Turunan fungsi trigonometri - Latihan 1 - LKS hal. 67 11F

1. Jika $f(x) = \sin(3x^2 + 1)$, turunan pertama dari $f(x)$ adalah $f'(x) = ...$

$f(x) = \sin(3x^2 + 1)$

$f'(x) = 6x \cdot \cos(3x^2 + 1)$

(Jawaban: d)

2. Turunan pertama dari $h(x) = \sin x \tan x$ adalah $h'(x) = ...$

$h(x) = \sin x \tan x$

$h'(x) = \sin(x)(\sec^2(x)) + \tan(x)\cos(x)$

$\Leftrightarrow \sin(x)(1 + \tan^2(x)) + \tan(x)\cos(x)$

$\Leftrightarrow \sin(x) + \sin(x)\tan^2(x) + \sin(x)$

$\Leftrightarrow \sin(x)(1 + \tan^2(x) + 1)$

$\Leftrightarrow \sin(x)(2 + \tan^2(x))$

(Jawaban: c)

3. Jika $f(x) = x^2 \cos(1 - 2x)$, turunan pertama dari $f(x)$ adalah $f'(x) = ...$

$f(x) = x^2 \cos(1 - 2x)$

$f'(x) = (x^2)(-\sin(1 - 2x)(-2)) + \cos(1 - 2x)(2x)$

$\Leftrightarrow 2x^2 \sin(1 - 2x) + 2x\cos(1 - 2x)$

(Jawaban: b)

4. Turunan pertama dari $g(x) = \dfrac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)}$ adalah $g'(x) = ...$

$$g(x) = \frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)}$$

$$\Leftrightarrow \frac{(\sin(x) + \cos(x))(\cos(x) + \sin(x)) - (\sin(x) - \cos(x))(\cos(x) - \sin(x))}{(\sin(x) + \cos(x))^2}$$

$$\Leftrightarrow \frac{\cancel{\sin(x)\cos(x)} + \sin^2(x) + \cos^2(x) + \cancel{\sin(x)\cos(x)} - \cancel{\sin(x)\cos(x)} + \sin^2(x) + \cos^2(x) - \cancel{\sin(x)\cos(x)}}{\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x)}$$

$$\Leftrightarrow \frac{2(\sin^2(x) + \cos^2(x))}{1 + \sin(2x)}$$

$$\Leftrightarrow \frac{2}{1 + \sin(2x)}$$

(Jawaban: d)

5. Diketahui $g(x) = \tan^2(\pi - 3x)$. Nilai $\frac{d}{dx}g(\frac{\pi}{4}) = ...$

$g(x) = \tan^2(\pi - 3x)$

$\dfrac{d}{dx}g(x) = 2\tan(\pi - 3x)(\sec^2(\pi - 3x)(-3))$

$\dfrac{d}{dx}g\left(\dfrac{\pi}{4}\right) = 2\tan(\pi - 3\left(\dfrac{\pi}{4}\right))(\sec^2(\pi - 3\left(\dfrac{\pi}{4}\right))(-3))$

$$\Leftrightarrow 2(1)\left(\frac{-3}{\cos^2(\pi - 3(\frac{\pi}{4}))}\right)$$

$$\Leftrightarrow -6 \cdot \frac{1}{\frac{1}{2}}$$

$\Leftrightarrow -12$

(Jawaban: a)

6. Jika $f(x^2 - \frac{\pi}{2}) = \cos(\frac{1}{2}x^2 - \pi)$, nilai $f'(\frac{\pi}{2}) = ...$

Misalkan $z = x^2 - \frac{\pi}{2}$, maka:

$$f(z) = \cos(\frac{1}{2}(\sqrt{z + \frac{\pi}{2}})^2 - \pi)$$
$$f(x) = \cos(\frac{1}{2}(x + \frac{\pi}{2}) - \pi)$$
$$f(x) = \cos(\frac{x}{2} + \frac{\pi}{4} - \pi)$$
$$f'(x) = (\frac{1}{2}) \cdot -\sin(\frac{x}{2} + \frac{\pi}{4} - \pi)$$
$$f'(\frac{\pi}{2}) = (\frac{1}{2}) \cdot -\sin(\frac{\frac{\pi}{2}}{2} + \frac{\pi}{4} - \pi)$$
$$\Leftrightarrow (\frac{1}{2}) \cdot -\sin(-\frac{\pi}{2}) \Rightarrow (\frac{1}{2}) \cdot (1) \Rightarrow \frac{1}{2}$$

