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1) $\frac{d}{dx}f(x) = f'(x)$

2) $\frac{d}{dx}\sin x = \cos x$

3) $\frac{d}{dx}\cos x = -\sin x$

4) $\frac{d}{dx}\tan x = sec^2 x$

6) $\frac{d}{dx}\sec x = \sec x \tan x$

7) $\frac{d}{dx}\cot x = -cosec^2 x$

8) $\frac{d}{dx}e^x = e^x$

9) $\frac{d}{dx}a^x = a^x log a$

10) $\frac{d}{dx}\log x = \frac{1}{x}$

11) $\frac{d}{dx}\log_a x = \frac{1}{x \log a}$

12) $\frac{d}{dx}x^n = n\ x^{n-1}$

13) $\frac{d}{dx}x = 1$

14) $\frac{d}{dx}x^2 = 2x$

15) $\frac{d}{dx}x^3 = 3x^2$

16) $\frac{d}{dx}x^4 = 4x^3$

17) $\frac{d}{dx}x^5 =$

18) $\frac{d}{dx}x^{\frac{5}{7}} =$

19) $\frac{d}{dx}x^{-\frac{5}{7}} =$

20) $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

21) $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$

22) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

22) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

23) $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$

24) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$

25) $\dfrac{d}{dx}\sec^{-1}x = \dfrac{1}{x\sqrt{x^2-1}}$

26) $\dfrac{d}{dx}\operatorname{cosec}^{-1}x = \dfrac{-1}{x\sqrt{x^2-1}}$

27) $\dfrac{d}{dx}k = 0$

$(where\ k\ is\ constant)$

28) $\dfrac{d}{dx}a = 0$

29) $\dfrac{d}{dx}(-234) = 0$

30) $\dfrac{d}{dx}1crore = 0$

31) $\dfrac{d}{dx}\pi = 0$

32) $\dfrac{d}{dx}a = 0$

33) $\dfrac{d}{dx}b = 0$

Rules of Differentiation:

i. $\dfrac{d}{dx}(u+v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$

ii. $\dfrac{d}{dx}(u-v) = \dfrac{du}{dx} - \dfrac{dv}{dx}$

iii. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

iv. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\cdot\dfrac{du}{dx} - u\cdot\dfrac{dv}{dx}}{v^2},\ v \neq 0$

v. $\dfrac{d}{dx}(ku) = k\dfrac{du}{dx}$

vi. $\dfrac{d}{dx}(uvw) = uv\dfrac{dw}{dx} + uw\dfrac{dv}{dx} + vw\dfrac{du}{dx}$

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Formulae

Formulae for Indefinite Integrals

(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + k,\ n \neq -1$

(b) $\int \frac{1}{x} dx = \log|x| + k$

(c) $\int a^x dx = \frac{1}{\log a} a^x + k$

(d) $\int e^{ax} dx = \frac{1}{a} e^{ax} + k$

(e) $\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + k$

(f) $\int \cos(ax) dx = \frac{1}{a}\sin(ax) + k$

(g) $\int \tan x dx = \log|\sec x| + k$ or $-\log|\cos x| + k$

(h) $\int \cot x dx = \log|\sin x| + k$ or $-\log|\operatorname{cosec} x| + k$

(i) $\int \sec x dx = \log|\sec x + \tan x| + k$ or $\log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + k$

(j) $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + k$ or $\log\left|\tan\frac{x}{2}\right| + k$

(k) $\int \sec^2 x dx = \tan x + k$

(l) $\int \operatorname{cosec}^2 x dx = -\cot x + k$

(m) $\int \sec x.\tan x dx = \sec x + k$

(n) $\int \operatorname{cosec} x.\cot x dx = -\operatorname{cosec} x + k$

(o) $\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + k$

(p) $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + k$

(q) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + k$

(r) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + k$

(s) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + k$

(t) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + k$

(u) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + k$

(v) $\int \frac{1}{ax+b} dx = \frac{1}{a}\log|ax+b| + k$, where '$a$' is any constant (and obviously, k is the integral constant).

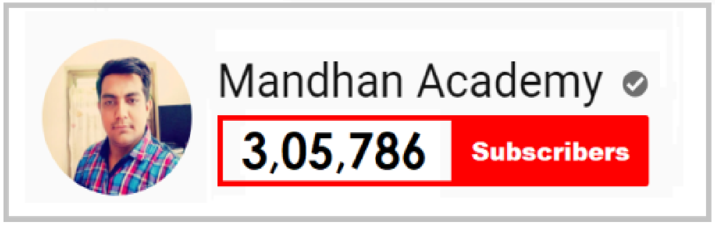
(w) $\int \lambda dx = \lambda x + k$, where '$\lambda$' is a constant (and k is the integral constant).

