LAB 2 (Oct. 5-23)
Functions and Arrays

This lab is worth 3% of the course mark. The number in square brackets at the beginning of each question shows the number of points the question is worth. The total number of points is 30.
Question 5 is a bonus question.

You will receive a mark only for the codes that are demonstrated in front of a TA by the end of the lab. With that in mind, please complete as many problems as you can before you come to lab. Work not completed by the end of the lab will not be marked. TAs will leave the lab at 5:20pm sharp.

General Requirements on Your Programs

A. Your programs should be written in a good programming style, including instructive comments and well-formatted, well-indented code. Use self-explanatory names for variables as much as possible. (~5% of the mark)
B. When outputting the results, include in the output an explanatory message. When inputting values display a message prompting the user to input the appropriate values. (~10% of the mark)

Lab Questions

Assume the character code used is ASCII.
You are allowed to use from the C standard library only functions for input and output *(e.g. printf, scanf, getchar)* **and math library functions. The use of global variables is not permitted.**

1. [7] Consider representing vectors with ***n*** floating point components using arrays. Develop a library for vector operations, that includes the following functions.
   - A function print_vector() to print the vector components, on a single line, separated by commas. The function prototype has to be:
     - void print_vector(double array[], int size);

     Parameter size represents the size of parameter array and the vector dimension.
   - A function add_vectors() to add two vectors of the same size, with prototype:
     - void add_vectors(double vector1[], double vector2[], double vector3[], int n);

     vector3 should store the sum of vector1 and vector2. You may assume that all three arrays have the size equal to n, which equals the vector dimension. (In other words, assume that the calling function ensures that the arrays passed in satisfy this condition.)
   - A function scalar_prod() that returns the scalar product of two vectors of the same dimension. You may assume that the passed in arrays have the same size.
   - A function norm2(), which returns the L2 norm of a vector. The L2 norm is defined as the square root of the scalar product of the vector with itself. Function norm2() should call function scalar_prod().

   Write a program to test this library. You **are allowed** to use math library functions.
   Attention: When you pass an array (which is not a string) to a function, you also need to pass to the function the size of the array.
   Note: Consider vectors x=(x(0), x(1), …, x(n-1)) and y=(y(0), y(1), …, y(n-1)).
   The **sum** of x and y is the vector z=(z(0),z(1), …, z(n-1)), where z(i)=x(i)+y(i) for every 0<=i<n.
   The **scalar produ**ct of x and y is the value x(0)y(0)+x(1)y(1)+…+x(n-1)y(n-1).

Example: Assume n=3 and vectors x=(2,4,6) and y=(0,1,2). Then the sum of vectors x and y is the vector sum=(2,5,8). The scalar product of vectors x and y is the number 0+4+12=16.

2. a) [4] A diagonally dominant matrix is a matrix A such that for each row, the absolute value of the diagonal element on that row is strictly larger than the sum of the absolute values of all other elements in the row. That is, for each row i=0,1,…, n-1, the following holds:

$$|a_{ii}| > \sum_{j=0, j\neq i}^{n-1} |a_{ij}|$$

Write a function is_diag_dom() that determines if an N-by-N matrix mat is diagonally dominant (it returns 1 if the matrix is diagonally dominant and 0 otherwise). The function prototype has to be

- int is_diag_dom(int mat[ ][N]).

You may use the function fabs()with prototype

- double fabs(double x),

from the C standard math library, which returns the absolute value of x. Write a program to test this function. Note that N represents a constant. To set a value to N use the define directive at the beginning of the file (E. g.: #define N 20 replaces N by 20 all over the file).

b) [4] In linear algebra, a Toeplitz matrix or diagonal-constant matrix, is a matrix in which every descending diagonal from left to right is constant (in other words, has all its elements equal). Such matrices are encountered in applications in signal processing and communications. For instance, the following 4-by-5 matrix is a Toeplitz matrix:

9 2 1 0 4
7 9 2 1 0
3 7 9 2 1
5 3 7 9 2

Write a C function with prototype

- int is_toeplitz(int a[][N], int m)

which determines if the m-by-N matrix a is a Toeplitz matrix or not. The function has to return 1 if the answer is positive and 0 otherwise.

c) [4] Write a function which prints all elements of a squared matrix in a diagonal scan order, starting at the top left corner. For instance, for the following matrix

1 12 13 49
5 16 17 81
9 10 11 20
2 45 19 14

the output has to be: 1 5 12 9 16 13 2 10 17 49 45 11 81 19 20 14

3. [4] Write a function with prototype

- void string_copy(const char source[], char destination[], int n);

This function copies string source to string destination. Parameter n represents the size of array destination. If the latter array is not sufficiently large to hold the whole source string then only the prefix of the string which has room in the latter array should be copied. Note that after copying, the null character should also be included to mark the end of string destination.

Write a program to test your functions.

You are not allowed to use any function declared in string.h.

You may use the function which returns the length of a string given in class. **Recall that a string is a**

char array with the null character marking the end. The length of the string is the number of characters in the array appearing before the null character.

4. [7] A sparse vector is a vector whose most components are zero. To store a sparse vector efficiently it is enough to store only its non-zero components and their index (position in the vector). The components of a vector are indexed starting from 0, like in C arrays. Precisely, to store a sparse vector with **n** components, only **k** of which are non-zero, we can use two arrays: **val** and **pos**, each of size **k**. For example, if the sparse vector **x** with 8 components is the following

   0 0 23 0 -7 0 0 48

   then **k**=3 and

   val contains 23 -7 48

   pos contains 2 4 7

   Notice that the elements of array **pos** are in increasing order. We will assume that each vector contains at least one non-zero element.

   Write a function efficient() with prototype

   - void efficient(const int source[], int val[], int pos[], int size)

   which computes the efficient representation of vector source, by filling the arrays val and pos. Parameter size represents the number of components of vector source (i.e., the size of the array). Assume that the size of arrays pos and val equals the number of non-zero values of vector source.

   Additionally, write a function reconstruct() with prototype

   - void reconstruct(int source[], int m, const int val[], const int pos[], int n)

   which reconstructs vector source from the efficient representation stored in arrays val and pos. Parameter n represents the size of arrays val and pos. Parameter m represents the size of array source, which equals the dimension of the vector.

   Write a program to test the two functions.

5. [5] **Bonus Question**.
   Additionally, write a function with prototype

   - int addEff(int val1[], int val2[], int val3[],int pos1[], int pos2[],int pos3[], int k1, int k2)

   where val1, pos1 and val2, pos2 represent two sparse vectors of integers, stored efficiently. k1 is the number of non-zero elements of vector 1 and k2 is the number of non-zero elements of vector 2. Function addEff() has to add the two vectors and store the result in efficient representation as well, using val3, pos3. Assume that the size of arrays val3 and pos3 equals the number of non-zero elements in the sum vector. **The function is not allowed to allocate any array**, in other words only a constant amount of variables may be allocated during the function execution. **No mark is awarded if this requirement is not satisfied.** **Note:** Pay attention to the case when two non-zero elements sum up to 0. You may assume that the two vectors, as well as their sum, are not equal to 0.