

Homework 3

1. Plot your results and show your MATLAB program in text in your answer sheets. (1 for UG students. 1& 2 for Graduate students)

Synapse is the way neurons pass activation signal from one to another. Here we simulate synapse caused conductance change in response to a train of uniformly spread input spikes. The conductance g (in unit is described by:

$$\frac{d^2 g}{dt^2} + \frac{2}{\tau} \frac{dg}{dt} + \frac{g}{\tau^2} = G_{norm} u(t)$$

Which is equivalent to the following two differential equations:

$$\frac{dz}{dt} = \frac{-z}{\tau} + G_{norm} u(t)$$

And

$$\frac{dg}{dt} = \frac{-g}{\tau} + z(t)$$

The constant G_{norm} is related to the peak conductivity g_{peak} by:

$$G_{norm} = \frac{g_{peak}}{\left(\frac{\tau}{e}\right)}$$

τ is synaptic time and $e = 2.718$

The impulse response is the function:

$$g(t) = G_{norm} t e^{-\frac{t}{\tau}} = \frac{g_{peak}}{\left(\frac{\tau}{e}\right)} t e^{-t/\tau}$$

the peak of the conductance waveform occurs at $t = \tau$:

$$g(\tau) = g_{peak}$$

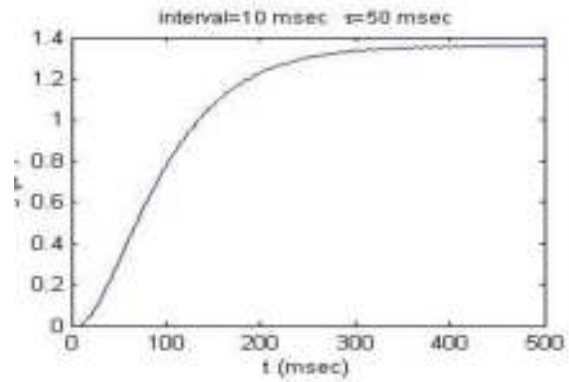
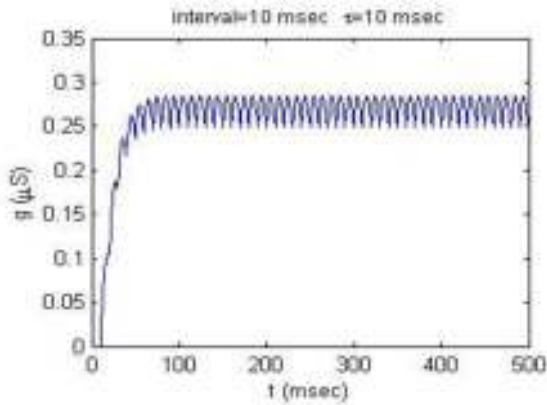
I. Write a MATLAB program that use the two first order differential equations to simulate conductance changes in response to a train of uniformly space input spikes. Model the spike input as:

$$u(t) = \begin{cases} \frac{1}{\Delta t} & \text{when a spike occurs} \\ 0 & \text{otherwise} \end{cases}$$

Δt = input spike interval, Simulate the following four cases and turn in plots of conductance vs. time using $T_{max}=500\text{ms}$, $dt=1\text{ ms}$, and $g_{peak}=0.1\text{ }\mu\text{S}$.

(a) $\Delta t=50\text{ms}$, $\tau=10\text{ms}$, (b) $\Delta t=50\text{ms}$, $\tau=50\text{ms}$, (c) $\Delta t=10\text{ms}$, $\tau=10\text{ms}$, (d) $\Delta t=10\text{ms}$, $\tau=50\text{ms}$. - show your answer (a) and (b)

Hint: your answer (c) and (d) shall look like the following [only grade (a) and (b)]



II. Now, we insert the synapse from the previous problem into a neuron and simulate the postsynaptic response to an input train of uniformly spaced action potentials. the differential equations describing the model are:

