



Mathematical Morphology

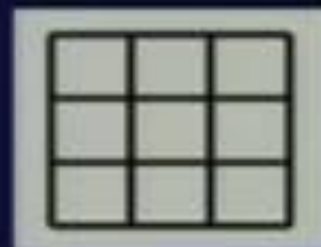
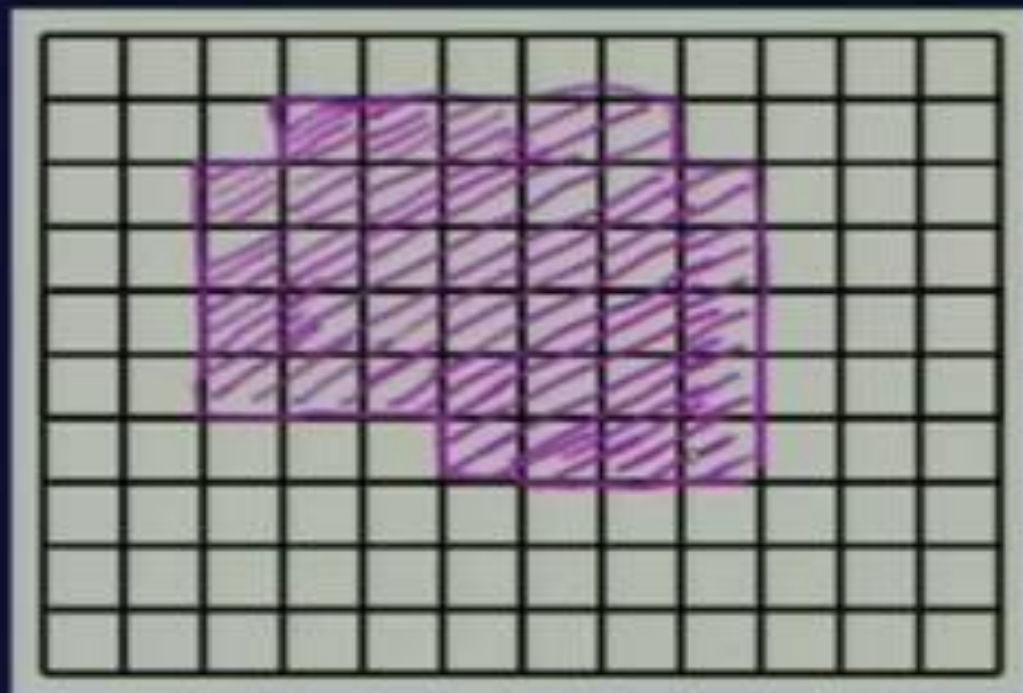
- On completion the students will learn and be able to implement

Applications of Morphological techniques

- Boundary extraction
- Region filling
- Extraction of connected components
- Convex hull
- Thinning
- Thickening

