

# 1 The Photon

## 1.1 constants

$c \left[ \frac{\text{m}}{\text{s}} \right]$	speed of light	$c = 2.998 \cdot 10^8 \left[ \frac{\text{m}}{\text{s}} \right]$
$h \left[ \frac{\text{m}^2 \text{kg}}{\text{s}} \right]$	planc's constant	$h = 6.626 \cdot 10^{-34} \left[ \frac{\text{m}^2 \text{kg}}{\text{s}} \right]$
$e \text{ [C]}$	electorn charge	$\hbar = \frac{h}{2\pi}$
$m_e \text{ [kg]}$	electron mass	$e = 1.602 \cdot 10^{-19} \text{ [C]}$
$k_B \left[ \frac{\text{m}^2 \text{kg}}{\text{s}^2 \text{K}} \right]$	bolzmann constant	$m_e = 9.109 \cdot 10^{-31} \text{ [kg]}$
$\epsilon_0 \left[ \frac{\text{F}}{\text{m}} \right]$	vacuum permittivity	$k_B = 1.381 \cdot 10^{-23} \left[ \frac{\text{m}^2 \text{kg}}{\text{s}^2 \text{K}} \right]$
$\lambda \text{ [m]}$	Wavelength	$\epsilon_0 = 8.854 \cdot 10^{-12} \left[ \frac{\text{F}}{\text{m}} \right]$
$\nu \left[ \frac{1}{\text{s}} \right]$	Frequency	$1 \text{ [eV]} = 1.602 \cdot 10^{-19} \text{ [J]}$
$\omega \left[ \frac{\text{rad}}{\text{s}} \right]$	Radial frequency	$\lambda = \frac{c}{\nu} \quad \nu = \frac{c}{\lambda} \quad \omega = 2\pi\nu$
$E \text{ [J]}$	Energy	$E = h \cdot \nu$

## 1.2 Photoelectric effect

$V \text{ [V]}$	Voltage
$\phi_0 \text{ [eV]}$	Work function
$I \text{ [A]}$	Photo-current
$n \left[ \text{m}^{-3} \right]$	Volume density of electrons
$A \left[ \text{m}^2 \right]$	Area
$v \left[ \frac{\text{m}}{\text{s}} \right]$	velocity of electrons

$$h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$V(\nu) = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

$$I = nAve$$

## 1.3 Blackbody Radiation

$L \text{ [m]}$	length of blackbody cube	$k_i$	wave constants
$E_x$	Electric field in x-direction	$\langle E \rangle$	Average Energy
$N$	Number of states	$D$	Density of states
$u$	Blackbody radiation	$I$	Power radiated

$$E_x(x, y, z) = E_{0x} \cos(k_x x) \sin(k_y y) \sin(k_z z)$$

$$k_x = n \frac{\pi}{L} \quad k_y = m \frac{\pi}{L} \quad k_z = l \frac{\pi}{L} \quad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$N(k) = \frac{1}{3\pi^2} k^3 L^3 \quad D(k) = \frac{k^2}{\pi^2}$$

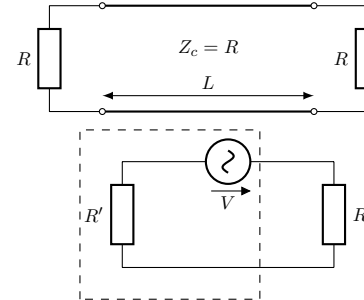
$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \cdot \frac{\hbar \omega}{\exp\left(\frac{\hbar \omega}{kT}\right) - 1} d\omega \quad u(\nu) = \frac{8\pi h \nu^3}{c^3 \left( \exp\left(\frac{h\nu}{kT}\right) - 1 \right)} d\nu$$

$$I(\omega) = c \cdot u(\omega)$$

**Equipartition-Theorem:** Each degree of Freedom has an energy of  $kT$

## 1.4 Johnson-Noise

This is the noise created in a one-dimensional circuit (like a coax-cable).



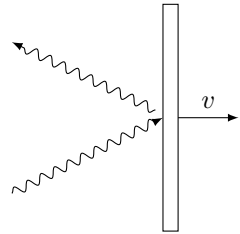
$\langle V^2 \rangle$  Noise Voltage

$\Delta\nu$  Bandwidth

$$E = E_0 \cdot \sin(k_x \cdot x)$$

$$\langle V^2 \rangle = 4R \cdot k_B T \cdot \Delta\nu$$

## 1.5 Momentum of a photon

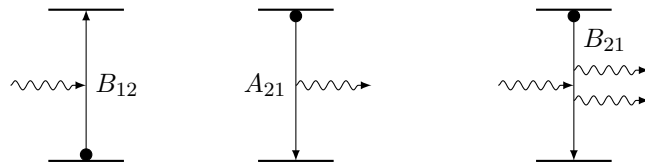


$$p \left[ \frac{\text{kg m}}{\text{s}} \right] \text{ momentum}$$

$$p_{\text{absorbing}} = \frac{h\nu}{c} = m \cdot v$$

$$p_{\text{reflecting}} = 2 \cdot \frac{h\nu}{c}$$

## 1.6 Absorption, spontaneous and stimulated emission



absorbtion    spontaneous emission    stimulated emission

$n_1$     Number of electrons in the lower energy state

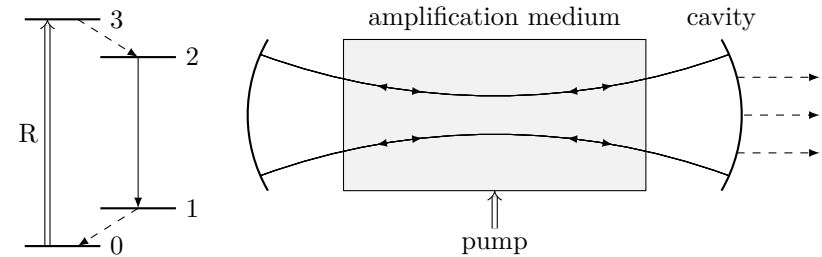
$n_2$     Number of electrons in the higher energy state

