

1. $f(x) = 5x^3 - 3x^2 - 5x + 3$. $f'(2)$?

$$f(x) = 5x^3 - 3x^2 - 5x + 3$$

$$f'(x) = 15x^2 - 6x - 5$$

$$f'(2) = 15(2)^2 - 6(2) - 5$$

$$f'(2) = 15 \cdot (4) - 12 - 5$$

$$f'(2) = 60 - 17$$

$$f'(2) = 43$$

(Jawaban: d)

2. $f(x) = \frac{2x-5}{3x-4}$. $f'(1)$?

$$f(x) = \frac{2x-5}{3x-4}$$

$$f'(x) = \frac{(3x-4)(2) - (2x-5)(3)}{(3x-4)^2}$$

$$f'(1) = \frac{(3(1)-4)(2) - (2(1)-5)(3)}{(3(1)-4)^2}$$

$$f'(1) = \frac{(-1)(2) - (-3)(3)}{(-1)^2}$$

$$f'(1) = \frac{-2+9}{1}$$

$$f'(1) = 7$$

(Jawaban: d)

3. $y = 2x^3 - 4x^2 + 2$. y' ?

$$y = 2x^3 - 4x^2 + 2$$

$$y' = 6x - 8x$$

(Jawaban: d)

4. $f(x) = (3x^2 - 7)^4$

$$f(x) = (3x^2 - 7)^4$$

$$f'(x) = 4 \cdot (3x^2 - 7)^3 \cdot (6x)$$

$$f'(x) = 24x(3x^2 - 7)^3$$

(Jawaban: c)

5. $f(x) = (2x^2 - 3x + 1)^4$. $f'(x)$?

$$f(x) = (2x^2 - 3x + 1)^4$$

$$f'(x) = 4 \cdot (2x^2 - 3x + 1)^3 \cdot (4x - 3)$$

$$f'(x) = (16x - 12)(2x^2 - 3x + 1)^3$$

(Jawaban: e)

6. $h(x) = (2x - 1)(1 - 4x)^5$. $h'(x)$?

$$h(x) = (2x - 1)(1 - 4x)^5$$

$$h'(x) = (2x - 1)d(1 - 4x)^5 + (1 - 4x)^5d(2x - 1)$$

$$h'(x) = (2x - 1)(5 \cdot (1 - 4x)^4 \cdot (-4)) + (1 - 4x)^5(2)$$

$$h'(x) = (2x - 1)(-20(1 - 4x)^4) + 2(1 - 4x)^5$$

$$h'(x) = (1 - 4x)^4(-20 \cdot (2x - 1) + 2(1 - 4x))$$

$$h'(x) = (1 - 4x)^4(-40x + 20 + 2 - 8x)$$

$$h'(x) = (22 - 48x)(1 - 4x)^4$$

(Jawaban: d)

7. $f(x) = \sqrt{x} + x$. $f'(4)$?

$$f(x) = \sqrt{x} + x$$

$$f'(x) = \frac{1}{2\sqrt{x}} + 1$$

$$f'(4) = \frac{1}{4} + 1$$

$$f'(4) = \frac{5}{4}$$

(Jawaban: c)

8. $f(x) = \sqrt[3]{(2x^3 + 3x + 3)^2}$. $f'(1)$?

$$f(x) = \sqrt[3]{(2x^3 + 3x + 3)^2}$$

$$f(x) = (2x^3 + 3x + 3)^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3}(2x^3 + 3x + 3)^{-\frac{1}{3}}(6x^2 + 3)$$

$$f'(1) = \frac{2}{3}(2 + 3 + 3)^{-\frac{1}{3}}(6 + 3)$$

$$f'(1) = \frac{2}{3 \cdot 2}(9)$$

$$f'(1) = 3$$

(Jawaban: a)

9. Jika $f(x+1) = 3x^2 + 5x + 7$ dan $f'(x-1) = -x^2$, nilai x yang memenuhi?

$$f(x+1) = 3x^2 + 5x + 7$$

Misalkan $z = x + 1$, maka:

$$f(z) = 3(z-1)^2 + 5(z-1) + 7$$

Nilai z diganti dengan x , maka:

$$f(x) = 3(x-1)^2 + 5(x-1) + 7$$

$$f(x) = 3(x^2 - 2x + 1) + 5x - 5 + 7$$

$$f(x) = 3x^2 - 6x + 3 + 5x - 5 + 7$$

$$f(x) = 3x^2 - x + 5$$

$$f(x-1) = 3(x-1)^2 - (x-1) + 5$$

$$f(x-1) = 3(x^2 - 2x + 1) - x + 1 + 5$$

$$f(x-1) = 3x^2 - 6x + 3 - x + 6$$

$$f'(x-1) = 6x - 7$$

$$-x^2 = 6x - 7$$

$$x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0$$

$$x = -7 \vee x = 1$$

THE QUEEN:

$$\begin{aligned}
 f(x+1) &= 3x^2 + 5x + 7 \\
 f'(x+1) &= 6x + 5 \\
 f'(x-2+1) &= 6(x-2) + 5 \\
 f'(x-1) &= 6x - 7 \\
 -x^2 &= 6x - 7 \\
 x^2 - 6x + 7 &= 0 \\
 (x-7)(x+1) &= 0 \\
 x = 7 \vee x = -1
 \end{aligned}$$

