Support Vector Machine
(SVM)

C_1 $\qquad$ C_2

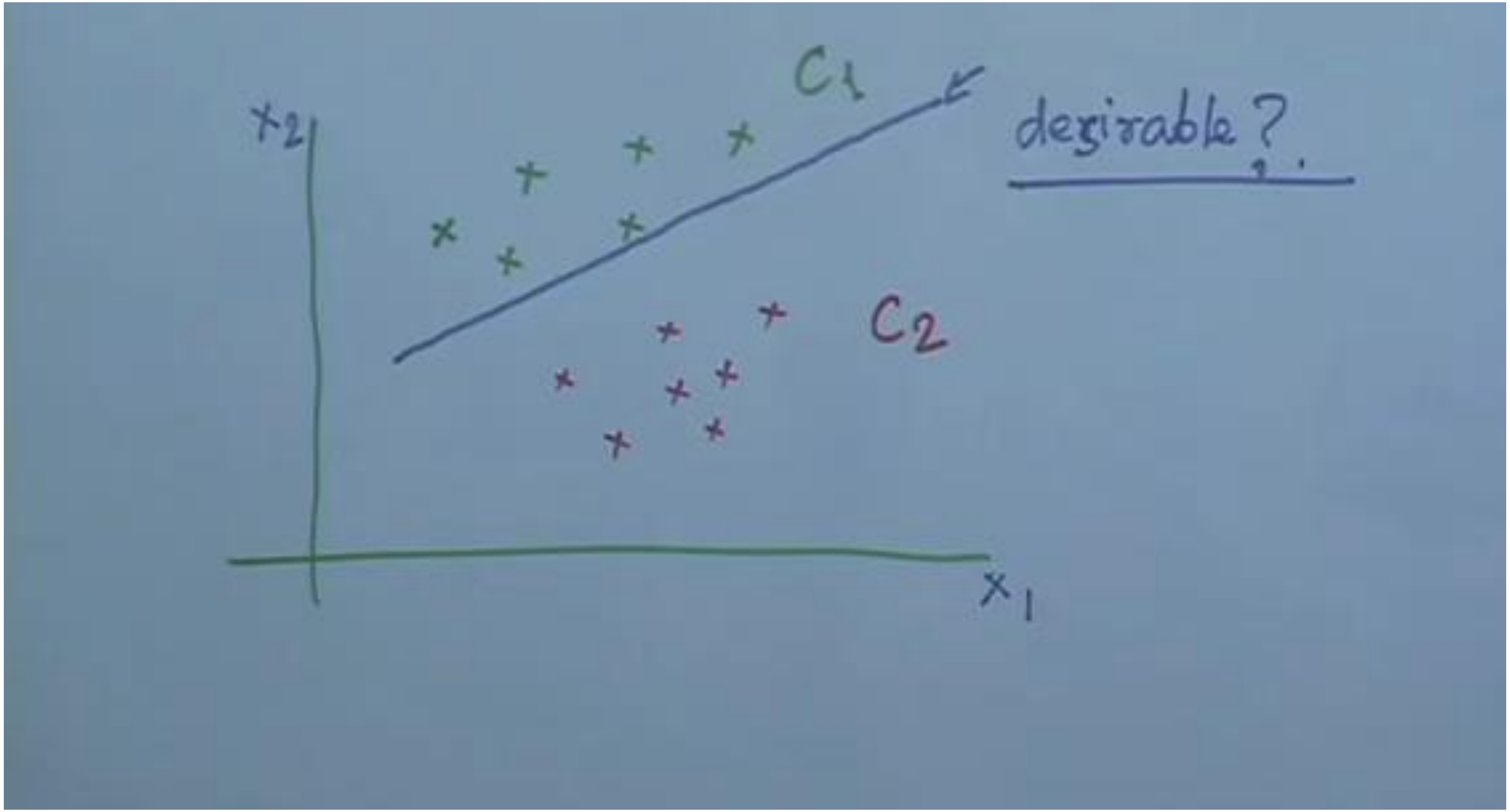
$$g(\underline{\underline{x}}) = \underline{\underline{W}}^t \underline{\underline{x}} + \underline{\underline{w_0}}.$$