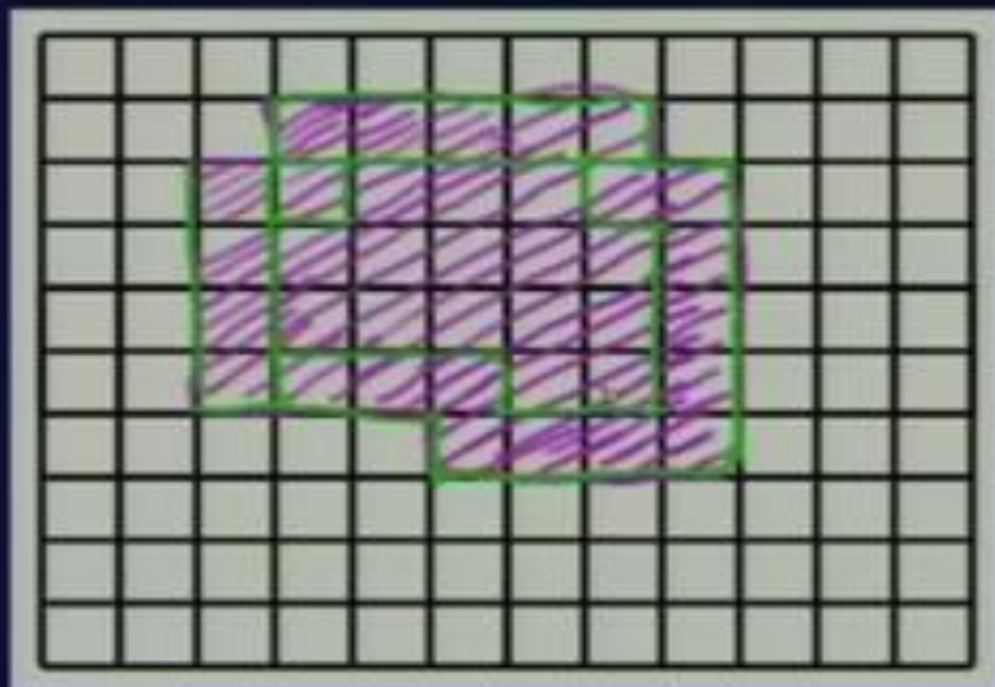


Boundary Extraction





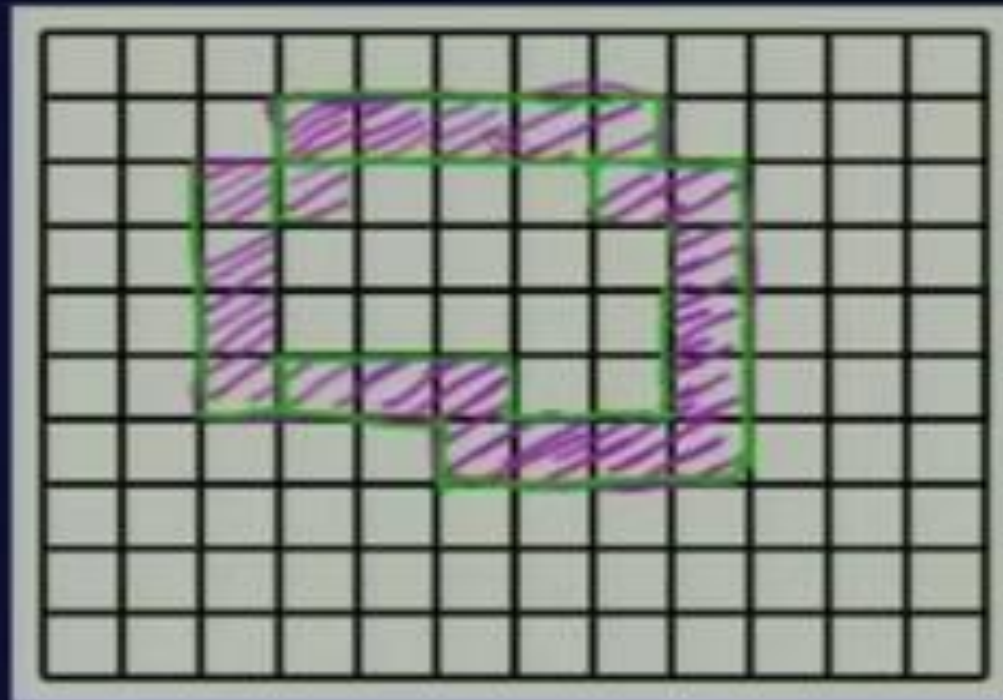
Boundary Extraction



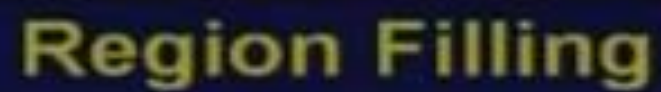
$$A \Rightarrow \beta(A)$$
$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction



$$A \Rightarrow \beta(A)$$
$$\beta(A) = A - (A \ominus B)$$



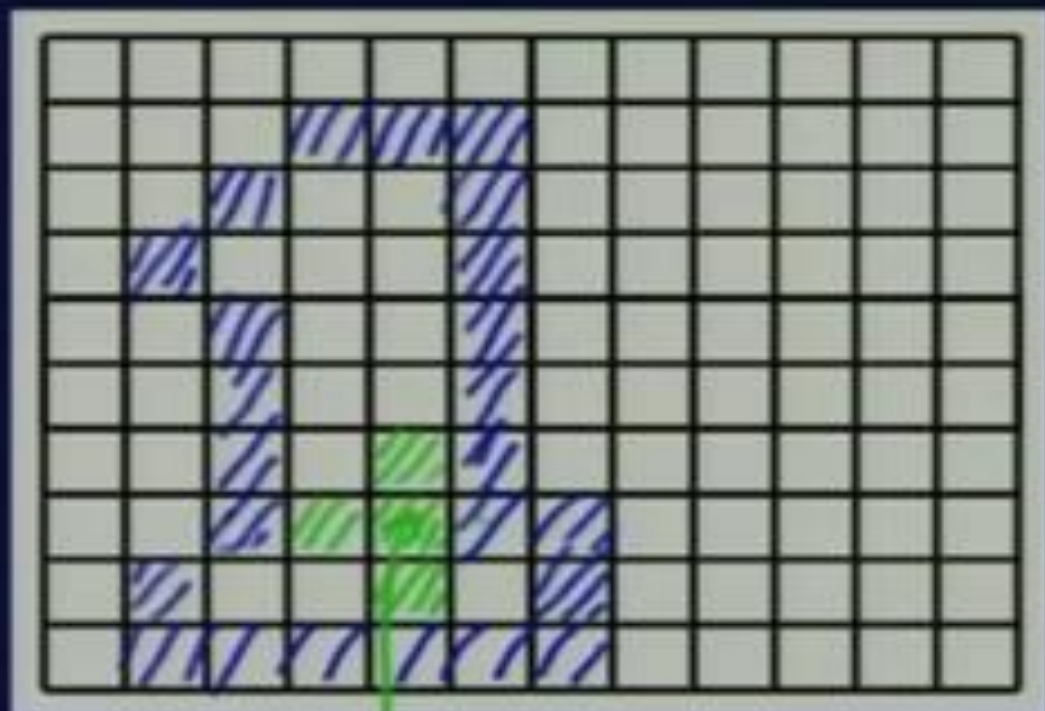
8

$$x_0 = p$$

$$x_k = (x_{k-1} \oplus b) \cap A^c$$



Region Filling



p



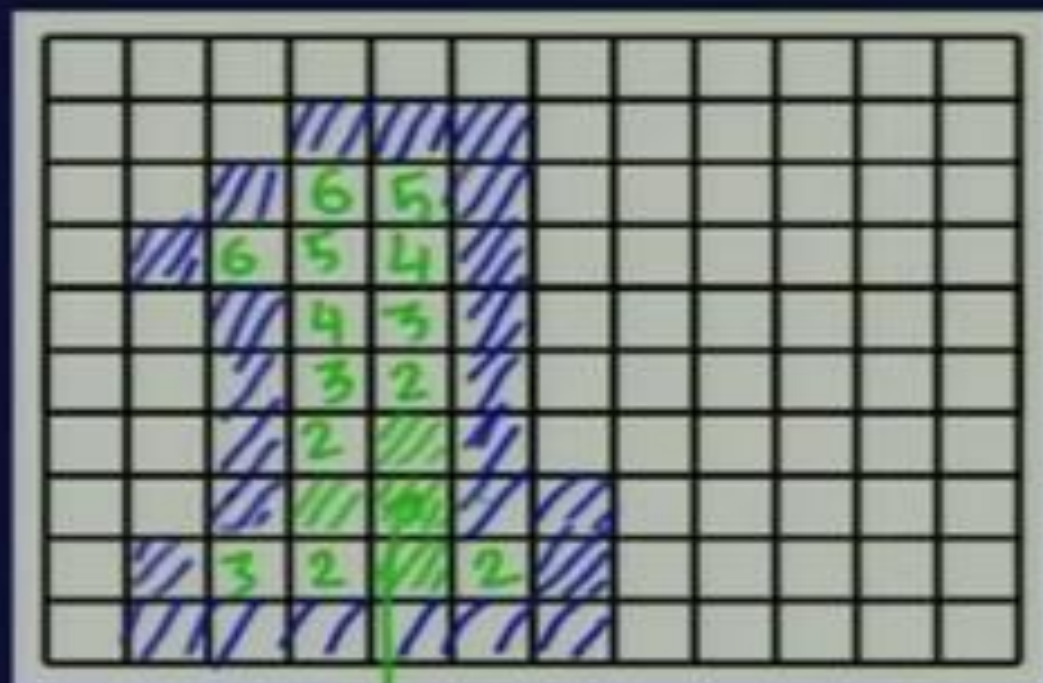
B

$$x_0 = p$$

$$x_k = (x_{k-1} \oplus B) \cap A^c$$



Region Filling



$$x_0 = p$$

$$x_k = (x_{k-1} \oplus B) \cap A^c$$



Region Filling



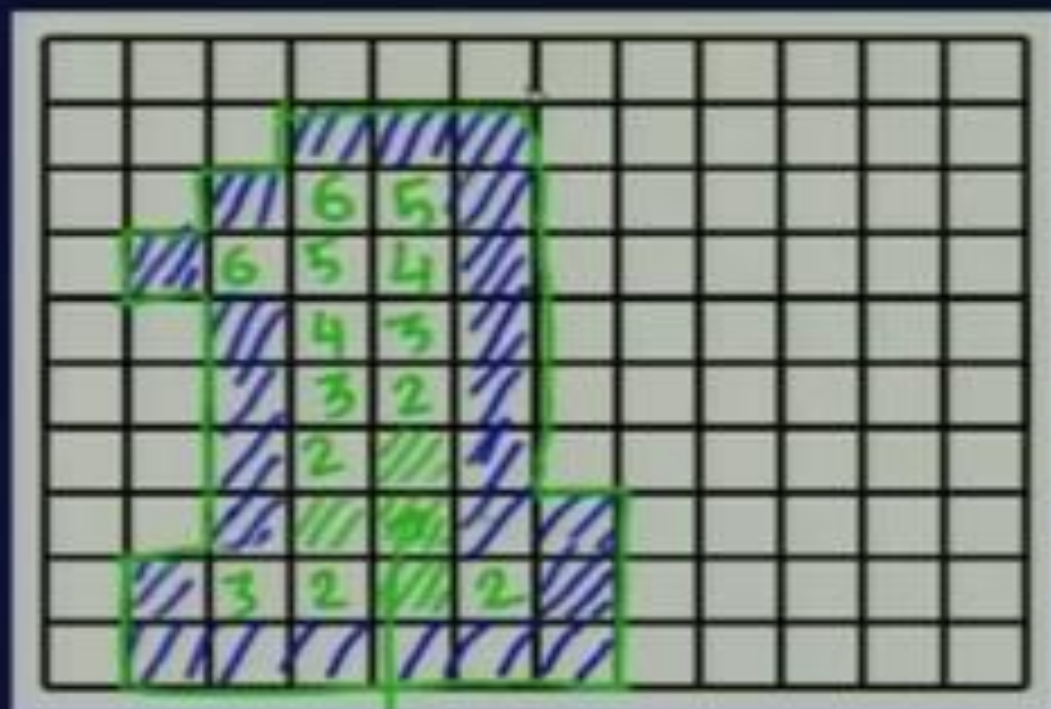
$$x_0 = p$$

$$x_k = (x_{k-1} \oplus B) \cap A^c$$

$$x_k = x_{k-1}$$



Region Filling



$$x_0 = p$$

$$x_k = (x_{k-1} \oplus B) \cap A^c$$

$$x_k = x_{k-1}$$

$$x_k \cup A$$

Connected Component Extraction

A (Y)