$$\frac{dn_2}{dt} = \underbrace{n_1 \cdot u(\nu) \cdot B_{12}}_{\text{absorbtion}} - \underbrace{n_2 \cdot u(\nu) \cdot B_{21}}_{\text{stimulated emission}} - \underbrace{n_2 \cdot A_{21}}_{\text{spontaneous emission}}$$

$$\frac{n_2}{n_1} = e^{-\frac{h\nu}{k_B T}} = \frac{u(\nu) B_{12}}{u(\nu) B_{21} + A_{21}}$$

$$B_{21} = B_{12} = B \quad A_{21} = \frac{8\pi h\nu^3}{c^3}$$

## 1.7 Laser-optical amplification



Electrons are excited from the ground state “0” to the level “3” by pumping through incoherent radiation. The electrons then fall onto a long-lived state  $n_2$  (State “2”) from level “3”. The pumping can be done either optically by shining a strong incoherent light or by passing a current. It is also assumed that the lower state is quickly emptied by a fast process with lifetime  $\tau_1$ . As a result, the population in state “2” is:

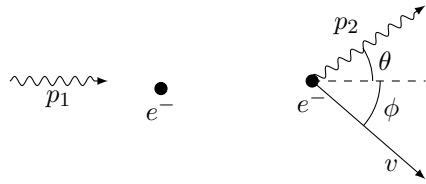
$$n_2 = \frac{R}{A_{21}} \quad \text{whereas} \quad n_1 \approx 0 \quad \text{because} \quad A_{21} < \frac{1}{\tau_1}$$

We have rherefore a population inversion between the two states. The likelihood of a stimulated emission process is larger than the one of absorbtion. If we enclose the system in an optical cavity, we can achieve self-sustained oscillation at the frequency  $\nu$ .

## 2 Wave mechanics

	frequency	wavelength	momentum	energy
Particle		$\lambda_b = \frac{h}{p}$	$p = mv$	$E = \frac{1}{2}mv^2$
Wave	$\omega$	$\lambda = \frac{2\pi c}{\omega}$	$p = \frac{\hbar\omega}{c}$	$E = \hbar\omega$

## 2.1 Compton Scattering

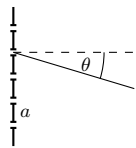


$$p_1 = \frac{h\nu_1}{c} \quad p' = \frac{h\nu_2}{c}$$

$$\nu_2 = \nu_1 - \frac{P_e^2}{2m_e h}$$

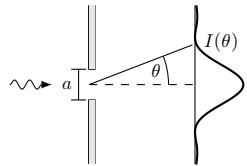
$$\lambda_2 - \lambda_1 = \frac{h}{m_e c} (1 - \cos \theta);$$

## 2.2 Bragg diffraction



$$\sin \theta = \frac{n\lambda}{a}$$

## 2.3 Single slit



$$I(\theta) = I_0 \frac{\sin^2 \theta}{\theta^2}$$

$$\sin \theta = \frac{\lambda}{a}$$

## 2.4 Bohr-Sommerfeld equalization

Every single particle must satisfy the following equation. The quantized energy levels below relate to the hydrogen atom

$p$  Momentum of particle  
 $E_n$  Energy of the nth state  
 $E_{ry}$  Constant (energy)  
 $a_0$  Constant (Bohr-radius)

$$\int_{length} p \cdot ds = n \cdot h \quad n \in \mathbb{N}$$

$$E_n = \frac{-1}{n^2} \frac{m_e e^2}{8\epsilon_0^2 h^2} = \frac{-1}{n^2} E_{ry} = \frac{-1}{n^2}$$

$$r_n = n^2 \cdot \frac{2\epsilon_0 h}{m_e e^2} = n^2 \cdot a_0$$

$$E_{ry} = 13.6 \text{ [eV]}$$

$$a_0 = 5.292 \cdot 10^{-11} \text{ [m]}$$

## 3 Quantum Mechanics

### 3.1 Wave function

$$\psi(\vec{r}, t) : \mathbb{R}^4 \rightarrow \mathbb{C} \quad \iiint |\psi(\vec{r}, t)|^2 d^3r = 1$$

$$\psi(\vec{r}, t) = a\psi_1(\vec{r}, t) + b\psi_2(\vec{r}, t), \quad |a|^2 + |b|^2 = 1$$

### 3.2 The Schrödinger equation

$V(x, t)$	potential	$m$	mass
$\psi(x, t)$	1-dimensional wave function		

$$i\hbar \cdot \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x, t)}{\partial x^2} + V(x, t)\psi(x, t)$$