Normalized Bispectrum Mapping for Probing Primordial Non-Gaussianity in Redshift Space

Understanding the Large-Scale Structure of the Universe

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Introduction to the Large-Scale Structure

- The universe's structure forms a complex network known as the cosmic web.
- These structures originated from small, predominantly Gaussian density fluctuations during inflation.
- Subtle deviations from gaussianity—known as primordial non-Gaussianity (PNG)—offer insights into the physics of inflation,
- giving clues about what caused the universe to expand.

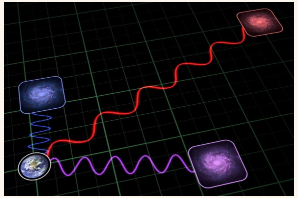
Understanding the Bispectrum

- The bispectrum helps us measure **how galaxies cluster** in the universe
- While the **power spectrum** captures only pairwise correlations (Gaussian information), the bispectrum checks **triplets** (3-point correlation), revealing more complex interactions.

- A non-zero bispectrum indicates deviations from Gaussianity
- It tells us if those points cluster in a way that deviates from randomness.

Redshift Space Distortions (RSD)

- When we observe galaxies, we estimate their distance using redshift—how much their light is stretched due to the universe's expansion.
- But galaxies also have their own motions (peculiar velocities), which distort their true positions in redshift space.
  - Galaxies moving towards us appear blueshifted (shorter wavelengths),
  - Galaxies moving away appear redshifted (longer wavelengths).



- These distortions affect the bispectrum, thus we must account for RSDs to extract accurate PNG information.

Primordial Non-Gaussianity (PNG) Templates

Local PNG	Equilateral PNG	Orthogonal PNG
Generated when fluctuations in different spots interact locally	when interactions involve high speeds or non-standard forces	A mix of local and equilateral shapes but with opposite patterns
Peaks in squeezed triangles	Maximized for equilateral triangles	Sensitive to flattened triangles

$$B_\Phi^{\mathrm{local}} = 2 f_{\mathrm{NL}} P_1 P_2 + \mathrm{cyc.}$$

$$\frac{1}{6 f_{\mathrm{NL}}} B_\Phi^{\mathrm{equil}} = -(P_1 P_2 + \mathrm{cyc.}) - 2 (P_1 P_2 P_3)^{\frac{2}{3}} + (P_1^{\frac{1}{3}} P_2^{\frac{2}{3}} P_3 + \mathrm{cyc.}) \tag{3}$$

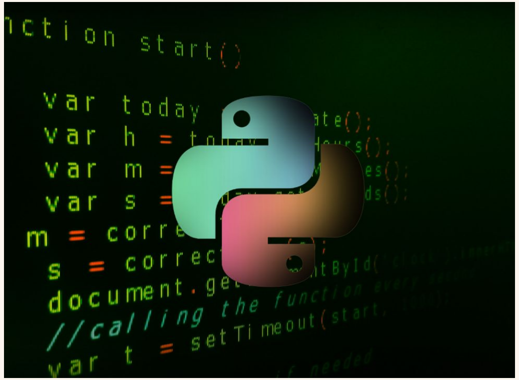
$$\frac{1}{6 f_{\mathrm{NL}}} B_\Phi^{\mathrm{ortho}} = -3 (P_1 P_2 + \mathrm{cyc.}) - 8 (P_1 P_2 P_3)^{\frac{2}{3}} + 3 (P_1^{\frac{1}{3}} P_2^{\frac{2}{3}} P_3 + \mathrm{cyc.}) \tag{4}$$

Methodology

Programming Language: Python 3.9

Libraries:

- NumPy. [Computations]
- Matplotlib. [Visualization]



Parameterizing Bispectrum

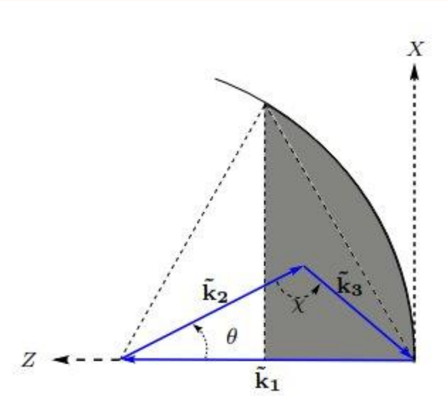
- B(k1, k2, k3)
- Real space bispectrum is independent of orientations
- Depends only upon shape and size of triangle
- Fix k1 (largest side) to specify shape
- Paramaterized using μ and t :
    - t= k2/k1 and μ= cos θ

Given μ-t , can you plot k-vectors triangle??