$$x_k = (x_{k-1} \oplus B) \cap A$$

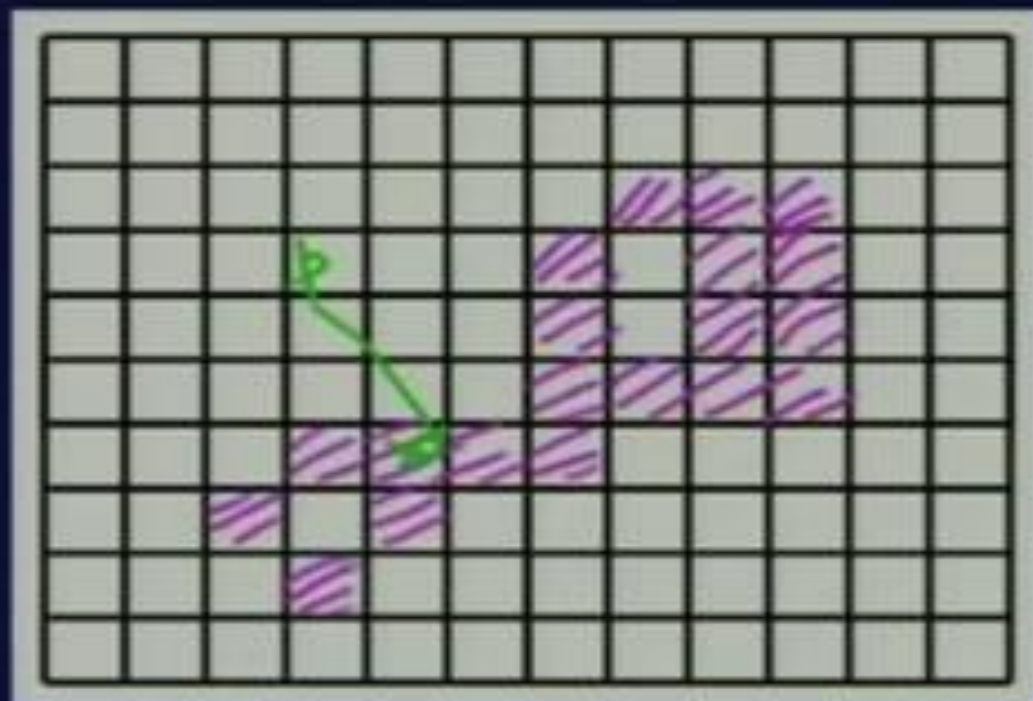
$k = 1, 2, 3, \dots$

$$x_k = x_{k-1}$$

$$\underline{\underline{Y}} = \underline{\underline{x_k}}$$



Connected Component Extraction

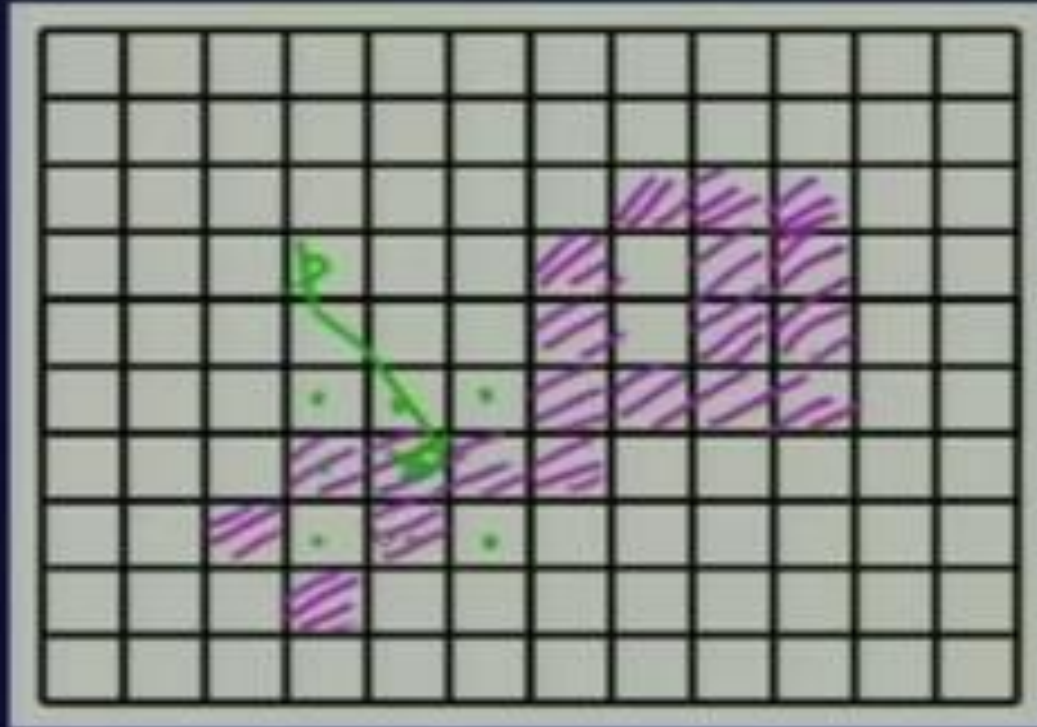


$$x_k = (x_{k-1} \oplus B) \cap A$$

$$x_0 = p$$



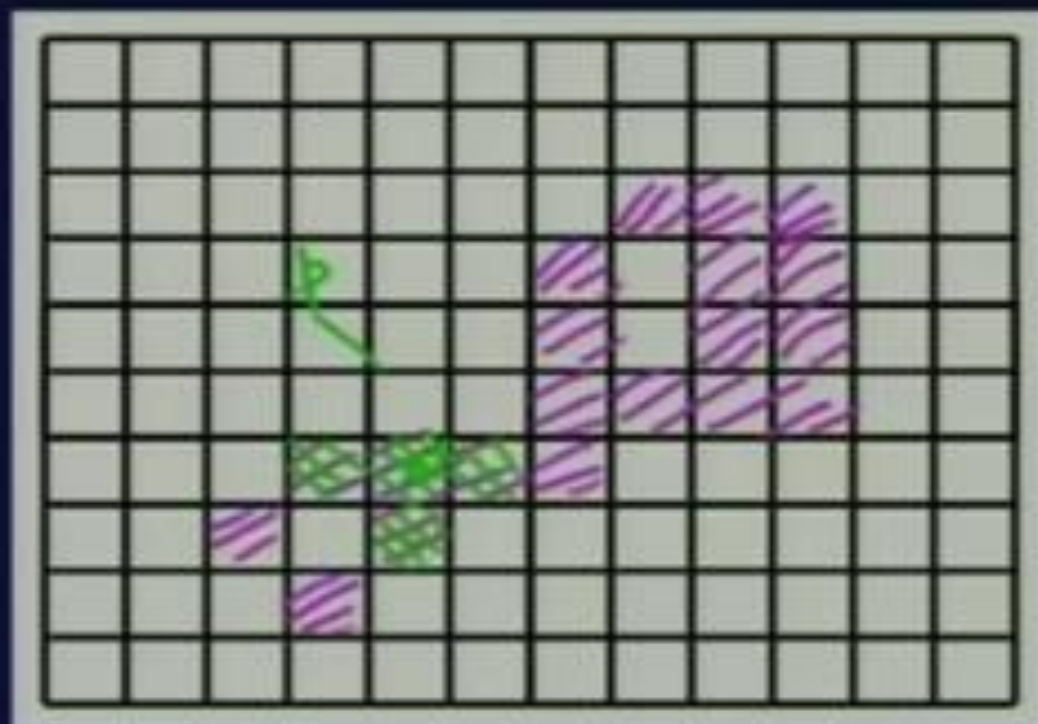
Connected Component Extraction



$$x_k = (x_{k-1} \oplus B) \cap A$$
$$x_0 = P$$



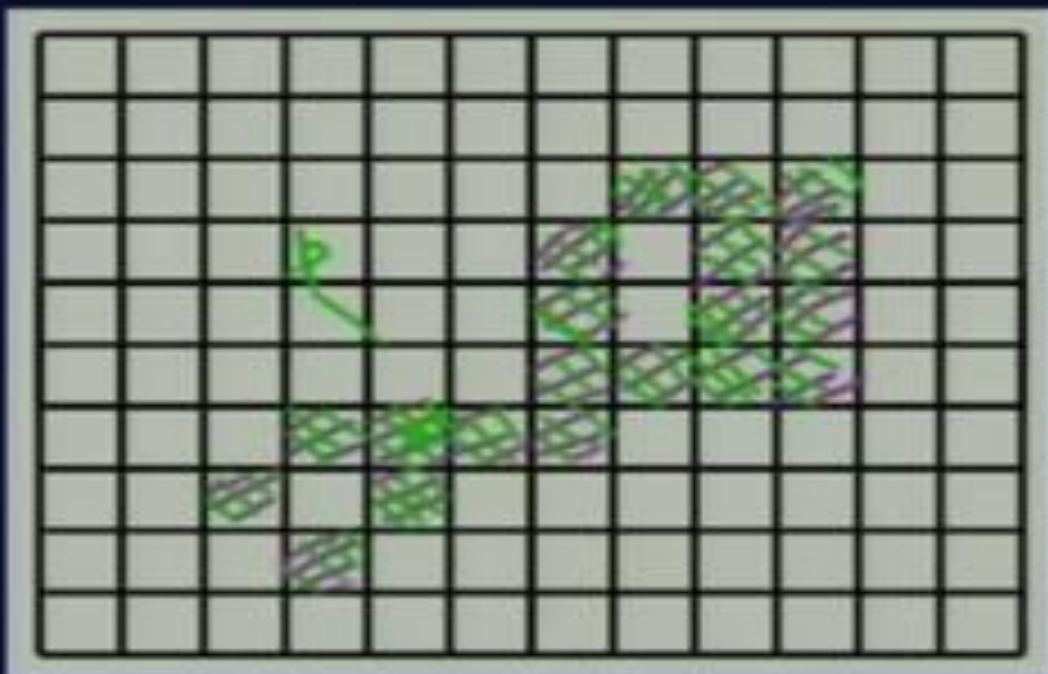
Connected Component Extraction



$$x_k = (x_{k-1} \oplus B) \cap A$$