(x) $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + k$

(y) $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + k$

(z) $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + k$

Limits & Continuity Formula-1



1) $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ *or* $\lim\limits_{x\to 0}\dfrac{x}{\sin x} = 1$
2) $\lim\limits_{x\to 0}\dfrac{\cos x}{x} \neq 0$ ……………………….. *Don't TRY THIS AT HOME*
3) $\lim\limits_{x\to 0}\dfrac{\tan x}{x} = 1$ *or* $\lim\limits_{x\to 0}\dfrac{x}{\tan x} = 1$
4) $\lim\limits_{x\to 0}\dfrac{\log_e(1+x)}{x} = +1$ *or* $\lim\limits_{x\to 0}\dfrac{\log(1+x)}{x} = +1$
5) $\lim\limits_{x\to 0}\dfrac{\log_e(1-x)}{x} = -1$ *or* $\lim\limits_{x\to 0}\dfrac{\log(1-x)}{x} = -1$
6) $\lim\limits_{x\to 0}\dfrac{e^x-1}{x} = \log(e)$ *or* $\lim\limits_{x\to 0}\dfrac{x}{e^x-1} = 1$
7) $\lim\limits_{x\to 0}\dfrac{a^x-1}{x} = \log a$ *or* $\lim\limits_{x\to 0}\dfrac{x}{a^x-1} = \dfrac{1}{\log a}$ e.g. $\lim\limits_{x\to 0}\dfrac{4^x-1}{x} = \log 4$
8) $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a} = na^{n-1}$ $na \ldots\ldots raise \ldots\ldots to \ldots\ldots n-1$
9) $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e$ *or* $\lim\limits_{x\to 0}\left(1+\dfrac{1}{x}\right)^{x} = e$ Note: power is reciprocal of term of 'x'.

NOTE : a is any constant (Number/ fixed value) : e.g. a , b , c , 5 , - 23 , k , $\frac{3}{2}$, log 5 , α , β , 1crore , π , 5^2, e , $\sqrt{a}$, $\sqrt{2}$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, sin α , cos β , sin ∅ , ∞ , ∝ , δ, σ, μ , θ, sin(π), sin 90°

etc. **Every place of "x" is subject to UPDATE** e.g.

$$\lim_{x\to 0}\frac{\sin 5x}{5x} = 1\ ,\quad \lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}} = 1\ ,\quad \lim_{x\to 0}\frac{\log(1+9x)}{9x} = 1\ ,\quad \lim_{x\to 0}\frac{e^{2x}-1}{2x} = 1\ ,\quad \lim_{x\to 0}\frac{e^{5x}-1}{5x} = 1$$

#)Use it , IMP formula

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$\therefore$ $\mathbf{1 - \cos 2\theta = 2\sin^2\theta}$ similarly , $\mathbf{1 + \cos 2\theta = 2\cos^2\theta}$

$\therefore$ $\mathbf{1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)}$ similarly , $\mathbf{1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)}$

$\mathbf{1 - \cos 3\theta =}$ $\mathbf{1 - \cos 6\theta =}$ $\mathbf{1 + \cos 3\theta =}$ $\mathbf{1 + \cos 6\theta =}$

$\mathbf{1 - \cos 4\theta =}$ $\mathbf{1 - \cos 7\theta =}$ $\mathbf{1 + \cos 4\theta =}$ $\mathbf{1 + \cos 7\theta =}$

$\mathbf{1 - \cos 5\theta =}$ $\mathbf{1 - \cos 8\theta =}$ $\mathbf{1 + \cos 5\theta =}$ $\mathbf{1 + \cos 8\theta =}$

9) Anything raise to zero becomes 1. **(Zero & infinity is exception)**

$e^0 = 1$; $5^0 = 1$; $a^0 = 1$; $(Donkey)^0 = 1$, $(Sunny\ Deol)^0 = 1$

10) Whenever you see $\sqrt{\quad}$ Rationalise it by taking conjugate.