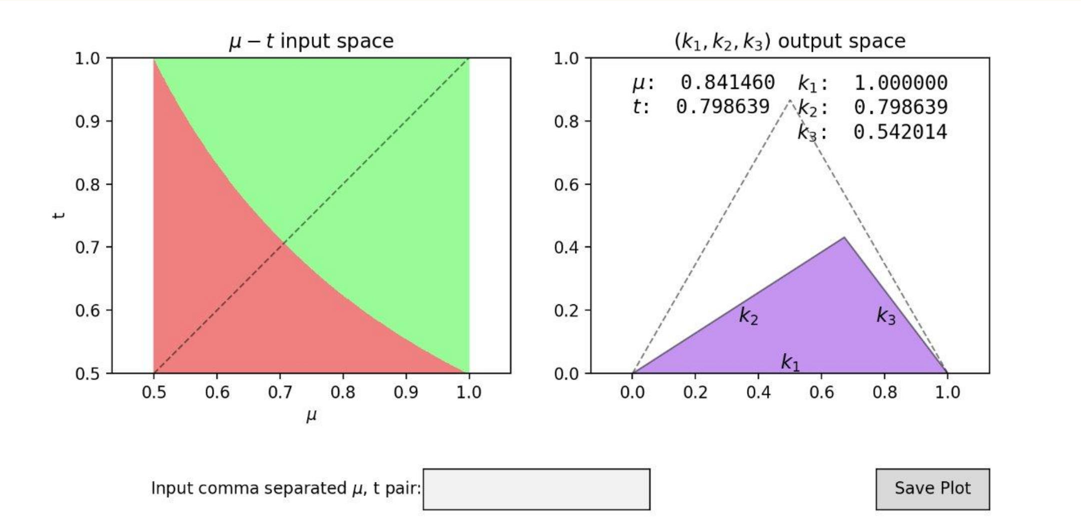
$$\tilde{\mathbf{k}}_1 = k_1\,\hat{\mathbf{z}}\,.$$

$$\tilde{\mathbf{k}}_2 = tk_1[-\mu\,\hat{\mathbf{z}} + \sqrt{1-\mu^2}\,\hat{\mathbf{x}}]$$

$$\mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2$$



$$0.5 \leqslant t, \mu \leqslant 1 \text{ with } t\mu \geqslant 0.5\,.$$



TriPlot

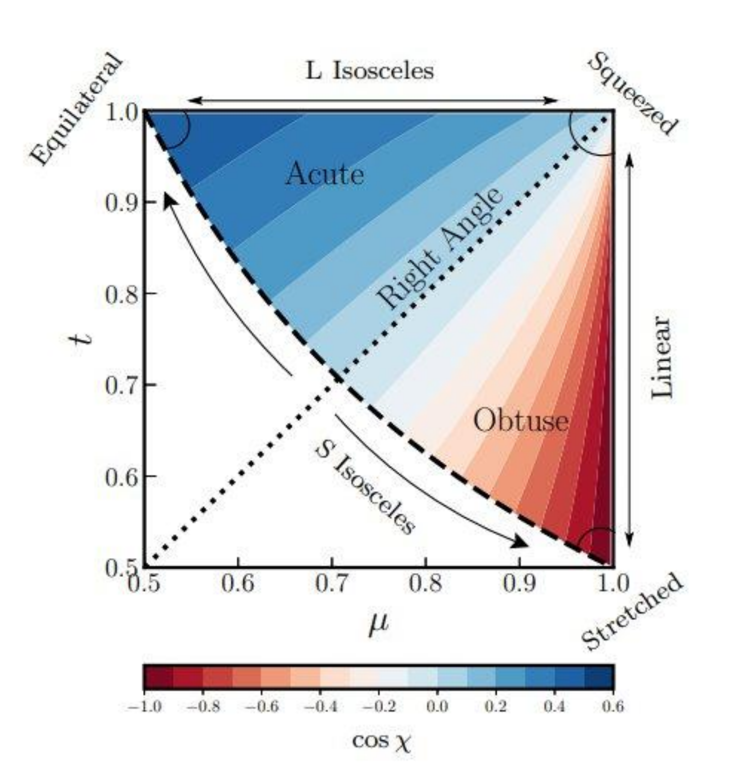
Visualizing triangle shapes for allowed μ-t space

Discussion

- μ=1 : Linear
  - (stretched -> squeezed)
- t = 1 => k1=k2 => Isosceles

- 2μt = 1 => lower boundary => k2=k3 => Isosceles

- μ=0.5, t=1 => equilateral



Orientation Dependence

- Quantifying Redshift Space Distortions
- Bispectrum also depend upon triangle orientations
  - B(p, k1, μ, t)
  - p: unit vector to denote orientation dependence

Given p, can you visualize k-vector triangle??