$$x_0 = p$$



Connected Component Extraction



$$x_k = (x_{k-1} \oplus B) \cap A$$

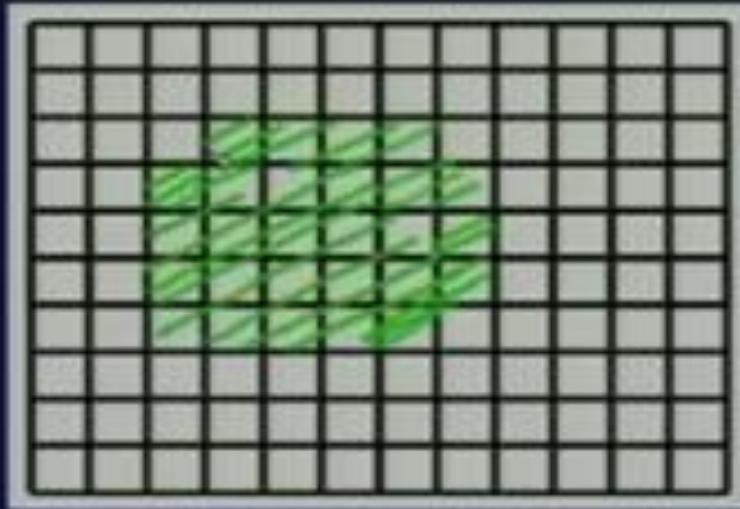
$$x_0 = P$$

$$x_6 = x_7$$

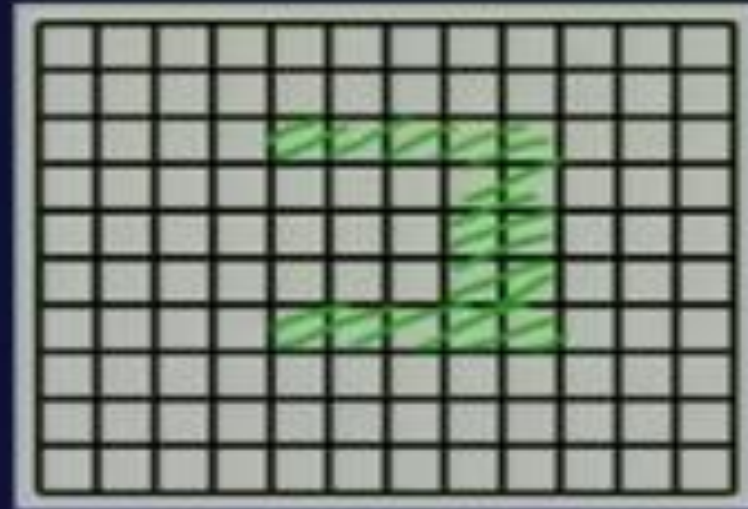
$$Y = x_7, x_6$$



Convex Set



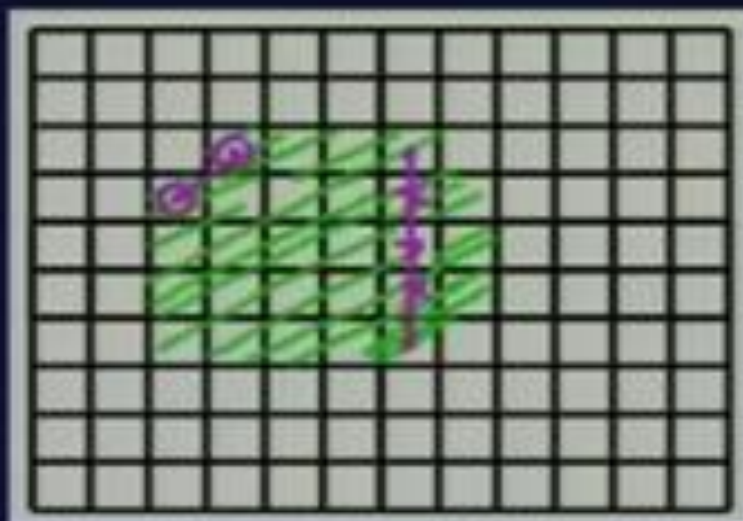
S_1



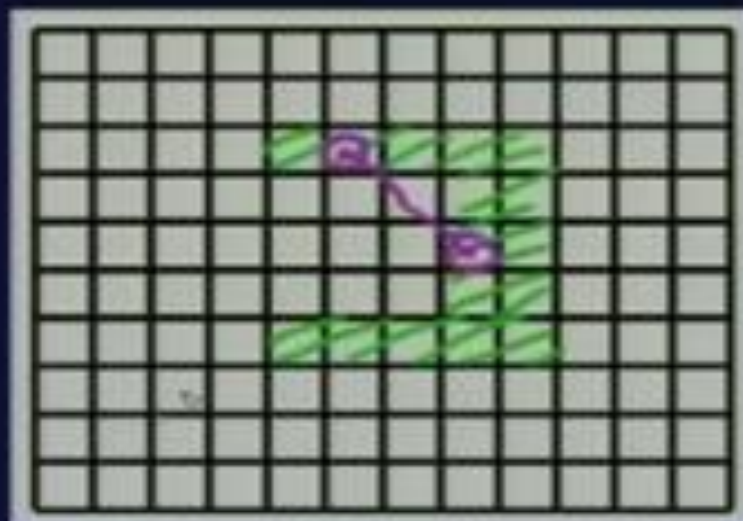
S_2



Convex Set



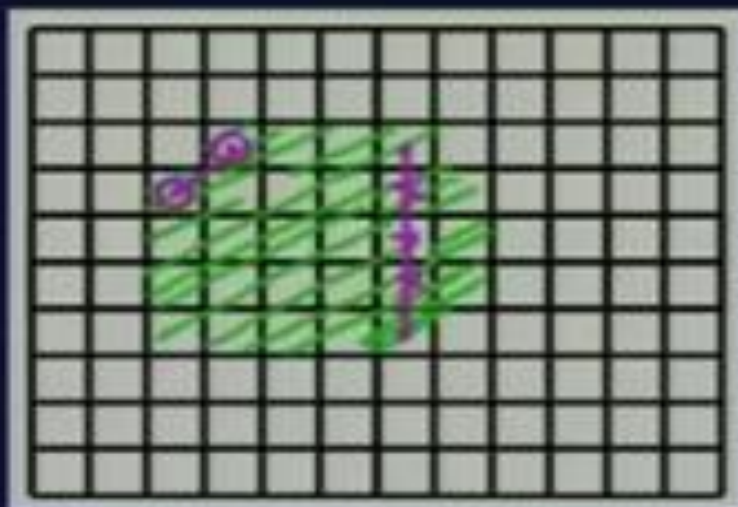
S_1



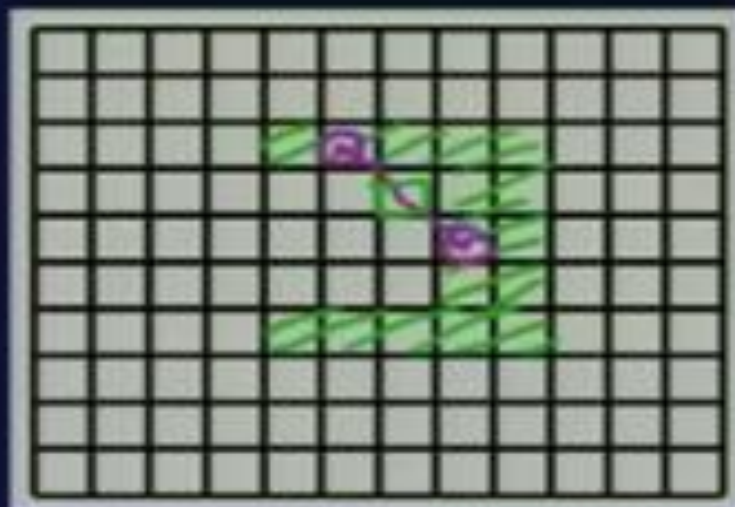
S_2



Convex Set



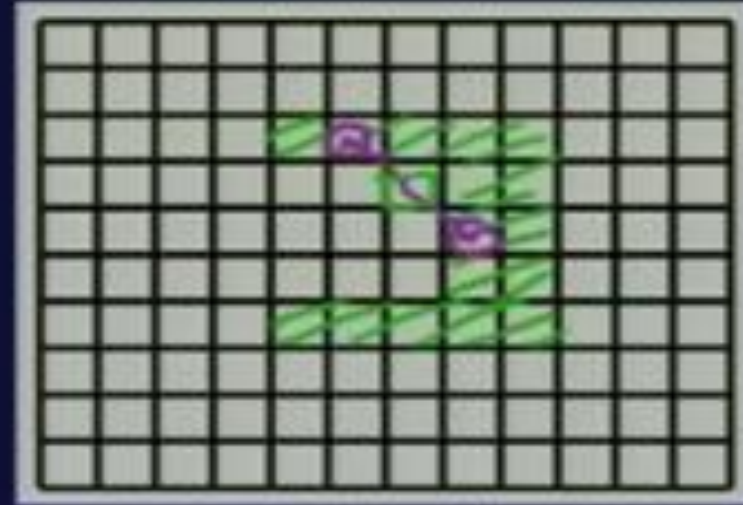
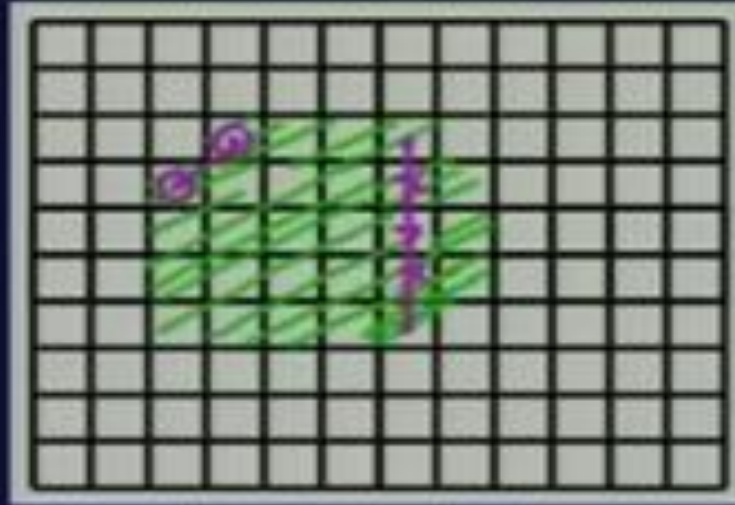
