1 The Photon

1.1 constants

c $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	speed of light
h $\left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}}\right]$	planc's constant
e [C]	electorn charge
m_e [kg]	electron mass
k_B $\left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}^2\,\mathrm{K}}\right]$	bolzmann constant
ϵ_0 $\left[\frac{\mathrm{F}}{\mathrm{m}}\right]$	vacuum permittivity
λ [m]	Wavelength
ν $\left[\frac{1}{\mathrm{s}}\right]$	Frequency
ω $\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]$	Radial frequency
E [J]	Energy

$$c = 2.998 \cdot 10^{8} \ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

$$h = 6.626 \cdot 10^{-34} \ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}}\right]$$

$$\hbar = \frac{h}{2\pi}$$

$$e = 1.602 \cdot 10^{-19} \ [\mathrm{C}]$$

$$m_e = 9.109 \cdot 10^{-31} \ [\mathrm{kg}]$$

$$k_B = 1.381 \cdot 10^{-23} \ \left[\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}^2\,\mathrm{K}}\right]$$

$$\epsilon_0 = 8.854 \cdot 10^{-12} \ \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$$

$$1 \ [\mathrm{eV}] = 1.602 \cdot 10^{-19} \ [\mathrm{J}]$$

$$\lambda = \frac{c}{\nu} \qquad \nu = \frac{c}{\lambda} \qquad \omega = 2\pi\nu$$

$$E = h \cdot \nu$$

1.2 Photoelectric effect

V [V]	Voltage
ϕ_0 [eV]	Work function
I [A]	Photo-current
n $\left[\mathrm{m}^{-3}\right]$	Volume density of electrons
A $\left[\mathrm{m}^{2}\right]$	Area
v $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$	velocity of electrons

$$h\nu - \phi_0 = \frac{1}{2}mv^2 = eV$$

$$V(\nu) = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

$$I = nAve$$

1.3 Blackbody Radiation

L [m]	length of blackbody cube	k_i	wave constants
E_x	Electric field in x-direction	$< E >$	Average Energy
N	Number of states	D	Density of states
u	Blackbody radiation	I	Power radiated

$$E_x(x,y,z) = E_{0x}\cos(k_x x) sin(k_y y) sin(k_z z)$$

$$k_x = n\frac{\pi}{L} \quad k_y = m\frac{\pi}{L} \quad k_z = l\frac{\pi}{L} \qquad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$N(k) = \frac{1}{3\pi^2}k^3L^3 \qquad D(k) = \frac{k^2}{\pi^2}$$

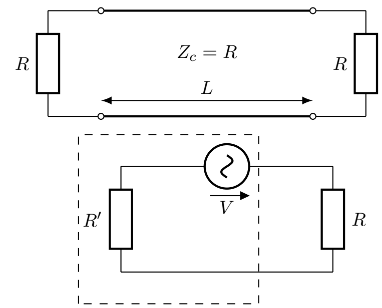
$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \cdot \frac{\hbar\omega}{\exp\left(\frac{-\hbar\omega}{kT}\right) - 1} d\omega \qquad u(\nu) = \frac{8\pi h \nu^3}{c^3\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)} d\nu$$

$$I(\omega) = c \cdot u(\omega)$$

Equipartition-Theorem: Each degree of Freedom has an energy of kT

1.4 Johnson-Noise

This is the noise created in a one-dimensional circuit (like a coax-cable).

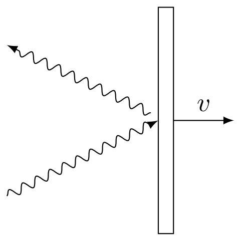


$\langle V^2 \rangle$	Noise Voltage
$\Delta\nu$	Bandwidth

$$E = E_0 \cdot \sin(k_x \cdot x)$$

$$\langle V^2 \rangle = 4R \cdot k_B T \cdot \Delta\nu$$

1.5 Momentum of a photon

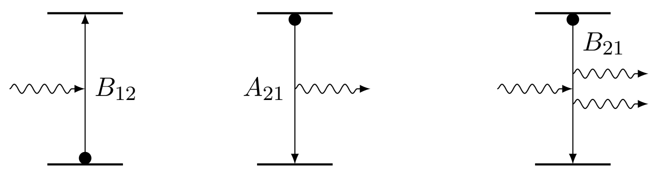


p	momentum

$$p_{absorbing} = \frac{h\nu}{c} = m \cdot v$$

$$p_{reflecting} = 2 \cdot \frac{h\nu}{c}$$

1.6 Absorption, spontaneous and stimulated emission



absorbtion spontaneous emission stimulated emission

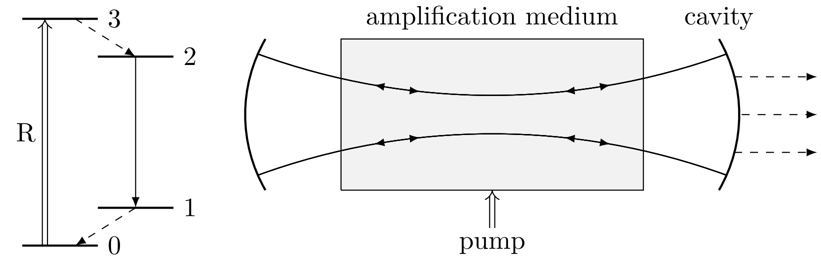
n_1	Number of electrons in the lower energy state
n_2	Number of electrons in the higher energy state

