

Tutorial 4

Q.1) $T(n) = 3T(n/2) + n^2$

A.1) $a = 3$

$$b = 2$$

as $a > 1, b > 1$

$$\Rightarrow C = \log_b a = \log_2 3 = 1.58$$

$$\Rightarrow n^C = n^{1.58}$$

as $f(n) > n^C$

$$T(n) = \Theta(f(n))$$

$$= \Theta(n^2). \text{ Ans}$$

Q.2) $T(n) = 4T(n/2) + n^2$

A.2) $a = 4$

$$b = 2.$$

as $a > 1, b > 1$

$$\Rightarrow C = \log_b a = \log_2 4 = 2. \Rightarrow n^C = n^2$$

so as $f(n) = n^C$

$$\Rightarrow T(n) = \Theta(n^2 \log n) \text{ Ans}$$

Q.3) $T(n) = T(n/2) + 2^n$

$$a = 1$$

$$b = 2.$$

\because the following (f) is exponential
 \therefore not polynomial it can't be solved.

Q.4) $T(n) = 2^n T(n/2) + n^n$

- Master theorem does not apply here
 because a is not constant.

$$Q5) T(n) = 16T(n/4) + n$$

$$a = 16$$

$$b = 4$$

$$\Rightarrow c = \log_b a = \log_4 16 = \frac{1.204}{0.60} = 2$$

$$\text{so } n^c = n^2$$

$$\text{as } n^c > f(n)$$

$$\Rightarrow T(n) = \Theta(n^2)$$

$$Q6) T(n) = 2T(n/2) + n \log n$$

$$a = 2$$

$$b = 2$$

$$a > 1, b > 1 \Rightarrow c = \log_b a = \log_2 2 = 1 \Rightarrow n^c = 1.$$

$$\Rightarrow n^c = 1.$$

assuming $f(n) = n \log(n) > -1$.

$$\Rightarrow T(n) = \Theta(n \log_b \log^{p+1} n)$$

$$= \Theta(n^1 \log^{1+p} n)$$

$$= \underline{\underline{\Theta(n \log^n n)}}$$

$$Q7) T(n) = 2T(n/2) + n/\log n$$

\therefore Master theorem applies to form that are polynomial, $n/\log n$ is not polynomial so master theorem does not apply.

Q.8) $T(n) = 2T(n/4) + n^{0.51}$

A.8) $a=2$
 $b=4$.

$$\Rightarrow c = \log_b a = \log_4 2 = \frac{\log(2)}{\log(4)} = \frac{0.30}{0.60} = 0.5.$$

as $f(n) > n^c$

$$\Rightarrow T(n) = \Theta(f(n)) = \Theta(n^{0.51}) \text{ Ans}$$

Q.9) $T(n) = 0.5T(n/2) + 1/n$

- Does not apply $\because a < 1$.

Q.10) $T(n) = 16T(n/4) + n!$

$a = 16$

$b = 4$

$$\Rightarrow c = \log_b a = \log_4 16 = \frac{\log(16)}{\log 4} = \frac{4 \log 2}{\log 4}$$

as $n^c = n^2$

$$= \frac{4 \times 0.30}{0.60} = 2.0$$

but $f(n) > n^c$

$$\Rightarrow T(n) = \Theta(f(n)) \\ = \Theta(n!) \text{ Ans}$$

Q.11) $T(n) = \sqrt{2}T(n/2) + \log n$

$$a = \sqrt{2}$$

$$b = 2$$

$$c = \log_b a = \log_2(\sqrt{2}) = \frac{\log 2}{\log 2} = \frac{1}{2} = 0.5,$$

$$\text{so } n^c = n^{0.5}.$$

$$\Rightarrow f(n) = \log n$$

as $n^c > f(n)$

$$\Rightarrow T(n) = \Theta(n^c) = \Theta(n^{0.5})$$

Q.12) $T(n) = \sqrt{n} T(n/2) + \log n$

$\therefore a$ is not a constant, following form
cannot be solved.

Q.13) $T(n) = 3T(n/2) + n$

$$a = 3$$

$$b = 2$$

$$\Rightarrow c = \log_b a = \log_2 3 = \frac{0.69}{0.30} = 2.3.$$

as $n^c > f(n)$

$$\Rightarrow T(n) = \Theta(n^c) = \Theta(n^{2.3})$$

Q.14) $T(n) = 3T(n/3) + \sqrt{n}$

$$a = 3$$

$$b = 3$$

$$\Rightarrow c = \log_b a = \log_3 3 = 1 \Rightarrow n^c = n.$$

as $n^c \leq f(n)$; i.e. $n^1 \leq n^{1/2}$

$$\Rightarrow T(n) = \Theta(f(n)) \\ = \Theta(n^{\underline{2}}) \text{ by } .$$

Q.15) $T(n) = 4T(n/2) + cn$

A.15) $a=4 \ b=2$

$$c = \log_2 4 = 2. \\ n^c = n^2 \geq f(n) \\ T(n) = \Theta(n^2).$$

Q.16) $T(n) = 3T(n/4) + n \log n$

$$a=3, b=4$$

$$c = \log_4 3 = 0.79. \\ n^c = n^{0.79} < f(n)$$

$$T(n) = \Theta(n \log n)$$

Q.17) $T(n) = 3T(n/3) + n/2$

$$a=3 \ b=3$$

$$c = \log_3 3 = 1 \\ n^c = n \geq f(n) \\ T(n) = \Theta(n)$$

Q.18) $T(n) = 6T(n/3) + n^2 \log n$

$$a=6, b=3$$

$$c = \log_2 6 = 1.63$$

$$n^c = n^{1.63} < f(n)$$

$$T(n) = O(n^2 \log n).$$

Q. 19) $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

A. 19) $a=4, b=2$

$$c = \log_2 4 = 2$$

$$n^c = n^2 > f(n)$$

$$T(n) = O(n^2)$$

Q. 20) $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

$$a=64, b=8$$

$$c = \log_8 64 = 2$$

$$n^c = n^2 < f(n)$$

$$T(n) = O(n^2 \log \frac{1}{n})$$

Q. 21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$$a=7, b=3$$

$$c = \log_3 7 = 1.77$$

$$n^c = n^{1.77} < f(n)$$

$$T(n) = O(n^2)$$

Q. 22) $T(n) = T\left(\frac{n}{2}\right) + n(2 \cos n)$

$$a=1, b=2, c = \log_2 1 = 0$$

$$n^c = n^0 = 1 < f(n)$$

$$T(n) = O(n(2 - \cos n))$$