



## Mathematical Morphology

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➤ On completion the students will learn and be able to implement

- What is Morphology?
- Morphological image processing techniques
  - Dilation
  - Erosion
  - Opening
  - Closing
- Applications



## Application

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- **Preprocessing**
  - Filtering
  - Shape simplification
- **Segmentation using object shape**
- **Object quantification**
  - Area, perimeter etc.
- **Enhancing object structure**
  - Skeletonization, thinning, thickening, convex hull, object marking etc.



## Mathematical Morphology

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### What is Morphology?

Commonly denotes a branch of biology that deals with the form and structure of animals and plants

Tool for extracting image components that are useful for representation and description of region shape, boundary, skeleton, convex hull etc.



## Images as Point Sets



$$X = \{(2,0), (2,1), (1,2), (3,2), \dots\}$$

$$X^c$$

→  $A$  in  $\mathbb{Z}^2$   
→  $a = (a_1, a_2)$

$$a \in A$$

$$b \notin A$$

Subset

$$A \subseteq B$$

Union

$$A \cup B$$

Intersection.

$$A \cap B$$

Complement

$$A^c = \{b \mid b \notin A\}$$

Difference.

$$A - B = \{w \mid w \in A \text{ and } w \notin B\}$$

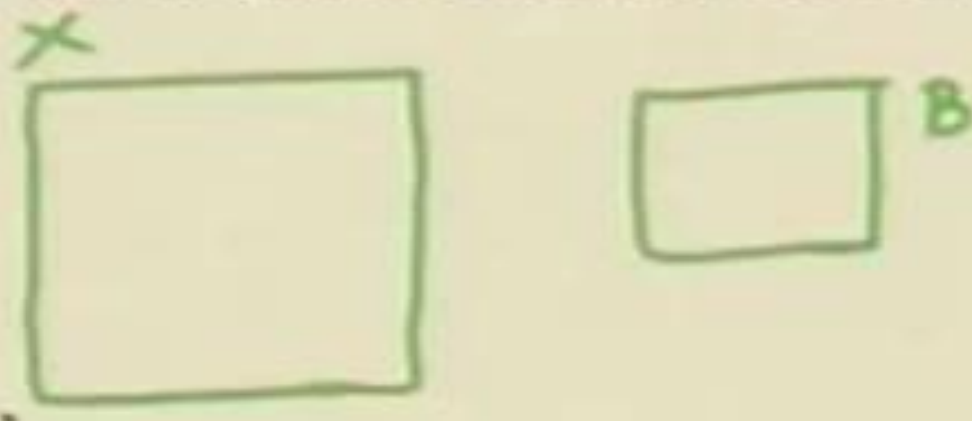




## Morphological transformation

### Morphological Transformation $\psi$

Gives a relation of the image  $X$ , with another small point set  $B$ , called structuring element



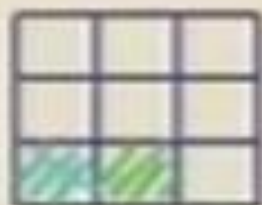
Dilation  $\oplus$

$$X \oplus B = \left\{ p \in \mathbb{Z}^2 \mid \begin{array}{l} p = x + b, \\ x \in X, b \in B \end{array} \right\}$$



