Q. 1.				
Q.1. (a)	**3**	**2**	Consider the recurrence T(n)= T(n/5) + c, for n>1 and T(1)= 1, where c is an absolute constant. Give best possible asymptotic upper bound on T(n). Express your answer using Big Oh asymptotic notation.	**1 marks**
Q.1. (b)	**1,3**	**2, 3**	While expressing asymptotic time complexities whenever logarithmic factor arises, typically, we do not specify the base of the logarithm used. Give concrete argument justifying it.	**1 marks**
Q.1. (c)	**1,3**	**2**	Define inversion in a permutation. Given an array A of size n, with n distinct elements what is the smallest and the largest number of inversions A can have?	**1 marks**
Q.1. (d)	**3**	**2**	Define Big Oh asymptotic notation. Prove that log n is Big Oh of n^{1/2}.	**2 marks**
Q.1. (e)	**1,3**	**3, 4**	Suppose you are given co-ordinates c1, c2, ..., cn of n cities located on a straight-line path. We want to build a petrol-pump on this straight-line path such that sum of the distances from cities to the petrol-pump is minimized, that is we want to find real number x such that the sum |x-c1| + |x-c2| + ... |x-cn| is minimized. Where |a| denotes absolute value of the number a. Note that c1, c2, ..., cn need not be in sorted order. Give an efficient algorithm to find x. Prove the correctness of your algorithm and analyze its time complexity. [Note: 3 marks are for correctness proof]	**5 marks**
Q. 2.				
Q. 2. (a)	**2, 3**	**2, 3**	Consider Josephus game with N players. Players are numbered from 1 to N. Players 1 to N are standing round a circle in clockwise direction with the first player holding a sword. Player 1 kills player 2 and passes the sword to player 3. Player 3 kills player 4 and passes sword to player 5. This goes on. Any player having the sword kills the next alive person in clockwise direction and passes the sword to the next alive person in clockwise direction. The game continues till only one player is left, and this player is called the winner of the game. Who is the winner for Josephus game with **N = 2^{2^100}+ 2^100 - 1** many players? (1 mark for the correct approach, 1 mark for final correct answer)	**2 marks**
Q. 2. (b)	**2, 3**	**2, 3**	Consider a chocolate bar of size **m x n** your task is to divide it in pieces of size **1 x 1** by cutting the bar. Every valid move involves cutting a single piece along any one grid line. (E.g. suppose you have bar of size **3 x 4** to begin with, then a single horizontal cut at y =1 grid line results into two pieces of dimensions **1 x 4**, **2 x 4** respectively). Note that in any move you are only allowed to cut a single piece. What is the minimum number of moves required for this task?	**2 marks**
Q. 2. (c)	**3**	**3**	Consider the recurrence T(n)= T(n-1) + n for n>1 and T(1)=1. Give correct asymptotic estimate of T(n) using Big Theta notation.	**1 mark**

Q. 2 (d) ii	1, 2, 3	3, 4	Let A be n x n array of integers such that the numbers in every row and column of A are in sorted order. On input an integer a, give an efficient algorithm to search a in A. Give proper pseudocode and brief argument justifying correctness of the algorithm. Analyze the time complexity of the algorithm. (Your algorithm should be asymptotically strictly better than straightforward *O(n log n)* algorithm.)	
Q. 3.				
Q. 3 (a)	1,2	4	What is the remainder you will get when you divide 11^58 by 59? (1 mark for correct answer, 1 mark for correct justification)	**2 marks**
Q. 3 (b)	1, 2	5	Compare divide and conquer and Dynamic Programming strategies. For each of the strategies give an example of a computational problem where the strategy can be effectively applied.	**2 marks**
Q. 3 (c)	1, 2, 3,	3, 4	Let A be an array of size n consisting of distinct integers. For any element m in A, rank(m) is the index at which m occurs in the sorted version of A (e.g. if m is the smallest element of A then rank(m)=1, if m is the largest element of A then rank(m)=n, etc). Assume that n is divisible by 14. Divide A into t=n/7 blocks each of size 7, call these subarrays as A_1, A_2, ..., A_t. Let m_i denote median of A_i for 1 <= i <= t. Let m be median of m_1, m_2, ..., m_t. (For array of size 2k assume that k th rank element is the median) Give best possible lower bound and upper bound on rank(m) in A. Justify your answer.	**3 marks**
Q. 3 (d)	2, 3	2	Consider a text over alphabet {a,b,c,d,e,f} relative frequencies of the characters are 5, 15, 16, 18, 22, 24 respectively. Find Huffman encoding for each of the six characters. Suppose original text contains total 10,00,000 characters what will be size of the text in bits after encoding.	**3 marks**