THE QUEEN

$$f(x^2 - \frac{\pi}{2}) = \cos(\frac{1}{2}x^2 - \pi)$$
$$2x \cdot f'(x^2 - \frac{\pi}{2}) = x \cdot -\sin(\frac{1}{2}x^2 - \pi) \Rightarrow \text{ Substitusi } x \leftarrow \sqrt{\pi}$$
$$f'(\frac{\pi}{2}) = \frac{-\sin(\frac{\sqrt{\pi}^2}{2} - \pi)}{2} \Rightarrow \frac{-\sin(\frac{-\pi}{2})}{2} \Rightarrow \frac{1}{2}$$

(Jawaban: e)

7. Diketahui $f(x) = ax + b\cot x$, $f'(\frac{\pi}{6}) = -7$, dan $f'(\frac{\pi}{2}) = 2$. Nilai $a - b = ...$

$$f(x) = ax + b\cot x$$
$$f'(x) = a - b\csc^2(x)$$
$$f'(\frac{\pi}{6}) = a - b\csc^2(\frac{\pi}{6}) \quad \leftarrow \text{ bagian ini ga penting}$$
$$-7 = a - b(4) \quad \leftarrow \text{ bagian ini ga penting}$$
$$f'(\frac{\pi}{2}) = a - b\csc^2(\frac{\pi}{2})$$
$$2 = a - b$$

(Jawaban: b)

8. Turunan dari $y = \sin^2(5x)$ adalah $y' = (2a - 1)\sin(5bx)$. Nilai $a \cdot b = ...$

$$y = \sin^2(5x)$$
$$y' = 2\sin(5x)5\cos(5x)$$
$$(2a - 1)\sin(5bx) = 10\cos(5x)\sin(5x)$$
$$(2a - 1)\sin(5bx) = 10\cos(5x)\sin(5x)$$
$$(2a - 1)\sin(5bx) \equiv 5\sin(10x)$$
$$2a - 1 = 5 \text{ dan } 5bx = 10x$$
$$a = 3 \text{ dan } b = 2$$
$$a \cdot b = 6$$

(Jawaban: a)

9. Jika $f(x) = \tan(4x)$ dan $f'(x) = 4$ untuk $0 \leq x \leq \frac{\pi}{2}$, nilai x yang memenuhi...

$$\begin{aligned} f(x) &= \tan(4x) \\ f'(x) &= 4\sec^2(4x) \\ 4 &= \frac{4}{\cos^2(4x)} \\ \cos^2(4x) &= 1 \\ \cos(4x) &= \pm 1 \\ x &= 0, \frac{\pi}{4}, \frac{\pi}{3} \end{aligned}$$

(Jawaban: d)

10. Jika $W(t) = \sin(2t)$ dan $\frac{d}{dt}W(\frac{\pi}{4} + 2t) = \sqrt{3}$ untuk $0 \leq t \leq \frac{3\pi}{4}$, nilai t = ...

$$\begin{aligned} W(t) &= \sin(2t) \\ W'(t) &= 2\cos(2t) \\ \sqrt{3} &= 2\cos(2(\frac{\pi}{4} + 2t)) \\ \frac{\sqrt{3}}{2} &= \cos(2(\frac{\pi}{4} + 2t)) \\ 30 &= 90 + 4t \\ -60 &= 4t \\ t &= -15 \rightarrow \frac{\pi}{12} \rightarrow \frac{\pi}{3} \end{aligned}$$

(Jawaban: e)

11. Diketahui fungsi $f(x) = (a-1)\tan(x) + (b-2)x$. Jika $f'(\frac{\pi}{3}) = 9$ dan $f'(\frac{\pi}{4}) = 3$, nilai a + b = ...