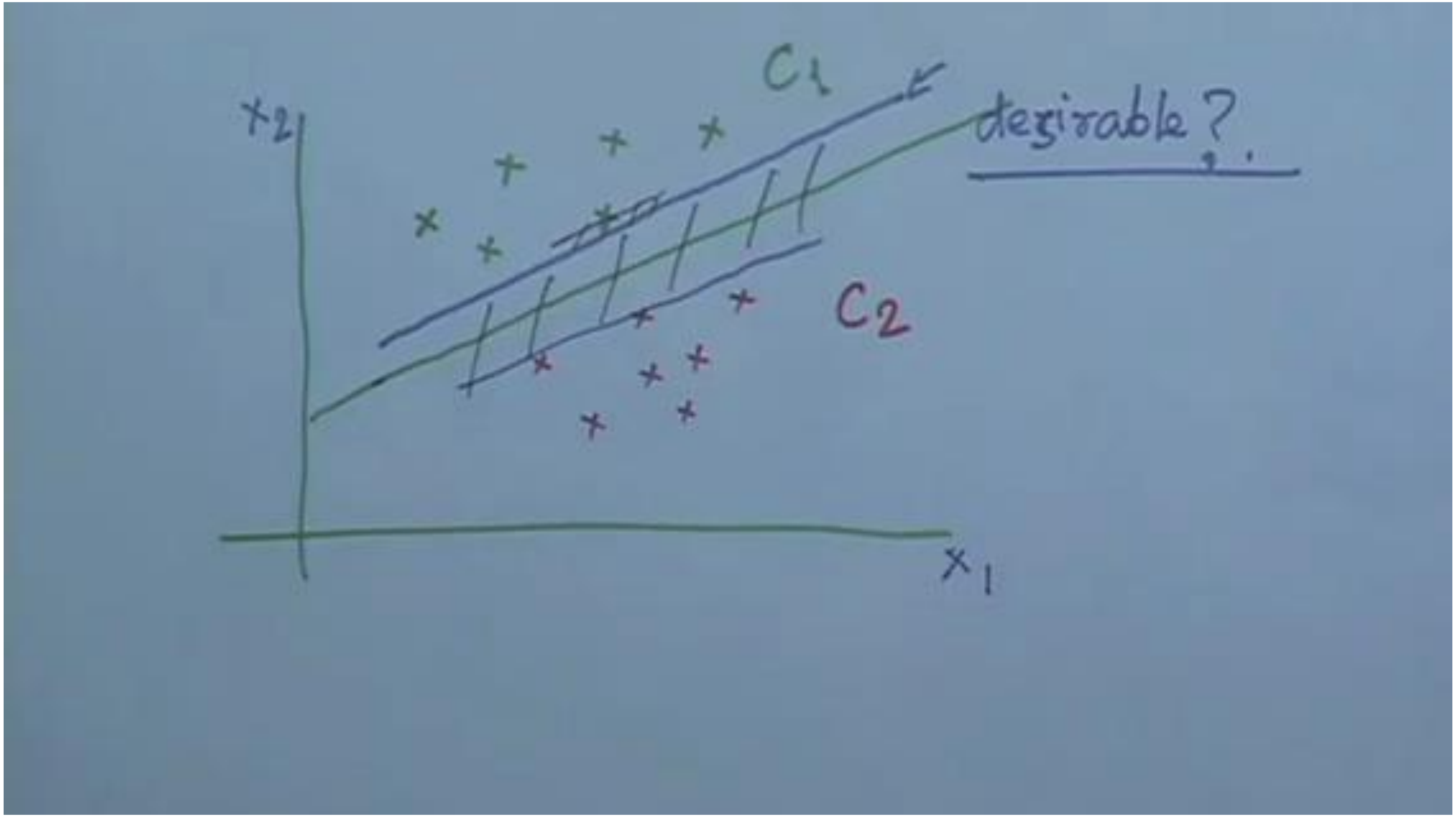
x

$$g(\underline{\underline{x}}) = \underline{\underline{W}}^t x + \underline{\underline{b}} = 0$$