'conjugate' matlab kya……………………..

e.g. $(a + b)$ **has conjugate** $(a - b)$

$(\sqrt{x} - \sqrt{y})$ **has conjugate** $(\sqrt{x} + \sqrt{y})$ **………&…so on……**

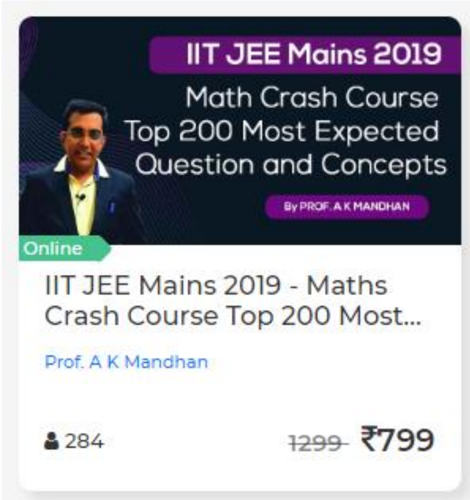
xii) Always slove in Radian , don't solve in degree ; Hint : *Multiply by* $\frac{\pi}{180}$

e.g $30° = \left(30 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{6}\right)^c$, $90° = \left(90 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{2}\right)^c$

$45° =$ $180° =$

$270° =$ $55° =$

$x° =$ $2x° =$



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3-D Concepts for Competitive Exams

IMPORTANT FORMULAE

1. If α, β, γ are the angles made by a line with positive directions of X-, Y- and Z-axes respectively, then its direction cosines are $\cos\alpha$, $\cos\beta$, $\cos\gamma$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

α, β, γ are called direction angles of the line.

2. If l, m, n are direction cosines of a line then any real numbers a, b, c such that $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$ are called direction ratios or direction numbers of that line.

3. If a, b, c are the direction ratios of a line, then its direction cosines are $\pm\frac{a}{\sqrt{a^2+b^2+c^2}}$, $\pm\frac{b}{\sqrt{a^2+b^2+c^2}}$, $\pm\frac{c}{\sqrt{a^2+b^2+c^2}}$.

4. If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, then the line PQ has direction ratios $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

5. If $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is a vector along any line, then a_1, a_2, a_3 are the direction ratios of the line.

6. If $\hat{e} = e_1\hat{i} + e_2\hat{j} + e_3\hat{k}$ is the unit vector along any line, then e_1, e_2, e_3 are the direction cosines of the line.

7. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of the two lines, then the acute angle θ between the lines is given by $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$.

8. If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of the two lines then the acute angle θ between the lines is given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\cdot\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}\right|.$$

9. Two lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are

(i) perpendicular, if $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(ii) parallel, if $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$.

10. Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are

(i) perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

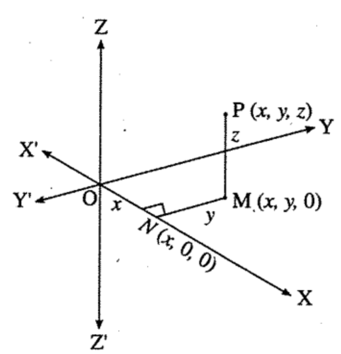
(ii) parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

INTRODUCTION

We know that an equation of a line in the plane is the general equation of the first degree, i.e., $ax + by + c = 0$, where a, b, c are real numbers and a, b both not zero simultaneously. The inclination of this line determines its direction. In this chapter, we shall study to determine the direction of a line in space.

We have studied that a point in space can be represented by means of ordered triplet (x, y, z) with reference to the three coordinate axes. Further we have seen that, if P is (x, y, z) then the position vector of P with respect to the origin O is given by $\bar{p} = x\hat{i} + y\hat{j} + z\hat{k}$, where $\hat{i}$, $\hat{j}$, $\hat{k}$ are the unit vectors along X-, Y-, Z-axes respectively.

Let us review these concepts which will be used in this chapter.



Consider the XY-plane bounded by the X- and Y-axes. Draw a line ZOZ′ perpendicular to the XY-plane and passing through the origin O.

4. If P is (x, y, z) then position vector of P with respect to the origin O is given by $\bar{p} = \overline{OP} = x\hat{i} + y\hat{j} + z\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along X, Y, Z-axes respectively, and

$$|\bar{p}| = |\overline{OP}| = l(OP) = \sqrt{x^2 + y^2 + z^2}.$$

Distance between P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) :

We have, $\overline{OP} = \bar{p} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and

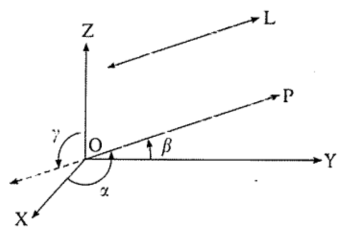
$$\overline{OQ} = \bar{q} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\therefore \overline{PQ} = \bar{q} - \bar{p} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\therefore |\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\therefore d(PQ) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$



$\therefore l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$.

