Integers

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Exercises

1 Let x be a real number, prove that there exists an integer q and a real number $0 \leq s \leq 1$ such that $x = q + s$ and q, s are uniquely determined.
Source: Lang, Undergraduate Algebra Chapter 1, exercise 5
Let q be an integer, the set of integers q such that $q \leq n$ is bounded from above. Then there exists an integer m such that $q < m$.

$$qx < (q+1)$$
$$0 \leq x - q < 1$$

Let $s = x - q$ then $o \leq 1$. This proves the existence of real numbers s and integer q. Now for the uniqueness, suppose that:

$$x_1 = q + s, 0 \leq s_1 \leq 1$$
$$x_2 = q + s, 0 \leq s_2 \leq 1$$

Then if $s_1 s_2$ and s_2 1 subtracting we get:

$$(q_1 - q_2) = s_2 - s_1$$

$s_2 - s_1 < 1$ and $s_2 - s_1 > 0$ but $(q_1 - q_2)$ is an integer then if $(q_1 - q_2) > 0$ we will have that $q_1 - q_2 \geq 1$ then $s_1 = s_2$ and therefore $q_1 - q_2 = 0$ thus $q_1 = q_2$