Implementation Discrete Choice Random Utility Model

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- Setup: N alternatives and $2^N - 1$ choice sets.
  - Example: For 3 products $\{A, B, C\}$, we have choice sets $\{A, B, C\}$, $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{A, B\}$, $\{A, B\}$, $\{A\}$, $\{B\}$ and $\{C\}$.
  - Comment: the choice sets $\{A\}$, $\{B\}$ and $\{C\}$ are not exactly necessary....
- Input: A choice probability vector for each choice set.
  - Example: We have the data p^{obs}:

$\{A, B, C\}$	A	0.5
	B	0.3
	C	0.2
$\{A, B\}$	A	0.6
	B	0.4
$\{A, C\}$	A	0.7
	C	0.3
$\{B, C\}$	B	0.4
	C	0.6
$\{A\}$	A	1
$\{B\}$	B	1
$\{C\}$	C	1

- Step 1: Create $N!$ deterministic choice vector and their associated probabilities $\{v_i\}_{i=1}^{N!}$
  - Example: With 3 choice we create 6 choice probability vector

		$A > B > C$	$A > C > B$	$B > A > C$	$B > C > A$	$C > B > A$	$C > A > B$
$\{A, B, C\}$	A	1	1	0	0	0	0
	B	0	0	1	1	0	0
	C	0	0	0	0	1	1
$\{A, B\}$	A	1	1	0	0	0	1
	B	0	0	1	1	1	0
$\{A, C\}$	A	1	1	1	0	0	0
	C	0	0	0	1	1	1
$\{B, C\}$	B	1	0	1	1	0	0
	C	0	1	0	0	1	1
$\{A\}$	A	1	1	1	1	1	1
$\{B\}$	B	1	1	1	1	1	1
$\{C\}$	C	1	1	1	1	1	1

- Implement the following optimization problem:

$$\min_p ||p - p^{\text{obs}}||_2^2 = \sum_{D \subset 2^X} \sum_{x \in D} \left(p(x, D) - p^{\text{obs}}(x, D)\right)^2, \tag{1}$$

subject to

$$p = \sum_{i=1}^{n} \lambda_i v_i \tag{2}$$

$$1 = \sum_{i=1}^{n} \lambda_i \tag{3}$$

$$\lambda_i \geq 0, \forall i \in [2^N], \tag{4}$$

where X is the set of alternatives and $p(x, D)$ is the probability of choosing x from choice set D.