocity

The phase velocity v_φ describes how fast the phase of the wave moves forward. The group velocity v_g describes how fast the energy is moving forward.

$$v_\varphi = \frac{\omega}{k} \qquad v_g = \frac{\partial\omega}{\partial k}$$

For a particle wave, the phase velocity v_φ is half the group velocity v_g

$$v_\varphi \cdot 2 = v_g$$

3.2.2 Stationary States

In a stationary state, the wave function is a product of a function $\varphi(\vec{x})$ independent of time and a function $\chi(t)$ independent of space.

$$\psi_n(\vec{x}, t) = \varphi_n(\vec{x}) \cdot \chi_n(t) = \varphi_n(\vec{x}) \cdot e^{-i\frac{E_n}{\hbar}t}$$

$$-\frac{\hbar}{2m}\nabla^2\varphi_n(\vec{x}) + V(\vec{x})\varphi_n(\vec{x}) = \varphi_n(\vec{x}) \cdot E_n$$

$$\iiint |\psi|^2 \, d^3\vec{x} = \iiint |\varphi|^2 \, d^3\vec{x} = 1$$

$$\psi(\vec{x}, t) = \sum a_n \varphi_n(\vec{x}) \cdot e^{-i\frac{E_n}{\hbar}t} \quad \sum |a_n|^2 = 1$$