$$\begin{aligned} f(x) &= (a-1)\tan(x) + (b-2)x \\ f'(x) &= (a-1)\sec^2(x) + (b-2) \\ f'(\frac{\pi}{3}) &= (a-1)4 + (b-2) \\ 9 &= 4a - 4 + b - 2 \\ 4a + b &= 15 \\ f'(\frac{\pi}{4}) &= (a-1)2 + (b-2) \\ 3 &= 2a - 2 + b - 2) \\ 2a + b &= 9 \\ a &= 3 \text{ dan } b = 3 \\ a + b &= 6 \end{aligned}$$

(Jawaban: 6 (tidak ada pilihan jawaban))

12. Jika $f(x) = -2\cos(x)$, nilai x yang memenuhi $f(x) + f'(x) = \sqrt{2}$ untuk $0° \leq x \leq 180°$ adalah...

$$
\begin{aligned}
f(x) &= -2\cos(x) \\
f'(x) &= 2\sin(x) \\
f(x) + f'(x) &= \sqrt{2} \\
-2\cos(x) + 2\sin(x) &= \sqrt{2} \\
2\sin(x) - 2\cos(x) &= \sqrt{2} \\
\sin(x) - \cos(x) &= \frac{\sqrt{2}}{2} \\
\sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x) &= \frac{1}{2} \\
1 - \sin(2x) &= \frac{1}{2} \\
\sin(2x) &= \frac{1}{2} \\
x &= 15^\circ \text{ atau } x = 75^\circ
\end{aligned}
$$

(Jawaban: a)

13. Diketahui $f(x) = \sin^2(x)$ dan $f'(x) = 1$ untuk $0^\circ \leq x \leq 180^\circ$. Nilai x yang memenuhi adalah...

$$
\begin{aligned}
f(x) &= \sin^2(x) \\
f'(x) &= 2\sin(x)\cos(x) \\
&\Leftrightarrow \sin(2x) \\
f''(x) &= 2\cos(2x) \\
-1 &= 2\cos(2x) \\
\cos(2x) &= -\frac{1}{2} \\
2x &= 120, 120 * 2, 120 * 4, 120 * 5, 120 * 7, 120 * 8... \\
x &= 60, 120, ...
\end{aligned}
$$

(Jawaban: d)

14. Besar gara Lorentz F (dalam Newton) yang dialami kawat berarus I (dalam Ampere) sepanjang l (dalam meter) dalam medan magnet B (dalam Tesla) memenuhi persamaan $F = i\ell B\sin(\theta)$. Diketahui θ merupakan sudut antara arah arus dan arah medan magnet. Jika i = 50 A, ℓ = 2 meter, B 0.04 Tesla, dan θ berubah dengan laju 0,02 rad/detik, laju perubahan gaya Lorentz yang dialami kawat pada saat $\theta = \frac{\pi}{3}$ adalah ... N/s

$$
\begin{aligned}
F &= i\ell B\sin(\theta) \\
F(\theta) &= (50)(2)(0.04)\sin(\theta) \\
\frac{dF}{d\theta}(\theta) &= 4\cos(\theta) \\
\frac{dF}{d\theta}(\frac{\pi}{3}) &= 2 \\
2 \cdot 0.02 &= 0.04 \text{ N/s}
\end{aligned}
$$

(Jawaban: d)

15. Sepanjang hari, kedalaman air H (dalam meter) dari dasar sebuah bendungan selalu berubah. Kedalaman air dari dasar bendungan suatu hari memenuhi persamaan $H = 8 + 2\cos(\frac{\pi t}{6})$, t (dalam jam) dan $0 \leq t \leq 24$. t = 0 dimulai pada tengah malam. Laju perubahan kedalaman air pada pukul 11.00 adalah ... m/jam

$$H(t) = 8 + 2\cos(\frac{\pi t}{6})$$
$$H'(t) = \frac{-2\pi}{6}\sin(\frac{\pi t}{6})$$
$$H'(11) = \frac{-2\pi}{6}\frac{-1}{2}$$
$$\Leftrightarrow \frac{1}{6}\pi$$

(Jawaban: b)