$$g(x_1) = w^t x_1 + b > 0 \Rightarrow x_1 \in C_1$$

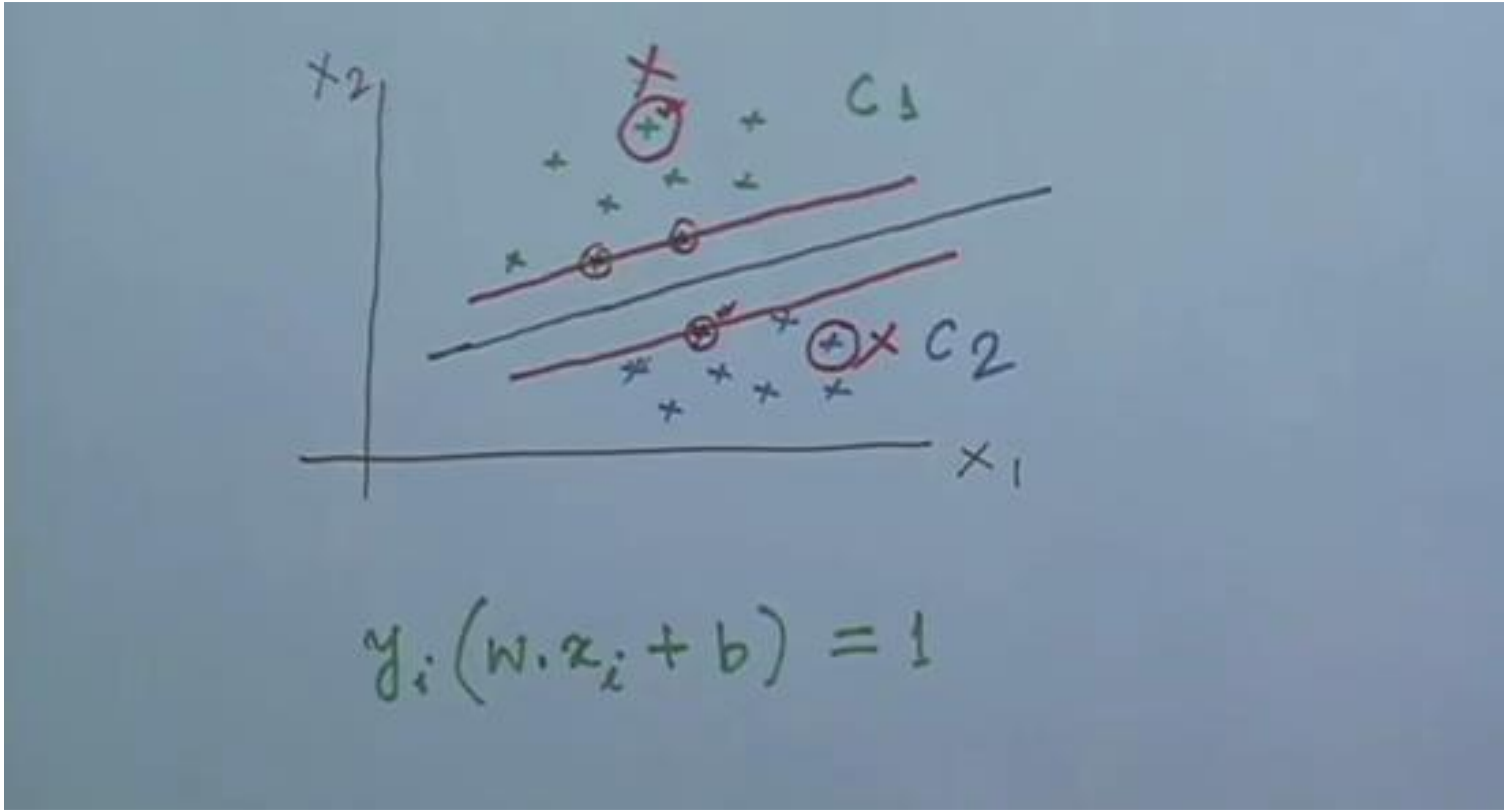
$$w^t x_1 + b < 0 \Rightarrow x_1 \in C_2$$


x_2
C_1
desirable ?
C_2
x_1



x_2
C_1
desirable?
C_2
x_1

Support vectors



x_i

$w^t x_i + b > 0$

$w \cdot x_i + b > 0 \longrightarrow x_i \in C_1$

$w \cdot x_i + b < 0 \longrightarrow x_i \in C_2$

x_i, y_i $\qquad y_i = \pm 1$

$y_i(\underline{\underline{w}} \cdot x_i + \underline{\underline{b}}) > 0$

$p \rightarrow$ unknown vector.