(Jawaban: e)

10. $f(x) = px^3 - x^2 + 5$. $f''(1) = 10$, $f'(-1)$?

$$\begin{aligned}
 f(x) &= px^3 - x^2 + 5 \\
 f''(x) &= 6px - 2 \\
 f''(1) &= 6p - 2 \\
 10 &= 6p - 2 \\
 p &= 2
 \end{aligned}$$

$$\begin{aligned}
 f'(-1) &= 3p(-1)^2 - 2(-1) \\
 f'(-1) &= 3(2)(1) + 2 \\
 f'(-1) &= 8
 \end{aligned}$$

(Jawaban: a)

11. $f(x) = ax^2 - (a+1)x + 8$, $a > 0$. $f'(a) = 14$, a ?

$$\begin{aligned}
 f(x) &= ax^2 - (a+1)x + 8 & , a > 0 \\
 f'(x) &= 2ax - a - 1 & , a > 0 \\
 f'(a) &= 2a(a) - a - 1 & , a > 0 \\
 14 &= 2a^2 - a - 1 & , a > 0 \\
 2a^2 - a - 15 &= 0 & , a > 0 \\
 a &= 3
 \end{aligned}$$

(Jawaban: c)

12. $f(x) = \frac{4}{3}x^3 + 9x^2 - 11x + 2$. $f'(a) = -1$. a ?

$$\begin{aligned}
 f(x) &= \frac{4}{3}x^3 + 9x^2 - 11x + 2 \\
 f'(x) &= 4x^2 + 18x - 11 \\
 f'(a) &= 4a^2 + 18a - 11 \\
 -1 &= 4a^2 + 18a - 11 \\
 4a^2 + 18a - 10 &= 0 \\
 a &= \frac{1}{2} \vee a = -5
 \end{aligned}$$

(Jawaban: c)

13. $f(x) = 2x^3 + nx^2 + 4x + 3$. $f''(-1) = -22$, n ?

$$\begin{aligned}
 f(x) &= 2x^3 + nx^2 + 4x + 3 \\
 f'(x) &= 6x^2 + 2nx + 4 \\
 f''(x) &= 12x + 2n \\
 f''(-1) &= -12 + 2n \\
 -22 &= -12 + 2n \\
 -10 &= 2n \\
 n &= -5
 \end{aligned}$$

(Jawaban: a)

14. $f(3-2x) = (1+3x)^4$. $f'(3)$?
Misalkan $z = 3-2x$, maka:

$$\begin{aligned}
 f(z) &= \left(1 + 3\left(\frac{z-3}{-2}\right)\right)^4 \\
 f(z) &= \left(1 + 3\left(\frac{3-z}{2}\right)\right)^4 \\
 f(x) &= \left(1 + 3\left(\frac{3-x}{2}\right)\right)^4 \\
 f'(x) &= 4 \cdot \left(1 + 3\left(\frac{3-x}{2}\right)\right)^3 \cdot \left(\frac{2(-3)-0}{4}\right) \\
 f'(3) &= 4 \cdot \left(1 + 3\left(\frac{3-3}{2}\right)\right)^3 \cdot \left(\frac{-3}{2}\right) \\
 f'(3) &= 4 \cdot (1)^3 \cdot \frac{-3}{2} \\
 f'(3) &= -6
 \end{aligned}$$

THE QUEEN:

$$\begin{aligned}
 f(3-2x) &= (1+3x)^4 \\
 f'(3-2x)(-2) &= 4(1+3x)^3(3) \\
 f'(3-2(0)) &= \frac{4(1+3(0))^3(3)}{-2} \\
 f'(3) &= -2 \cdot (1) \cdot (3) \\
 f'(3) &= -6
 \end{aligned}$$

(Jawaban: e)

15. $f(x) = \frac{2x-1}{\sqrt{x+1}}$, $f'(x-1)$?

