Wind Drift Calculation

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1 Introduction

Imagine an aircraft flying 95 kts at 2000 ft wanting to maintain a true track heading of 20°and a wind blows 12 kts from 120°. What heading does the aircraft have to maintain to compensate for the wind drift? This document explains how to calculate the heading when subject to a wind drift.

Two different calculation approaches are explained, via *Pythagoras's theorem* and the *cosine rule*. We will see that the former is a tad simpler than the latter, but both equally accurate.

Let's put forward a few variables to describe our wind drift problem:

- $\vec{a}$ is the aircraft velocity,
- $\vec{w}$ is the wind velocity,
- $\vec{r}$ is the resulting velocity $\vec{a} + \vec{w}$.

A velocity[1] has a length $\|\cdot\|$ and a given direction or heading $\theta(\cdot)$.

In the context of speeds and an aircraft subject to wind, $\|\vec{a}\|$ is also referred to as the *indicated airspeed* (IAS), and $\|\vec{r}\|$ is also known as the *ground speed* (GS), the *true track* (T_t) is the heading of $\vec{r}$ and the *true course* (T_c) is the heading of $\vec{a}$. The speeds can be in any unit, e.g., mph, kts, m/s or km/h, as long as the same unit is used consistently.

For the example in the introduction, the aircraft's true track is $T_\mathrm{t} = \theta(\vec{r}) = 20°$ and flying a ground speed of $\mathrm{GS} = \|\vec{r}\| = 96.35\,\mathrm{kts}$. The wind blows in the direction of $\theta(\vec{w}) = 300°$ blowing at $\|\vec{w}\| = 12\,\mathrm{kts}$. This results in a true course heading $T_\mathrm{t} = \theta(\vec{a}) = 27.15°$ and an indicated airspeed of $\mathrm{IAS} = \|\vec{a}\| = 95\,\mathrm{kts}$.

2 Calculation

The aim of the wind drift calculation is to obtain (1) our *ground speed* (GS) $\|\vec{r}\|$, and (2) find our *wind drift angle* α. Figure 1b shows the variables that we are going to use in the calculations. Via the wind drift angle $\angle$ we are able to calculate the heading to steer

$$T_\mathrm{t} = T_\mathrm{c} + \alpha \quad \text{or} \quad \theta(\vec{a}) = \theta(\vec{r}) + \alpha, \tag{1}$$

$$\mathrm{GS} = \|\vec{r}\| = \|\vec{r}_\mathrm{a}\| + \|\vec{r}_\mathrm{w}\|. \tag{2}$$

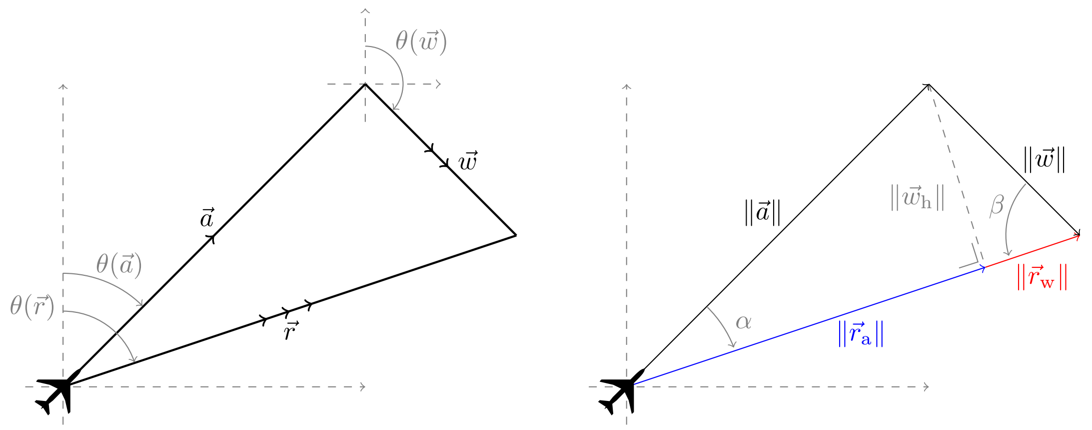
β is an angle given by the initial condition of the problem. We know the heading of the wind $\theta(\vec{w})$ and the direction in which we want to move $T_\mathrm{t} = \theta(\vec{r})$. In the case of Figure 1, the angle between the wind and the true track T_t is simply

$$\beta = \theta(\vec{w}) - \theta(\vec{r}), \tag{3}$$

as the left-hand side and the right-hand side of this equation are, in fact, vertically opposite angles.

Let's start the drift calculate for the example as shown in Figure 1. Later on we'll generalise the solution for cases where the wind blowes from other quadrants.

[1] Wikipedia: the *velocity* of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Velocity is equivalent to a specification of its speed and direction of motion (e.g. 60 km/h to the north).



(a) velocities and angles

(b) speeds and drift

Figure 1: Example of the true velocity $\vec{r}$ of an aircraft subject to wind $\vec{w}$ when following a course $\vec{a}$. The *true track* T_{t} is $\theta(\vec{r})$, while the *true course* T_{c} equals $\theta(\vec{a})$ and the winds blows in the direction T_{w}. $\|\vec{a}\|$ represents the *indicated airspeed* AIS and the *ground speed* GS is given by $\|\vec{r}\|$, which can be decomposed in $\vec{r}_{\mathrm{a}} + \vec{r}_{\mathrm{w}}$.