Q. 4.				
Q 4 (a)	2, 6	3	Let g(n) denotes the number of different binary search trees one can construct over the keys {1, 2, ..., n}. What is the value of g(7)? (Correct recurrence: 1 mark, correct value of g(7) 1 mark.) (Hint: try to observe the recursive substructure of the problem and obtain a recurrence for g(n))	**2 marks**
Q. 4 (b)	1, 2,3	3	Let M_1, M_2, ..., M_k be the matrices such that dimension of matrix M_i is n_i x n_{i+1} for 1<=i <= k. For j>=i let f(i,j) denote minimum number of integer multiplications needed to compute the product M_i M_{i+1} ... M_j (assume that we use standard matrix multiplication algorithm to find product of any two matrices). Give recurrence for f(i,j) with brief justifying argument of correctness. Specify time and space complexity of efficient dynamic programming solution to the problem based on your recurrence.	**3 marks**
Q. 4 (c)			Attempt **Any ONE** of the following	**5 marks**
Q 4 (c) i	1, 2,3 ,6	4, 6	Given integers n and k, along with p1 , . . . , pn ∈ [0, 1], you want to determine the probability of obtaining exactly k heads when n biased coins are tossed independently at random, where pi is the probability that the ith coin comes up heads. Give an O(n^2) algorithm for this task. Give an argument justifying correctness of the algorithm and time complexity. Assume you can multiply and add two numbers in [0, 1] in O(1) time.	
Q. 4 (c) ii	1, 2, 3, 6	4, 6	Let S be set of n integers each of which is less than k and greater than -k. You are given set S and number b, such that -nk < b <nk and goal is to check if there is a subset of S such that the addition of all the elements in the subset is b. Give an algorithm which has time complexity bounded above by O((nk)^c) for an absolute constant c. Give detailed pseudo-code and the complexity analysis.	

Q. 5.				
Q. 5. (a)	2,3	3	Prove the following property of the Huffman encoding scheme. If some character occurs with frequency strictly more than 2/5, then there is guaranteed to be a codeword of length 1.	3 marks
Q. 5. (b)	4, 6	1	Give prover-verifier based definition of complexity class NP and give two concrete examples of problems characterizing NP class.	4 marks
Q. 5. (c)	4, 1, 6	2, 3	Prove that any comparison based sorting algorithm must perform Omega(n log n) comparisons to sort a set of n numbers, where Omega denotes Big-Omega asymptotic notation.	3 marks
Q. 6.				
Q. 6. (a)	4, 5	1, 2	Compare Las Vegas and Monte Carlo randomized algorithms. Give an example for each.	2 marks
Q. 6. (b)	5	4	Consider the standard algorithm to compute maximum element of the array A[1-n]. We start with initializing a variable max with A[1] and do a pass on entire array A[2-n], in each step we compare max with A[i], if A[i] is larger than max we update max to A[i]. What is expected number of times the variable max gets updated if array A consists of a permutation over 1, 2, 3, …, n chosen uniformly at random.	4 marks
Q. 6. (c)	4, 6	2, 3	If P=NP prove that NP= CoNP	2 marks
Q. 6. (d)	4, 6	2, 3	We say that a Boolean formula F (over Boolean variables x1, x2, …, xn and Boolean operators AND, OR, NOT) is Tautology if formula F evaluates to TRUE for all Boolean assignments for the variables. Let P be the problem of testing if given Boolean formula F is a tautology or not. The Yes instance of the problem is when the formula F is a Tautology. Which of the following statement(s) is/are true. Justify your answer. 1. Problem P is in complexity class NP. 2. Problem P is in complexity class Co-NP.	2 marks