$S_1 \Rightarrow \text{Convex}$



$S_2 \Rightarrow \text{not convex}$



Convex Set



S

$S_1 \Rightarrow \text{Convex}$

$S_2 \Rightarrow \text{not convex}$

$H - S \Rightarrow \text{Convex deficiency}$

B^i

$i = 1, 2, 3, 4$

$$x_k^i = (x_{k-1}^i \oplus B^i) \cup A$$

$$x_0^i = A$$

$$x_k^i = x_{k-1}^i$$

$$D^i = x_{conv}^i$$

$$C(A) = \bigcup_{i=1}^4 D_i$$



Convex Hull

B^1

	X	X
		X
	X	X

B^2

X		X
X	X	X

B^3

X	X	
X		
X	X	

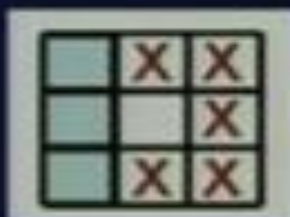
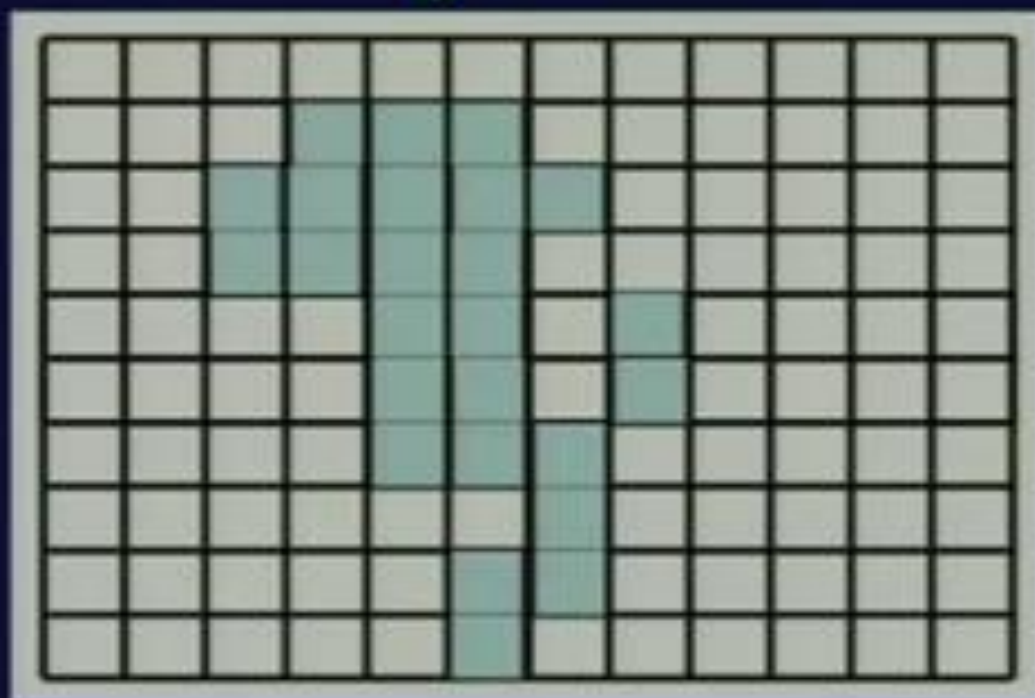
B^4