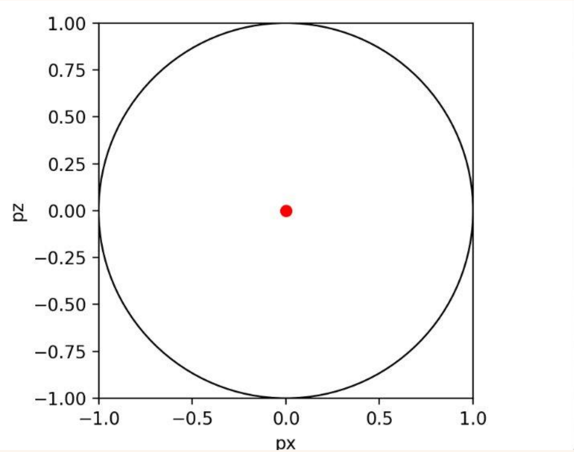
$$p_y = \sqrt{1 - p_x^2 - p_z^2}.$$

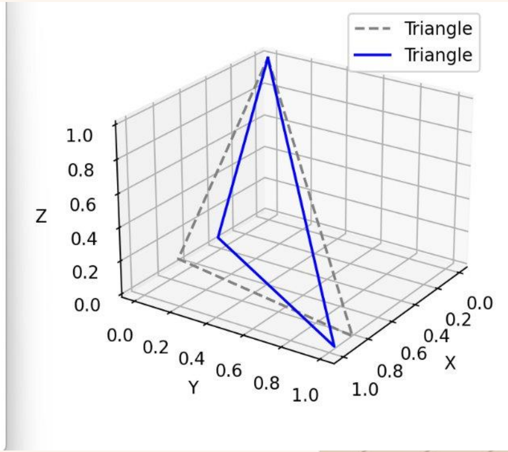
$$\mu_1 = p_z$$

$$\mu_2 = -\mu p_z + \sqrt{1-\mu^2}\,p_x$$

$$\mu_3 = \frac{-[(1-t\mu)p_z + t\sqrt{1-\mu^2}\,p_x]}{\sqrt{1-2t\mu+t^2}}$$

Cosine of angles b/w p and k-vectors

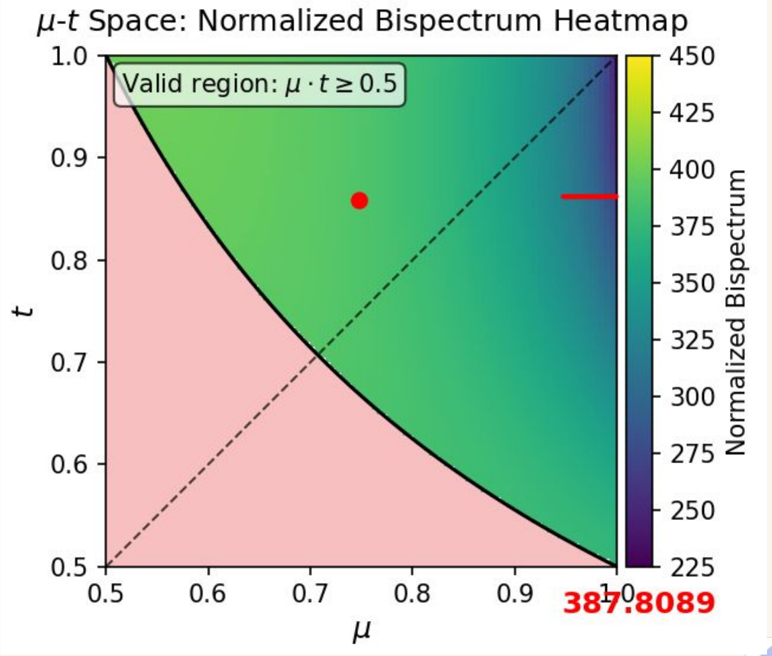




Normalized Bispectrum Heatmaps

$$Q(\mu, t) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)},$$

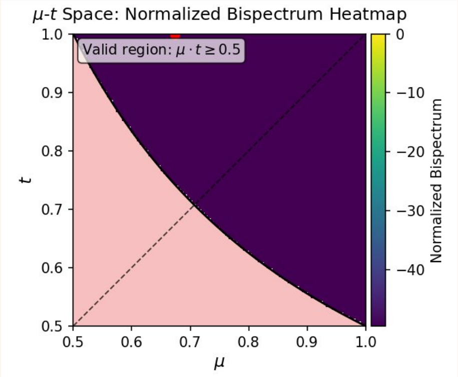
Separate heatmaps were generated for each primordial non-Gaussianity template.