From the above it is clear that parallel lines have the same direction cosines.

Also $0° \leqslant \alpha < 180°$, $0° \leqslant \beta < 180°$, $0° \leqslant \gamma < 180°$.

Theorem 1 :

If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.

Notes :

1. A line may have infinitely many direction ratios.
2. If P(x_1, y_1, z_1) and Q(x_2, y_2, z_2), then the direction ratios of the line PQ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Relation between direction ratios and direction cosines :

$$l = \frac{\pm a}{\sqrt{a^2 + b^2 + c^2}} \quad m = \frac{\pm b}{\sqrt{a^2 + b^2 + c^2}} \quad n = \frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}.$$

3. Parallel lines have proportional direction ratios and same direction cosines.

4. Let A(x_1, y_1, z_1) be the point on a line whose direction ratios are a, b, c. Let P(x, y, z) be any point on the line which is at a distance of d units from the point A.

   Then $a = x - x_1$, $b = y - y_1$ and $c = z - z_1$

   If l, m, n are the direction cosines of the line, then

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{a}{d}$$

$$m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{b}{d}$$

and $n = \pm\dfrac{c}{\sqrt{a^2 + b^2 + c^2}} = \pm\dfrac{c}{d}$

where

$$d = \sqrt{a^2 + b^2 + c^2} = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}$$

$$\therefore a = \pm ld, \; b = \pm md, \; c = \pm nd$$

$$\therefore x - x_1 = \pm ld, \; y - y_1 = \pm md, \; z - z_1 = \pm nd$$

$$\therefore x = x_1 \pm ld, \; y = y_1 \pm md, \; z = z_1 \pm nd$$

Hence, the coordinates of the points on the line which are at a distance of d units from the point A(x_1, y_1, z_1) are $(x_1 \pm ld, \; y_1 \pm md, \; z_1 \pm nd)$, where l, m, n are the direction cosines of the line.

Find the direction ratios of a line perpendicular to the two lines whose direction ratios are

(i) $-2, 1, -1$ and $-3, -4, 1$

Let $\bar{a}$ and $\bar{b}$ be the vectors along the lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$ respectively.

$\therefore \bar{a} = -2\hat{i} + \hat{j} - \hat{k}$ and $\bar{b} = -3\hat{i} - 4\hat{j} + \hat{k}$

A vector perpendicular to both $\bar{a}$ and $\bar{b}$ is given by

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ -3 & -4 & 1 \end{vmatrix}$$

$$= (1 - 4)\hat{i} - (-2 - 3)\hat{j} + (8 + 3)\hat{k}$$

$$= -3\hat{i} + 5\hat{j} + 11\hat{k}$$

$\therefore$ the direction ratios of the required line are $-3, 5, 11$.

3. The direction ratios of $\overline{AB}$ are $-2, 2, 1$. If $A \equiv (4, 1, 5)$ and $l(AB) = 6$ units, find the coordinates of B.

Solution : The direction ratio of $\overline{AB}$ are $-2, 2, 1$.

$\therefore$ the direction cosines of $\overline{AB}$ are

$$l = \frac{-2}{\sqrt{(-2)^2 + 2^2 + 1^2}} = \frac{-2}{3},$$

$$m = \frac{2}{\sqrt{(-2)^2 + 2^2 + 1^2}} = \frac{2}{3},$$

$$n = \frac{1}{\sqrt{(-2)^2 + 2^2 + 1^2}} = \frac{1}{3}.$$

i.e., $l = -\frac{2}{3}$, $m = \frac{2}{3}$, $n = \frac{1}{3}$

The coordinates of the points which are at a distance of d units from the point (x_1, y_1, z_1) are given by $(x_1 \pm ld, y_1 \pm md, z_1 \pm nd)$

Here, $x_1 = 4$, $y_1 = 1$, $z_1 = 5$, $d = 6$, $l = -\frac{2}{3}$, $m = \frac{2}{3}$, $n = \frac{1}{3}$

$\therefore$ the coordinates of the required points are

$$\left(4 \pm \left(-\frac{2}{3}\right)6, 1 \pm \frac{2}{3}(6), 5 \pm \frac{1}{3}(6)\right)$$

i.e., $(4 - 4, 1 + 4, 5 + 2)$ and $(4 + 4, 1 - 4, 5 - 2)$

i.e., $(0, 5, 7)$ and $(8, -3, 3)$.