X	X	X
X		X



Convex Hull

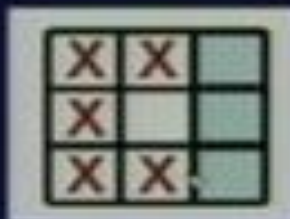
↓ A



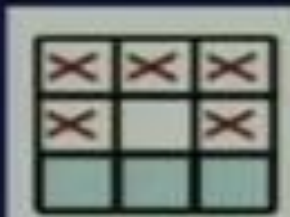
B¹



B²



B³

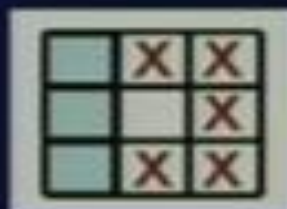
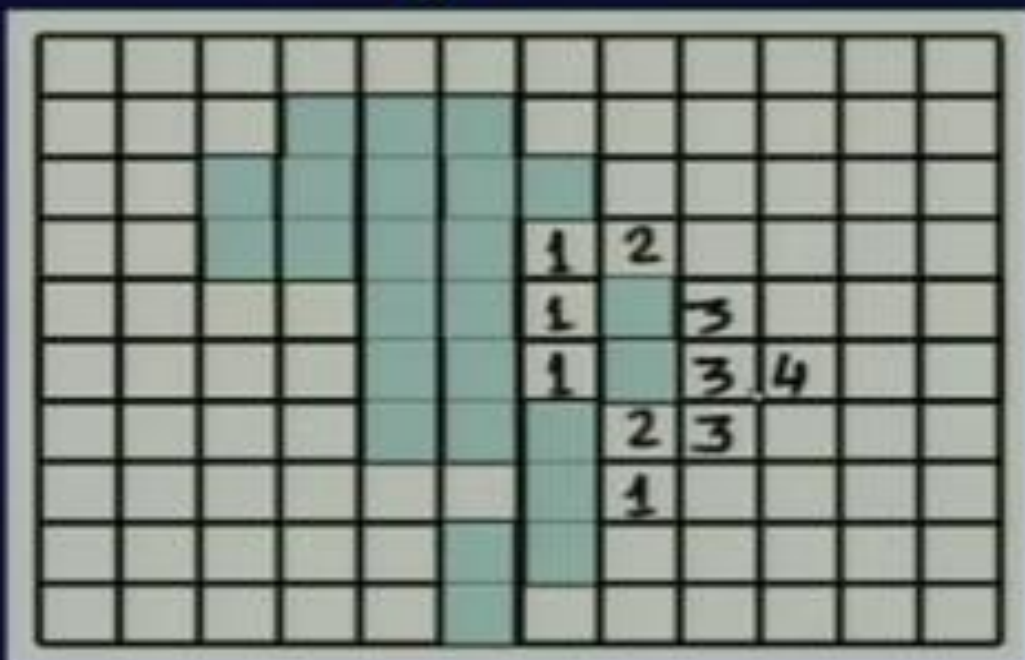