$$C \frac{dv}{dt} = \frac{-v}{R} - g_{ex}(v - E_{ex}) + I_{inject}$$

conductance change

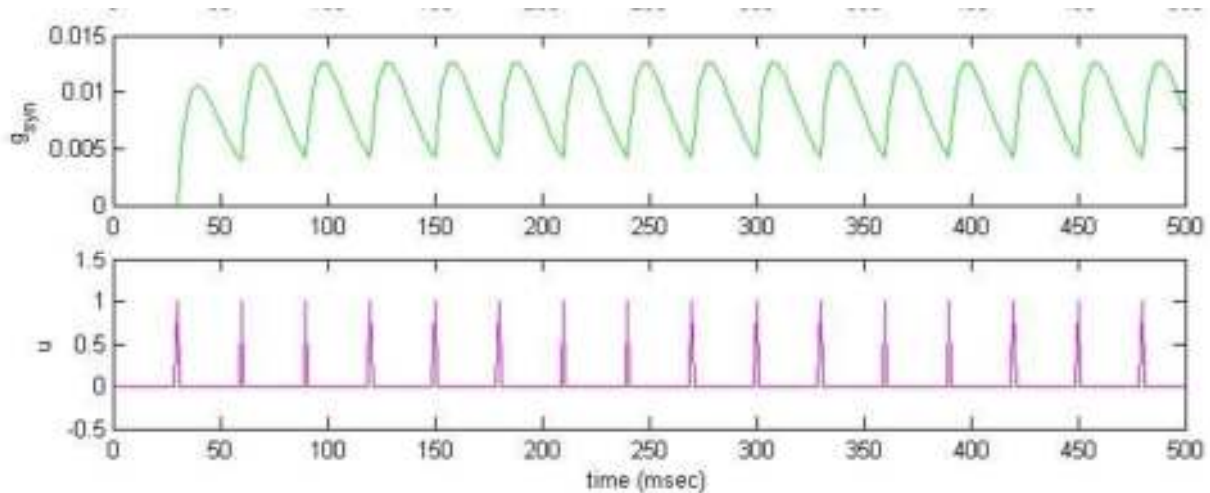
$$\frac{dz}{dt} = \frac{-z}{\tau_{syn}} + \frac{g_{peak}}{\left(\frac{\tau_{syn}}{e}\right)} u(t)$$

$$\frac{dg_{ex}}{dt} = \frac{-g_{ex}}{\tau_{syn}} + z(t)$$

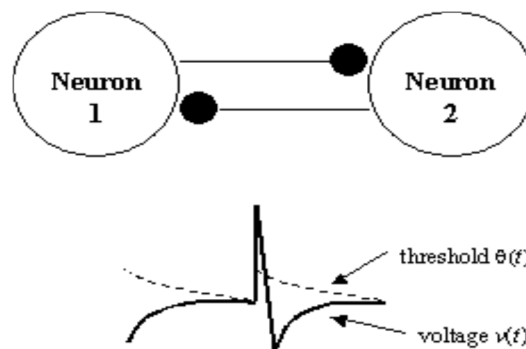
use the following parameter values in your calculation:

$C = 1$	membrane capacitance (nF)
$R = 10$	membrane resistance ($M\Omega$)
$V_{rest} = 0$	resting membrane potential (mV)
$V_{spk} = 70$	action potential amplitude (mV)
$V_{thr} = 5$	spike threshold (mV)
$E_{ex} = 70$	synaptic reversal potential (mV)
$\tau_{syn} = 10$	synaptic time constant (ms)
$g_{peak} = 0.01$	peak synaptic conductance (μS)
$T_{max} = 500$	total simulation time (ms)
$\Delta t = 1$	integration time step (ms)
$T_{ISI} = 30$	input interspike interval (ms)

generate plots of the following quantities verse time (a) input spike train $u(t)$, (b) synaptic conductance $g_{ex}(t)$, (c) synaptic current $I_{sync}(t)$, and (d) postsynaptic membrane voltage $v(t)$. Hint: your $u(t)$ and $g_{ex}(t)$, shall look like: [only grade (c) and (d)]



2. (graduate student only) III. Construct a two-neuron oscillator using reciprocal inhibition. the neurons will be modeled as leaky integrate-and-fire units with an adaptive threshold mechanism that generates firing-rate adaption and post-inhibitory rebound. the model structure is illustrated in the following diagram:



the update equations for each individual neuron are:

$$C \frac{dv}{dt} = \frac{-v}{R} - g_{ex}(v - E_{ex}) + I_{inject} \quad \text{Voltage update}$$

$$\frac{d\theta}{dt} = \frac{-\theta + v}{\tau_{thresh}} \quad \text{Threshold update}$$

$$\frac{dz}{dt} = \frac{-z}{\tau_{syn}} + \frac{g_{peak}}{(\frac{\tau_{syn}}{e})} u(t) \quad \text{Conductance Update I}$$

$$\frac{dg_{ex}}{dt} = \frac{-g}{\tau_{syn}} + z(t) \quad \text{Conductance Update II}$$

Remember that the input $u(t)$ comes from spike activity of the *pre-synaptic unit*.

The parameter values for the model are:

$C = 1$	membrane capacitance (nF)
$R = 10$	membrane resistance (M Ω)
$V_{rest} = 0$	resting membrane potential (mV)
$V_{spk} = 70$	action potential amplitude (mV)
$\tau_{thresh} = 50$	threshold time constant (ms)
$E_{inh} = -15$	synaptic reversal potential (mV)
$\tau_{syn} = 15$	synaptic time constant (ms)
$g_{peak} = 0.1$	peak synaptic conductance (μS)
$T_{max} = 1500$	total simulation time (ms)
$\Delta t = 1$	integration time step (ms)

both neurons should receive constant current injection. To break the symmetry of the model, inject slightly more current into neuron 1 than neuron 2. Specifically, inject 1.1nA into neuron 1 and 0.9 into neuron 2.

When the neuron fires an action potential, reset the membrane voltage to E_{inh} on the next time step (rather than 0) [Note: this is related to the adaptive threshold level, which can fall below zero in this model, but not below E_{inh} . We need to reset the membrane voltage to a value that is below the threshold level, hence we choose E_{inh} as the rest value.] plot your results of neuron 1, 2 , threshold level as function of time from 0-1500ms.