## Dilation

X



B

$$B = \{(0,0), (1,0)\}$$

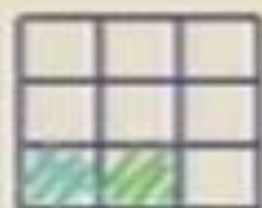
$$X = \{(3,2), (2,2), (3,1), \dots\}$$





## Dilation

X



B

$$B = \{(\underline{0}, \underline{0}), (1, 0)\}$$

$$X = \{(3, 2), (2, 2), (3, 1), \dots\}$$

$(3, 2) + (1, 0) = (4, 2)$



## Dilation

X



B

$$B = \{(\underline{0}, \underline{0}), (1, 0)\}$$

$$X = \{(3, 2), (2, 2), (3, 1) \dots\}$$
$$(3, 2) + (1, 0) = (4, 2)$$

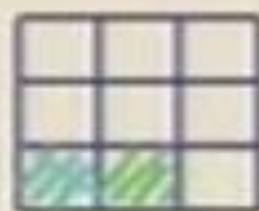


## Dilation

X



$X \oplus B$



B

$$B = \{(\underline{0}, \underline{0}), (\underline{1}, \underline{0})\}$$

$$X = \{(3, 2), (2, 2), (3, 1) \dots\}$$
$$(3, 2) + (1, 0) = (4, 2)$$



## Dilation

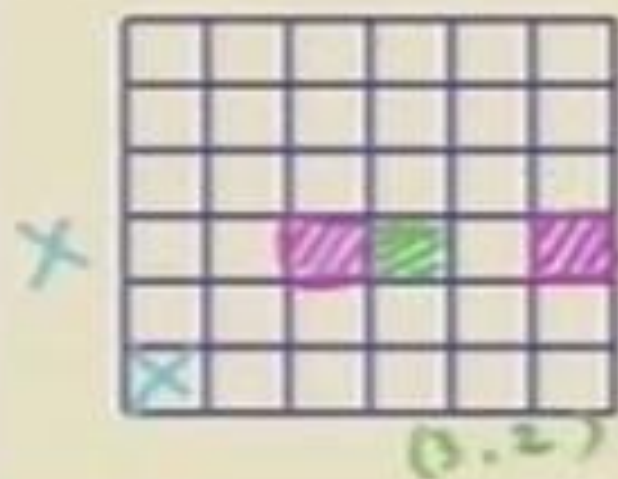


B

$$\{(-1, 0), (2, 0)\}$$



## Dilation

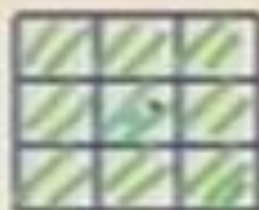
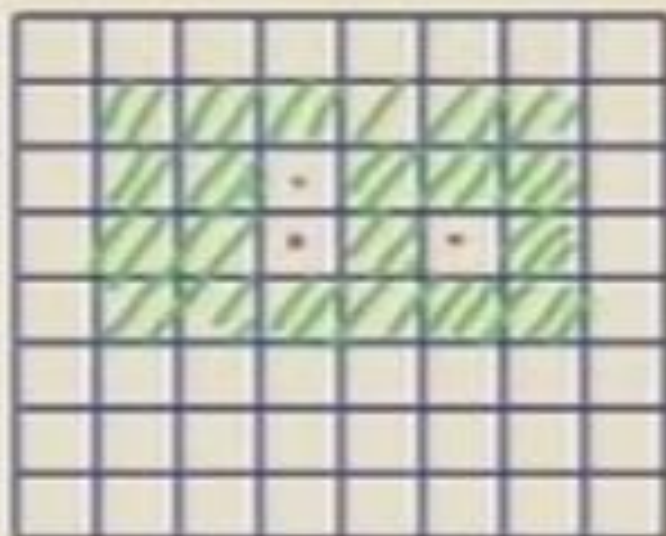


$B$   
 $\{(-1,0),$   
 $(2,0)\}$



## Dilation Application

x



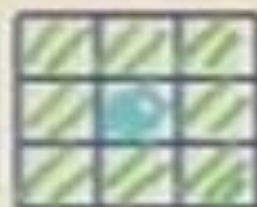
B





## Dilation Application

x

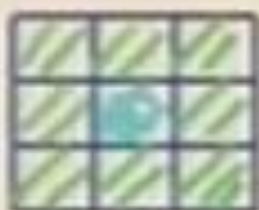


B



## Dilation Application

X



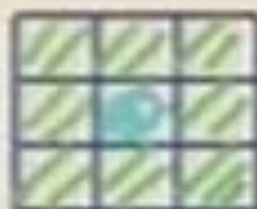
B

$$\underline{X \oplus B}$$



## Dilation Application

X



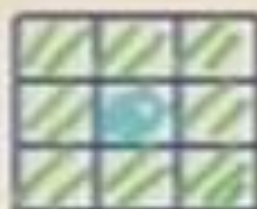
B

$X \oplus B$



## Dilation Application

X



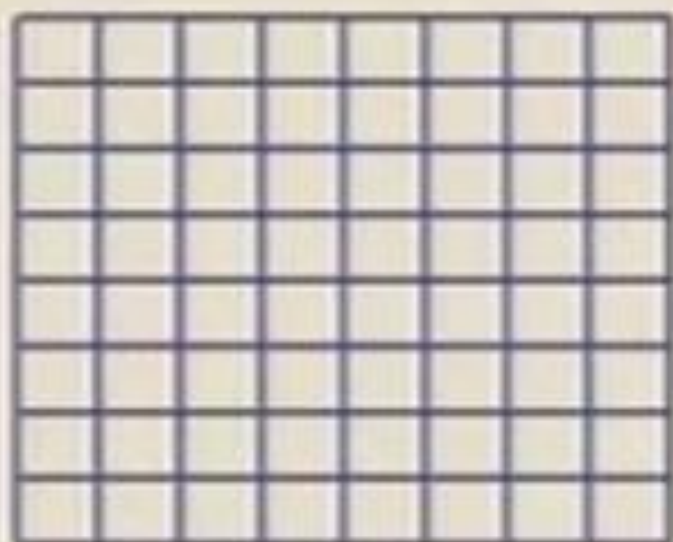
B

$$\underline{X \oplus B}$$



## Erosion

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$x$  $B$ 

$$x \ominus B = \{p \in \mathbb{Z}^2 \mid p + b \in x \text{ for every } b \in B\}$$