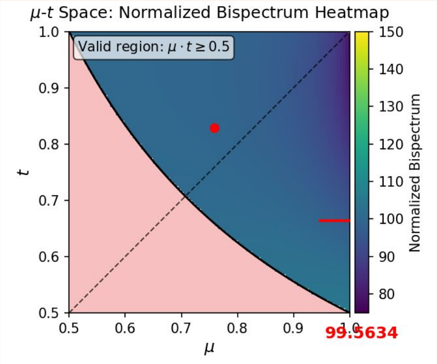
dimensionless

Discussion

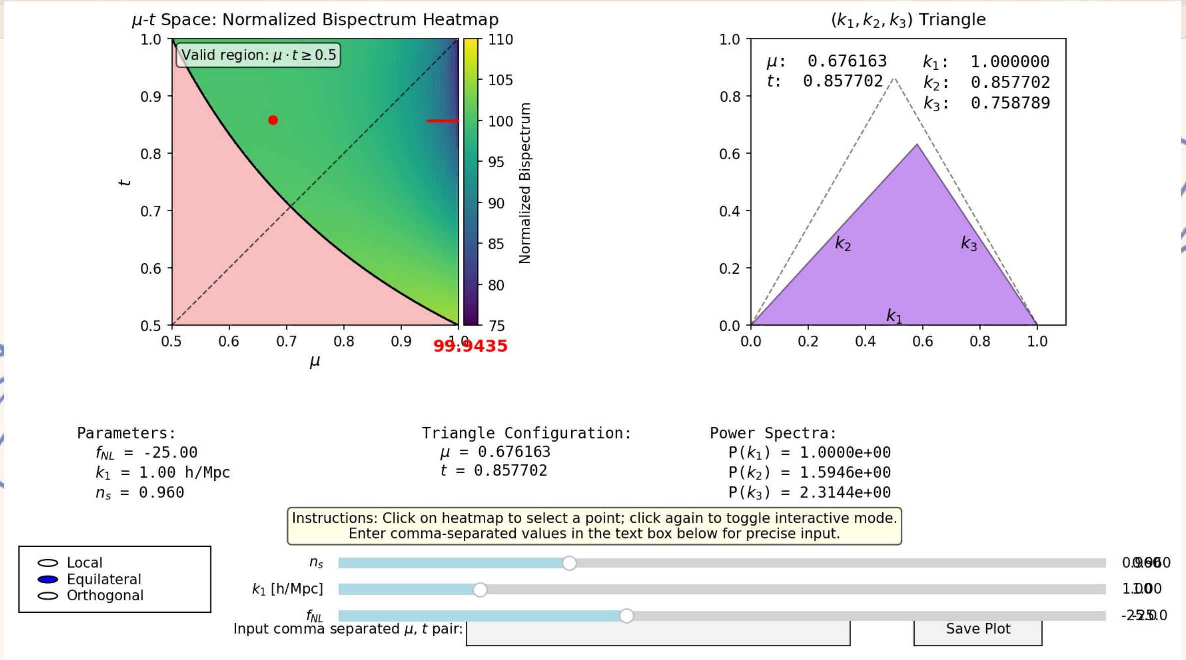
- Normalized Local PNG can be used to find fNL
- Differentiating between equilateral and orthogonal PNG



Local



Equilateral



TriPlot

A Bispectrum visualization tool

Future Prospects

Further research will focus on higher-order statistics and machine learning integration for faster computations.
Exploring non-linear effects and applying methods to mock catalogs will refine parameter estimates.

References

- Codes: https://github.com/aryabhatta0/TriPlot-BispectrumViz-MTP
- Bharadwaj, S., Mazumdar, A., and Sarkar, D. (2020). Quantifying the redshift space distortion of the bispectrum i: primordial non-gaussianity.
- Scoccimarro, R., Hui, L., Manera, M., and Chan, K. C. (2012). Large-scale bias and efficient generation of initial conditions for nonlocal primordial non-gaussianity.
- Forero-Romero, J. E. et al. (2009). A dynamical classification of the cosmic web. Monthly Notices of the Royal Astronomical Society.
- Mazumdar, A., Bharadwaj, S., and Sarkar, D. (2020). Quantifying the redshift space distortion of the bispectrum ii: induced non-gaussianity at second-order perturbation.

Thank you!

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