Cara I (Aturan pembagian):

$$f(x) = \frac{2x-1}{\sqrt{x+1}}$$

$$f'(x) = \frac{\sqrt{x+1}(2) - (2x-1)\left(\frac{1}{2\sqrt{x+1}}\right)}{x+1}$$

$$f'(x) = \frac{\sqrt{x+1}(2) - \left(\frac{2x-1}{2\sqrt{x+1}}\right)}{x+1}$$

$$f'(x-1) = \frac{\sqrt{(x-1)+1}(2) - \left(\frac{2(x-1)-1}{2\sqrt{(x-1)+1}}\right)}{(x-1)+1}$$

$$f'(x-1) = \frac{2\sqrt{x} - \left(\frac{2x-3}{2\sqrt{x}}\right)}{x}$$

$$f'(x-1) = \frac{\frac{4x}{2\sqrt{x}} - \left(\frac{2x-3}{2\sqrt{x}}\right)}{x}$$

$$f'(x-1) = \frac{\frac{4x-2x+3}{2\sqrt{x}}}{x}$$

$$f'(x-1) = \frac{2x+3}{2\sqrt{x}} \cdot \frac{1}{x}$$

$$f'(x-1) = \frac{2x+3}{2\sqrt{x^3}}$$

Cara II (Aturan perkalian):

$$f(x) = \frac{2x-1}{\sqrt{x+1}}$$

$$f(x) = (2x-1)(x+1)^{-\frac{1}{2}}$$

$$f'(x) = (2x-1)d(x+1)^{-\frac{1}{2}} + (x+1)^{-\frac{1}{2}}d(2x-1)$$

$$f'(x) = (2x-1)\left(-\frac{1}{2}\right)(x+1)^{-\frac{3}{2}}(1) + (x+1)^{-\frac{1}{2}}(2)$$

$$f'(x-1) = (2x-3)\left(-\frac{1}{2}\right)(x)^{-\frac{3}{2}}(1) + (x)^{-\frac{1}{2}}(2)$$

$$f'(x-1) = \left(\frac{-2x+3}{2(x)^{\frac{3}{2}}}\right) + \frac{2}{\sqrt{x}}$$

$$f'(x-1) = \left(\frac{-2x+3}{2(x)^{\frac{3}{2}}}\right) + \frac{4x}{2x^{\frac{3}{2}}}$$

$$f'(x-1) = \left(\frac{-2x+3+4x}{2(x)^{\frac{3}{2}}}\right)$$

$$f'(x-1) = \frac{2x+3}{2\sqrt{x^3}}$$

(Jawaban: a)

$$16. f(x) = 2x^2 + 4x, g(x) = \sqrt{x-3}, h(x) = (g \circ f)(x). h'(x)?$$

$$h(x) = (g \circ f)(x)$$

$$h(x) = (4x+4) \frac{1}{2\sqrt{(2x^2+4x)-3}}$$

$$h(x) = \frac{(4x+4)}{2\sqrt{2x^2+4x-3}}$$

$$h(x) = \frac{2x+2}{\sqrt{2x^2+4x-3}}$$

(Jawaban: b)

$$17. \frac{f(x)}{g(x)} = 2x - x^2. g(1) = 2, f'(1) = f(1), g'(1) = f'(1)$$

$$\frac{f(x)}{g(x)} = 2x - x^2$$

$$\frac{g(x)}{f(x)} = \frac{1}{2x - x^2}$$

$$g(x) = \frac{f(x)}{2x - x^2}$$

$$g(1) = \frac{f(1)}{1}$$

$$2 = f(1)$$

$$f(1) = f'(1) = g'(1)$$

$$f(1) = g'(1)$$

$$g'(1) = 2$$

(Jawaban: a)

$$18. y = \sqrt{x}, t = \sqrt{x-1}, \frac{dy}{dt}?$$

$$y = \sqrt{x}$$

$$y = \sqrt{t^2 + 1}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{t^2 + 1}}(2t)$$

$$\frac{dy}{dt} = \frac{t}{\sqrt{t^2 + 1}}$$

(Jawaban: d)

$$19. y = t^3 - 2t^2, x = \sqrt{t} + 1, \frac{dy}{dx}?$$

$$y = t^3 - 2t^2$$

$$y = (x-1)^6 - 2(x-1)^4$$

$$\frac{dy}{dx} = 6(x-1)^5 - 8(x-1)^3$$

$$\frac{dy}{dx} = (x-1)^3(6(x-1)^2 - 8)$$

$$\frac{dy}{dx} = (x-1)^3(6(x^2 - 2x + 1) - 8)$$

$$\frac{dy}{dx} = (x-1)^3(6x^2 - 12x - 2)$$

(Jawaban: d)

$$20. g(x) = \sqrt{x - \sqrt{x+1}}. g'(3)?$$

$$g(x) = (x - (x+1)^{1/2})^{1/2}$$

$$g'(x) = \frac{1}{2}(x - (x+1)^{1/2})^{-1/2}(1 - (\frac{1}{2}(x+1)^{-1/2}))$$

$$g'(3) = \frac{1}{2}(3 - 4^{1/2})^{-1/2}(1 - (\frac{1}{2}(4)^{-1/2}))$$

$$g'(3) = \frac{1}{2}(3 - 2)^{-1/2}(1 - (\frac{1}{4}))$$

$$g'(3) = \frac{1}{2}(\frac{3}{4})$$

$$g'(3) = \frac{3}{8}$$

(Jawaban: c)