$w \cdot p + b$

x_i

$w^t x_i + b > 0$

$w^t x_i + b > \underline{\underline{\nu}}$

$W \cdot x_i + b > 0 \longrightarrow x_i \in C_1$

$W \cdot x_i + b < 0 \longrightarrow x_i \in C_2$

x_i, y_i

$y_i = \pm 1$

$y_i(\underline{\underline{W}} \cdot x_i + \underline{\underline{b}}) > 0$

unknown vector.

$$w \cdot x + b = 0$$

$$\frac{w \cdot x + b}{\|w\|} \geqslant \gamma$$

$$w \cdot x + b = 0$$

$$\frac{w \cdot x + b}{\|w\|} \geqslant \nu$$

$$w \cdot x + b \geqslant \underbrace{\nu \cdot \|w\|}_{} \quad = 1$$

$$w \cdot x + b \geqslant 1 \quad \text{if } x \in C_1$$

$$\leqslant -1 \quad \text{if } x \in C_2$$

$$y_i(w \cdot \underline{x_i} + b) \geq 1$$

w

$$\phi(w) = w^t w = \frac{1}{2} w \cdot w$$

$$\frac{1}{2} w \cdot w.$$

$$y_i (w \cdot x_i + b) = 1 \leftarrow \text{Constraint.}$$

$$L(w,b) = \frac{1}{2}(w \cdot w) - \sum \alpha_i \left[y_i \left[w \cdot x_i + b \right] - 1 \right]$$

$\alpha_i \rightarrow$ Lagrangian Multiplier.

$$\frac{\partial L}{\partial b} = ?$$

$$L(w,b) = \frac{1}{2}(w \cdot w) - \sum \alpha_i y_i (w \cdot x_i) - \underbrace{\sum \alpha_i y_i b} + \sum \alpha_i$$

$$\frac{\partial L}{\partial b} = - \sum \alpha_i y_i$$

$\alpha_i \rightarrow$ Lagrangian Multiplier.

$$\frac{\partial L}{\partial b} = ?$$

$$L(w, b) = \frac{1}{2}(w \cdot w) - \sum \alpha_i y_i (w \cdot x_i) - \underbrace{\sum \alpha_i y_i b} + \sum \alpha_i$$

$$\frac{\partial L}{\partial b} = -\sum \alpha_i y_i = 0$$

$$\boxed{\sum_{i=1}^{m} \alpha_i y_i = 0}$$

$$L(w,b) = \frac{1}{2}(w \cdot w) - \sum \alpha_i y_i (w \cdot x_i) - \sum \alpha_i y_i b + \sum \alpha_i$$

$$\frac{\partial L}{\partial w} = w - \sum \alpha_i y_i x_i = 0$$

$$\boxed{w = \sum_{i=1}^{m} \alpha_i y_i x_i}$$

$$L = \frac{1}{2} w \cdot w - \cancel{\sum \alpha_i y_i} \cdot b - \sum \alpha_i y_i w \cdot x_i + \sum \alpha_i$$

$$= \frac{1}{2} \sum \alpha_i \alpha_j y_i y_j (x_i \cdot x_j) - \sum \alpha_i \alpha_j y_i y_j (x_i \cdot x_j) + \sum \alpha_i$$

$$= \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum \alpha_i \alpha_j y_i y_j (x_j \cdot x_i).$$

$$L = \frac{1}{2} w \cdot w - \cancel{\sum \alpha_i y_i} \cdot b - \sum \alpha_i y_i w \cdot x_i + \sum \alpha_i$$

$$= \frac{1}{2} \sum \alpha_i \alpha_j y_i y_j (x_i \cdot x_j) - \sum \alpha_i \cdot \alpha_j \cdot y_i \cdot y_j \cdot (x_i \cdot x_j) + \sum \alpha_i$$

$$= \boxed{\sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum \alpha_i \cdot \alpha_j \, y_i \, y_j \cdot (x_j \cdot x_i)}$$

$$\alpha_i \geq 0 \qquad \sum_{i=1}^{m} \alpha_i \cdot y_i = 0$$

$$D(Z) = \operatorname{sgn}\left(\sum_{j=1}^{m} \alpha_j \, y_j \cdot x_j \cdot \underline{\underline{Z}} + b\right)$$