

# Theory of Formal Languages and Automata

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# A Substitution Rule

$$S \rightarrow aB$$

$$A \rightarrow aaA$$

$$A \rightarrow abBc$$

$$B \rightarrow aA$$

$$B \rightarrow b$$

Substitute  
 $B \rightarrow b$

Equivalent  
grammar

$$S \rightarrow aB \mid ab$$

$$A \rightarrow aaA$$

$$A \rightarrow abBc \mid abbc$$

$$B \rightarrow aA$$

# A Substitution Rule

$$S \rightarrow aB \mid ab$$

$$A \rightarrow aaA$$

$$A \rightarrow abBc \mid abbc$$

$$B \rightarrow aA$$

Substitute

$$B \rightarrow aA$$

$$S \rightarrow \cancel{aB} \mid ab \mid aaA$$

$$A \rightarrow aaA$$

$$A \rightarrow \cancel{abBc} \mid abbc \mid abaAc$$

Equivalent  
grammar

In general:

$$A \rightarrow xBz$$

$$B \rightarrow y_1$$

Substitute

$$B \rightarrow y_1$$

$$A \rightarrow xBz \mid xy_1z$$

equivalent  
grammar

# Nullable Variables

$\lambda$  – production :  $A \rightarrow \lambda$

Nullable Variable:  $A \Rightarrow \dots \Rightarrow \lambda$

# Removing Nullable Variables

Example Grammar:

$$S \rightarrow aMb$$

$$M \rightarrow aMb$$

$$M \rightarrow \lambda$$

Nullable variable



## Final Grammar

$$S \rightarrow aMb$$

$$M \rightarrow aMb$$

~~$$M \rightarrow \lambda$$~~

Substitute

$$M \rightarrow \lambda$$

$$S \rightarrow aMb$$

$$S \rightarrow ab$$

$$M \rightarrow aMb$$

$$M \rightarrow ab$$

# Unit-Productions

Unit Production:  $A \rightarrow B$

(a single variable in both sides)



# Removing Unit Productions

Observation:

$$A \rightarrow A$$

Is removed immediately

## Example Grammar:

$$S \rightarrow aA$$

$$A \rightarrow a$$

$$A \rightarrow B$$

$$B \rightarrow A$$

$$B \rightarrow bb$$

$$S \rightarrow aA$$

$$A \rightarrow a$$

~~$$A \rightarrow B$$~~

$$B \rightarrow A$$

$$B \rightarrow bb$$

Substitute

$$A \rightarrow B$$

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

$$B \rightarrow A \mid B$$

$$B \rightarrow bb$$

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

$$B \rightarrow A \mid \cancel{B}$$

$$B \rightarrow bb$$

Remove

$$B \rightarrow B$$

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

$$B \rightarrow A$$

$$B \rightarrow bb$$

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

~~$$B \rightarrow A$$~~

$$B \rightarrow bb$$

Substitute

$$B \rightarrow A$$

$$S \rightarrow aA \mid aB \mid aA$$

$$A \rightarrow a$$

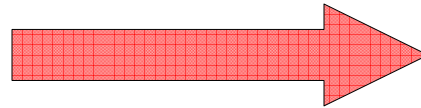
$$B \rightarrow bb$$

## Remove repeated productions

$$S \rightarrow aA \mid aB \mid \cancel{aA}$$

$$A \rightarrow a$$

$$B \rightarrow bb$$



## Final grammar

$$S \rightarrow aA \mid aB$$

$$A \rightarrow a$$

$$B \rightarrow bb$$

# Useless Productions

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$S \rightarrow A$$

$$A \rightarrow aA$$

Useless Production

Some derivations never terminate...

$$S \Rightarrow A \Rightarrow aA \Rightarrow aaA \Rightarrow \dots \Rightarrow aa \dots aA \Rightarrow \dots$$

Another grammar:

$$S \rightarrow A$$

$$A \rightarrow aA$$

$$A \rightarrow \lambda$$

$$B \rightarrow bA$$

Useless Production

Not reachable from S



In general:

contains only  
terminals

if  $S \Rightarrow \dots \Rightarrow xAy \Rightarrow \dots \Rightarrow w$

  $w \in L(G)$

then variable  $A$  is useful

otherwise, variable  $A$  is useless

A production  $A \rightarrow x$  is useless  
if any of its variables is useless

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Productions

Variables

$$S \rightarrow A$$

useless

useless

$$A \rightarrow aA$$

useless

useless

$$B \rightarrow C$$

useless

useless

$$C \rightarrow D$$

useless

# Removing Useless Productions

Example Grammar:

$$S \rightarrow aS \mid A \mid C$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$C \rightarrow aCb$$

**First:** find all variables that can produce strings with only terminals

$$S \rightarrow aS \mid A \mid C$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$C \rightarrow aCb$$

Round 1:  $\{A, B\}$

$$S \rightarrow A$$

Round 2:  $\{A, B, S\}$

Keep only the variables  
that produce terminal symbols:  $\{A, B, S\}$   
(the rest variables are useless)

$$S \rightarrow aS \mid A \mid \cancel{C}$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$\cancel{C \rightarrow aCb}$$



$$S \rightarrow aS \mid A$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

Remove useless productions

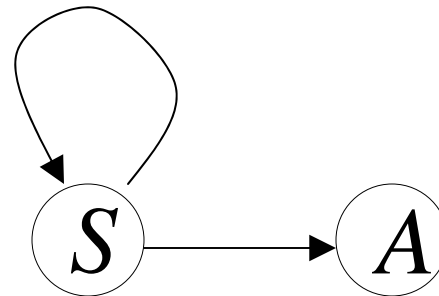
**Second:** Find all variables  
reachable from  $S$

Use a Dependency Graph

$S \rightarrow aS \mid A$

$A \rightarrow a$

$B \rightarrow aa$



not  
reachable

Keep only the variables  
reachable from  $S$

(the rest variables are useless)

Final Grammar

$$S \rightarrow aS \mid A$$

$$A \rightarrow a$$

~~$$B \rightarrow aa$$~~



$$S \rightarrow aS \mid A$$

$$A \rightarrow a$$

Remove useless productions

# Removing All

**Step 1:** Remove  $\lambda$ -productions

**Step 2:** Remove Unit-productions

**Step 3:** Remove Useless productions