$$\frac{dn_2}{dt} = \underbrace{n_1 \cdot u(\nu) \cdot B_{12}}_{\text{absorbtion}} - \underbrace{n_2 \cdot u(\nu) \cdot B_{21}}_{\text{stimulated emission}} - \underbrace{n_2 \cdot A_{21}}_{\text{spontaneous emission}}$$

$$\frac{n_2}{n_1} = e^{-\frac{h\nu}{k_B T}} = \frac{u(\nu)B_{12}}{u(\nu)B_{21} + A_{21}}$$

$$B_{21} = B_{12} = B \qquad A_{21} = \frac{8\pi h\nu^3}{c^3}$$

1.7 Laser-optical amplification



Electrons are excited from the ground state “0” to the level “3” by pumping through incoherent radiation. The electrons then fall onto a long-lived state n_2 (State “2”) from level “3”. The pumping can be done either optically by shining a strong incoherent light or by passing a current. It is also assumed that the lower state is quickly emptied by a fast process with lifetime τ_1. As a result, the population in state “2” is:

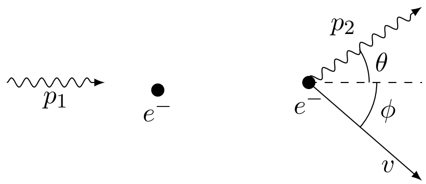
$$n_2 = \frac{R}{A_{21}} \quad \text{whereas} \quad n_1 \approx 0 \quad \text{because} \quad A_{21} < \frac{1}{\tau_1}$$

We have rherefore a population inversion between the two states. The likelihood of a stimulated emission process is larger than the one of absorbtion. If we enclose the system in an optical cavity, we can achieve self-sustained oscillation at the frequency ν.

2 Wave mechanics

	frequency	wavelength	momentum	energy
Particle		$\lambda_b = \frac{h}{p}$	$p = mv$	$E = \frac{1}{2}mv^2$
Wave	ω	$\lambda = \frac{2\pi c}{\omega}$	$p = \frac{\hbar\omega}{c}$	$E = \hbar\omega$

2.1 Compton Scattering

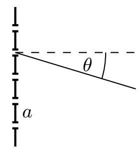


$$p_1 = \frac{h\nu_1}{c} \qquad p' = \frac{h\nu_2}{c}$$

$$\nu_2 = \nu_1 - \frac{P_e^2}{2m_e h}$$

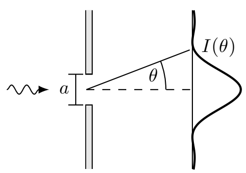
$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta);$$

2.2 Bragg diffraction



$$\sin\theta = \frac{n\lambda}{a}$$

2.3 Single slit



$$I(\theta) = I_0 \frac{\sin^2\theta}{\theta^2}$$

$$\sin\theta = \frac{\lambda}{a}$$

2.4 Bohr-Sommerfeld equalization

Every single particle must satisfy the following equation. The quantized energy levels below relate to the hydrogen atom

p	Momentum of particle
E_n	Energy of the nth state
E_{ry}	Constant (energy)
a_0	Constant (Bohr-radius)

$$\int_{length} p \cdot ds = n \cdot h \qquad n \in \mathbb{N}$$

$$E_n = \frac{-1}{n^2}\frac{m_e e^2}{8\epsilon_0^2 h^2} = \frac{-1}{n^2}E_{ry} = \frac{-1}{n^2}$$

$$r_n = n^2 \cdot \frac{2\epsilon_0 h}{m_e e^2} = n^2 \cdot a_0$$

$$E_{ry} = 13.6 \ \text{[eV]}$$

$$a_0 = 5.292 \cdot 10^{-11} \ \text{[m]}$$

3 Quantum Mechanics

3.1 Wave function

$$\psi(\vec{r}, t) : \mathbb{R}^4 \to \mathbb{C} \qquad \iiint \left|\psi(\vec{r}, t)\right|^2 d^3r = 1$$

$$\psi(\vec{r}, t) = a\psi_1(\vec{r}, t) + b\psi_2(\vec{r}, t), \qquad |a|^2 + |b|^2 = 1$$

3.2 The Schrödinger equation

$V(x,t)$	potential	m	mass
$\psi(x,t)$	1-dimensional wave function		

$$i\hbar \cdot \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t)$$