B⁴

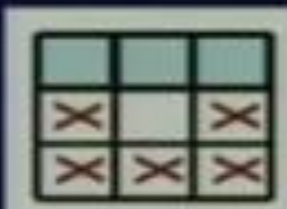


Convex Hull

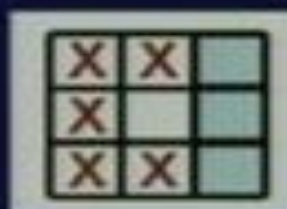
$\downarrow A$



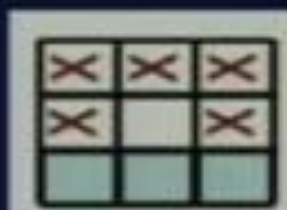
B^1



B^2



B^3

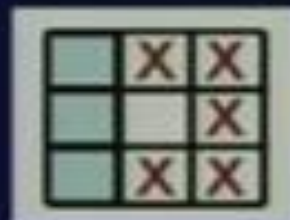


B^4

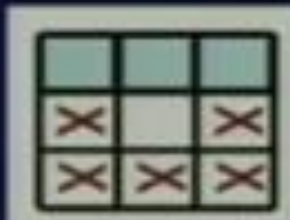


Convex Hull

\Downarrow^A



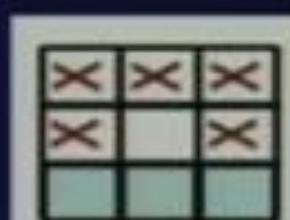
B^1



B^2



B^3

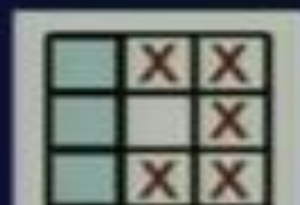


B^4

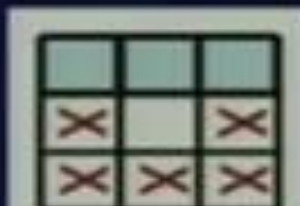


Convex Hull

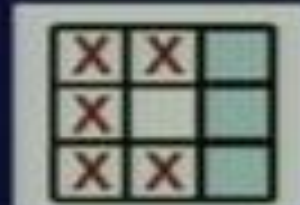
\Downarrow^A



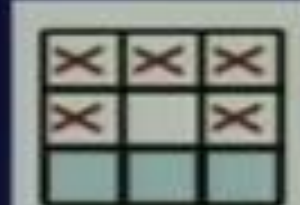
B^1



B^2



B^3

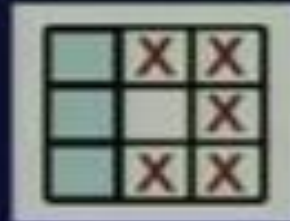


B^4



Convex Hull

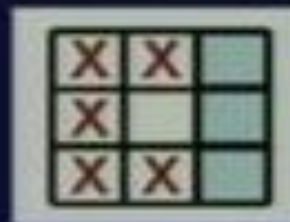
↓ A



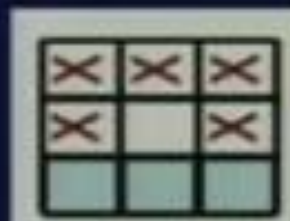
B^1



B^2



B^3



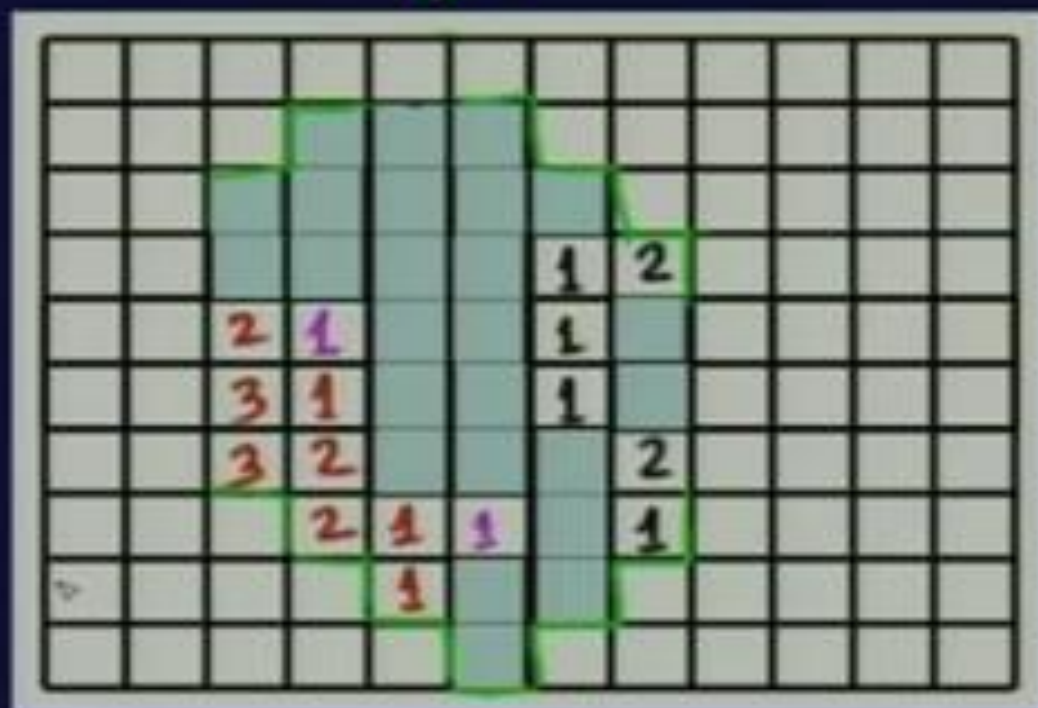
B^4





Convex Hull

↓ A



	X	X
		X
	X	X

B^1

X		X
X	X	X

B^2

X	X	
X		
X	X	

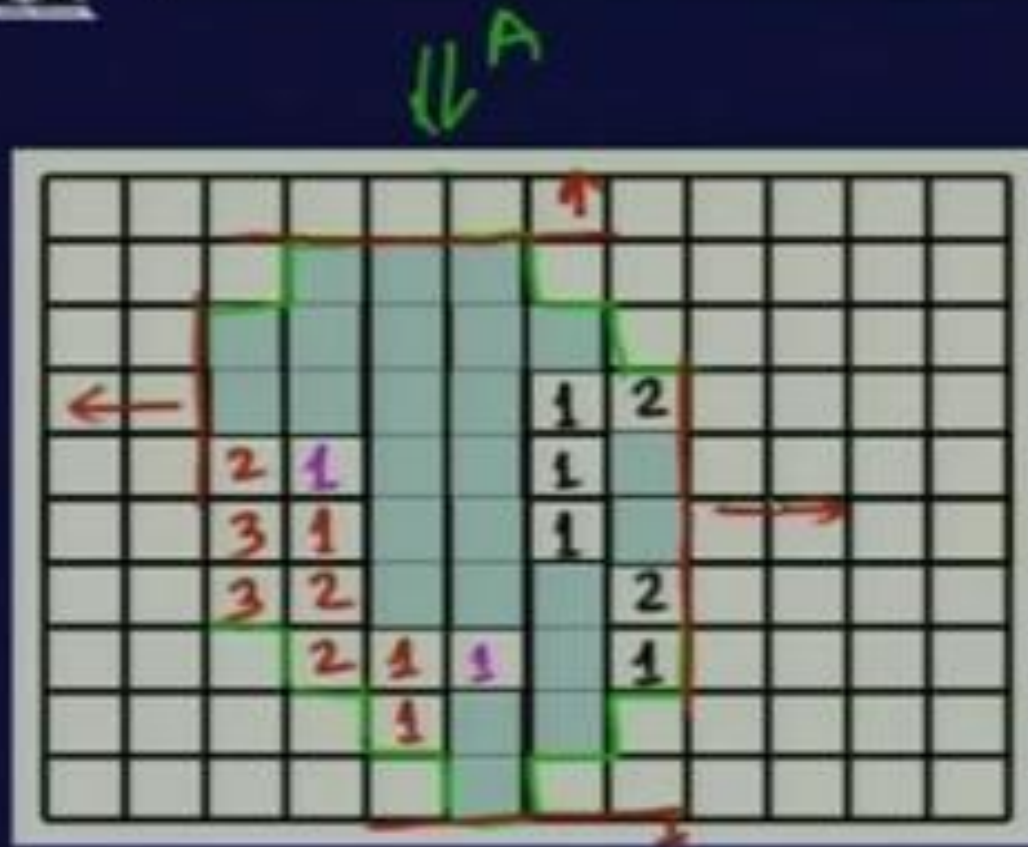
B^3

X	X	X
X		X

B^4



Convex Hull



	X	X
		X
	X	X

B^1

X		X
X	X	X

B^2

X	X	
X		
X	X	

B^3

X	X	X
X		X

B^4