2.1 Pythagoras

The Pythagoras is fairly simple, it requires twice the application of the sine rule, and once Pythagoras's theorem. The key to the solution is to calculate $\|\vec{w}_{\mathrm{h}}\|$ and $\|\vec{r}_{\mathrm{w}}\|$. $\vec{w}_{\mathrm{h}}$ and $\vec{r}_{\mathrm{w}}$ form the wind velocity

$$\vec{w} = \vec{w}_{\mathrm{h}} + \vec{r}_{\mathrm{w}}, \tag{4}$$

where $\vec{r}_{\mathrm{w}}$ is the projection of the wind velocity on the true track velocity. Knowing $\|\vec{w}_{\mathrm{h}}\|$, $\|\vec{a}\|$, β and $\|\vec{w}\|$, one can calculate α, $\|\vec{r}\|_{\mathrm{a}}$ and $\|\vec{r}\|_{\mathrm{b}}$.

$\|\vec{w}_{\mathrm{h}}\|$ is obtained by applying the sine rule to β:

$$\|\vec{w}_{\mathrm{h}}\| = \|\vec{w}\| \sin(\beta). \tag{5}$$

Now $\|\vec{w}_{\mathrm{h}}\|$ can be used to calculate α as we also know $\|\vec{a}\|$,

$$\sin(\alpha) = \frac{\|\vec{w}_{\mathrm{h}}\|}{\|\vec{a}\|}. \tag{6}$$

Replacing $\|\vec{w}_{\mathrm{h}}\|$ from Equation 5, and isolating α yields the drift angle α:

$$\alpha = \arcsin\left(\frac{\|\vec{w}\| \sin(\beta)}{\|\vec{a}\|}\right). \tag{7}$$

To calculate $\|\vec{r}\|$ we need to know $\|\vec{r}_{\mathrm{b}}\|$ and $\|\vec{w}_{\mathrm{w}}\|$. $\|\vec{r}_{\mathrm{b}}\|$ can be obtained by applying Pythagoras' theorem:

$$\|\vec{r}_{\mathrm{a}}\| = \sqrt{\|\vec{a}\|^2 - \|\vec{w}_{\mathrm{h}}\|^2} \tag{8}$$

Also, $\|\vec{r}_{\mathrm{w}}\|$ is obtained similar to $\|\vec{w}_{\mathrm{h}}\|$:

$$\|\vec{r}_{\mathrm{w}}\| = \|\vec{w}\| \cos(\beta). \tag{9}$$

Now that we know both $\|\vec{r}_{\mathrm{a}}\|$ and $\|\vec{r}_{\mathrm{w}}\|$ we can calculate $\|\vec{r}\|$

$$\|\vec{r}\| = \|\vec{r}_{\mathrm{a}}\| + \|\vec{r}_{\mathrm{w}}\|. \tag{10}$$

Merging Equation 8 and Equation 9 in Equation 10 yields the groundspeed

$$\|\vec{r}\| = \sqrt{\|\vec{a}\|^2 - (\|\vec{w}\| \sin(\beta))^2} + \|\vec{w}\| \cos(\beta). \tag{11}$$

Alternatively, the groundspeed can be calculated in function of the the wind drift angle

$$\|\vec{r}\| = \|\vec{a}\| \cos(\alpha) + \|\vec{w}\| \cos(\beta). \tag{12}$$

2.2 Cosine Rule

The cosine rule is a bit more complicated as we have to deal with the roots of a quadratic polynomial. Let's get started by writing the cosine rule for β:

$$\|\vec{a}\|^2 = \|\vec{w}\|^2 + \|\vec{r}\|^2 - 2\|\vec{w}\|\|\vec{r}\|\cos(\beta) \tag{13}$$

We are looking to find $\|\vec{r}\|$. In fact, if we look closer, this equation is a quadratic in $\|\vec{r}\|$:

$$0 = \|\vec{r}\|^2 - 2\|\vec{w}\|\cos(\beta)\|\vec{r}\| + (\|\vec{w}\|^2 - \|\vec{a}\|^2). \tag{14}$$

We can obtain $\|\vec{r}\|$ by finding the roots of the above equation. Let's define c_* as the coefficients of the second order polynomial:

$$c_2 = 1 \tag{15a}$$
$$c_1 = -2\|\vec{w}\|\cos(\beta) \tag{15b}$$
$$c_0 = \|\vec{w}\|^2 - \|\vec{a}\|^2 \tag{15c}$$

The well known solution for a quadratic equation is as follows

$$\|\vec{r}\| = \frac{-c_1 \pm \sqrt{c_1^2 - 4c_2c_0}}{2c_2}. \tag{16}$$

Inserting c_0, c_1 and c_2 yields the groundspeed

$$\|\vec{r}\| = \|\vec{w}\|\cos(\beta) \pm \sqrt{\frac{\|\vec{w}\|\cos(\beta)^2}{2} - (\|\vec{w}\|^2 - \|\vec{a}\|^2)}. \tag{17}$$

However, we obtain two roots, a negative and a positive one. Only the positive root applies to our real world. This corresponds to the summation case, and so we obtain the unambiguous groundspeed

$$\|\vec{r}\| = \|\vec{w}\|\cos(\beta) - \sqrt{\frac{\|\vec{w}\|\cos(\beta)^2}{2} - (\|\vec{w}\|^2 - \|\vec{a}\|^2)}. \tag{18}$$

At this point we know $\|\vec{a}\|$, $\|\vec{w}\|$, and $\|\vec{r}\|$. This allows us to calculate the wind drift angle α. Applying the cosine rule once more

$$\|\vec{w}\|^2 = \|\vec{r}\|^2 + \|\vec{a}\|^2 - 2\|\vec{r}\|\|\vec{a}\|\cos(\alpha), \tag{19}$$

and isolating α give us

$$\alpha = \arccos\left(\frac{\|\vec{w}\|^2 - (\|\vec{r}\|^2 + \|\vec{a}\|^2)}{-2\|\vec{r}\|\|\vec{a}\|}\right). \tag{20}$$

3 Example

TODO

4 Implementation

TODO