

Nick T. Thomopoulos

Essentials of Monte Carlo Simulation

Statistical Methods for Building
Simulation Models

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*For my wife,
my children,
and my grandchildren.*

Preface

I was fortunate to have a diverse career in industry and academia. This included working at International Harvester as supervisor of operations research in the corporate headquarters; at IIT Research Institute (IITRI) as a senior scientist with applications that spanned worldwide in industry and government; as a professor in the Industrial Engineering Department at the Illinois Institute of Technology (IIT), in the Stuart School of Business at IIT; at FIC Inc. as a consultant for a software house that specializes in supply chain applications; and the many years of consulting assignments with industry and government throughout the world. At IIT, I was fortunate to be assigned a broad array of courses, gaining a wide breadth with the variety of topics, and with the added knowledge I acquired from the students, and with every repeat of the course. I also was privileged to serve as the advisor to many bright Ph.D. students as they carried on their dissertation research. Bits of knowledge from the various courses and research helped me in the classroom, and also in my consulting assignments. I used my industry knowledge in classroom lectures so the students could see how some of the textbook methodologies actually are applied in industry. At the same time, the knowledge from the classroom helped to formulate and develop Monte Carlo solutions to industry applications as they unfolded. This variety of experience allowed the author to view how simulation can be used in industry. This book is based on this total experience.

Simulation has been a valuable tool in my professional life, and some of the applications are listed below. The simulations models were from real applications and were coded in various languages of FORTRAN, C++, Basic, and Visual Basic. Some models were coded in an hour, others in several hours, and some in many days, depending on the complexity of the system under study. The knowledge gained from the output of the simulation models proved to be invaluable to the research team and to the project that was in study. The simulation results allowed the team to confidently make the decisions needed for the applications at hand. For convenience, the models below are listed by type of application.

Time Series Forecasting

- Compare the accuracy of the horizontal forecast model when using 12, 24 or 36 months of history.
- Compare the accuracy of the trend forecast model when using 12, 24 or 36 months of history.
- Compare the accuracy of the seasonal forecast model when using 12, 24, or 36 months of history.
- Compare the accuracy of forecasts between weekly and monthly forecast intervals.
- Compare the accuracy benefit of forecasts when using month-to-date demands to revise monthly forecasts.
- Compare the accuracy of the horizontal forecast model with the choice of the alternative forecast parameters.
- Compare the accuracy of the trend forecast model with the choice of the alternative forecast parameters.
- Compare the accuracy of the seasonal forecast model with the choice of the alternative forecast parameters.
- In seasonal forecast models, measure how the size of the forecast error varies as the season changes from low-demand months to high-demand months.

Order Quantity

- Compare the inventory costs for parts (with horizontal, trend, and seasonal demand patterns) when stock is replenished by use of the following strategies: EOQ, Month-in-Buy or Least Unit Cost.
- Compare various strategies to determine the mix of shoe styles to have in a store that yields the desired service level and satisfies the store quota.
- Compare various strategies to determine the mix of shoe sizes for each style type to have in a store that yields the desired service level and satisfies the store quota for the style.
- Compare various strategies to find the initial-order-quantity that yields the least cost for a new part in a service parts distribution center.
- Compare various strategies to find the all-time-requirement that yields the least cost for a part in a service parts distribution center.
- Compare various ways to determine how to measure lost sales demand for an individual part in a dealer.
- Compare strategies, for a multi-distribution system, on how often to run a transfer routine that determines for each part when and how much stock to transfer from one location to another to avoid mal-distribution.

Safety Stock

- Compare the costs between the four basic methods of generating safety stock: month's supply, availability, service level and Lagrange.
- Compare how the service level for a part reacts as the size of the safety stock and the order quantity vary.
- Compare how a late delivery of stock by the supplier affects the service level of a part.
- Compare strategies on how to find the minimum amount of early stock to have available to offset the potential of late delivery by the supplier.
- Measure the relationship between the service level of a part and the amount of lost sales on the part.

Production

- In mixed-model (make-to-stock) assembly, compare various strategies on how to sequence the models down the line.
- In mixed-model (make-to-order) assembly, compare various strategies on how to sequence the individual jobs down the line.
- In job-shop operations, determine how many units to initially produce to satisfy the order needs and minimize the material, machine, and labor costs.
- In machine-loading operations, compare strategies on how to schedule the jobs through the shop to meet due dates and minimize machine idle times.
- Compare strategies on how to set the number of bays (for maintenance and repair) in a truck dealership that meets the customer needs and minimizes the dealer labor costs.

Other

- In the bivariate normal distribution, estimate the cumulative distribution function for any combination of observations when the means and variances are given, and the correlation varies between -1.0 and 1.0 .
- In the bivariate lognormal distribution, estimate the cumulative distribution function for any combination of observations when the means and variances of the transformed variables are known, and the correlation varies from -1.0 to 1.0 .
- In the multivariate normal distribution with k variables, an estimate of the cumulative distribution function is obtained, for any combination of observations when both the mean vector and the variance-covariance matrix are known.

- In an airport noise abatement study, noise measures were estimated, as in a contour map, for the airport and for all blocks surrounding the airport. The noise was measured with various combinations of: daily number of flights in and out, the type of aircraft and engines, and the direction of the runways in use.
- In a study for the navy, some very complex queuing systems were in consideration. Analytical solutions were developed, and when a level of doubt was present in the solution, simulation models were developed to verify the accuracy of the analytical solutions.
- A simulated database for part numbers were needed in the process of developing various routines in forecasting and inventory replenishment for software systems. These were for systems with one or more stocking locations. The database was essential to test the effectiveness of the routines in carrying out its functions in forecasting and inventory replenishments. The reader may note that many of the fields in the database were jointly related and thereby simulated in a correlated way.

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