ANGLE BETWEEN TWO LINES

Theorem 2 :

The acute angle θ between the two lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 is given by $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$.

Remark :

If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of the two lines whose direction cosines are l_1, m_1, n_1 and l_2, m_2, n_2 respectively, then

$$l_1 = \frac{\pm a_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}, \quad m_1 = \frac{\pm b_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}},$$

$$n_1 = \frac{\pm c_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$$

$$l_2 = \frac{\pm a_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}, \quad m_2 = \frac{\pm b_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}},$$

$$n_2 = \frac{\pm c_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\therefore$ the acute angle θ between the lines is given by

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$$

i.e., $$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

If θ is not mentioned as acute, then

$$\cos\theta = \pm\left(\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

Condition for parallel and perpendicular lines :

1. If two lines are perpendicular then $\theta = 90°$

   $\therefore \cos\theta = 0 \quad \therefore l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

   where l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of the lines.

   If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of the lines, then these lines are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$.

2. If the two lines are parallel then they make same angles with X-, Y-, Z- axes respectively. Therefore their direction cosines are same.

   Hence, if the two lines have their direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 respectively, then these lines are parallel, if

   $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$

   If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of the lines, then the above condition takes the form

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

   Thus the direction ratios are proportional, if the lines are parallel.

LINE

IMPORTANT FORMULAE

1. The vector equation of the line in parametric form passing through the point $A(\bar{a})$ and parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$.
2. The vector equation of the line in non-parametric form passing through the point $A(\bar{a})$ and parallel to $\bar{b}$ is $\bar{r}\times\bar{b}=\bar{a}\times\bar{b}$.
3. The equation of the line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$.
4. The coordinates of any point on the line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\lambda$ (say) are of the form $(x_1+a\lambda, y_1+b\lambda, z_1+c\lambda)$.
5. The vector equation of the line in parametric form passing through the points $A(\bar{a})$ and $B(\bar{b})$ is $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$.
6. The non-parametric vector equation of the line passing through the points $A(\bar{a})$ and $B(\bar{b})$ is $(\bar{r}-\bar{a})\times(\bar{b}-\bar{a})=\bar{0}$.
7. The equation of the line passing through the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$.
8. The length of the perpendicular from the point $P(a,b,c)$ to the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is given by
$$\sqrt{[(a-x_1)^2+(b-y_1)^2+(c-z_1)^2]-[(a-x_1)l+(b-y_1)m+(c-z_1)n]^2}$$
where l, m, n are the direction cosines of the line.
9. The length of perpendicular from the point $P(\bar{\alpha})$ to the line $\bar{r}=\bar{a}+\lambda\bar{b}$ is given by
$$\sqrt{|\bar{\alpha}-\bar{a}|^2-\left[\frac{(\bar{\alpha}-\bar{a})\cdot\bar{b}}{|\bar{b}|}\right]^2}$$
10. Two lines l_1 and l_2 which do not lie in the same plane are said to be **skew lines**, i.e., two lines in space are said to be skew if they are neither parallel nor intersecting.
11. The shortest distance between two skew lines l_1 and l_2 is the length of the segment AB with A lying on l_1 and B lying on l_2 such that AB is perpendicular to both l_1 and l_2.
12. The shortest distance between the lines $\bar{r}=\bar{a}_1+\lambda\bar{b}_1$ and $\bar{r}=\bar{a}_2+\mu\bar{b}_2$ is given by
$$d=\left|\frac{(\bar{b}_1\times\bar{b}_2)\cdot(\bar{a}_2-\bar{a}_1)}{|\bar{b}_1\times\bar{b}_2|}\right|$$
13. The shortest distance between the lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ is given by
$$d=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-l_1n_2)^2+(l_1m_2-l_2m_1)^2}}\right|$$
14. The distance between the parallel lines $\bar{r}=\bar{a}_1+\lambda\bar{b}$ and $\bar{r}=\bar{a}_2+\mu\bar{b}$ is given by
$$d=\left|\frac{(\bar{a}_2-\bar{a}_1)\times\bar{b}}{|\bar{b}|}\right|=|(\bar{a}_2-\bar{a}_1)\times\hat{b}|.$$

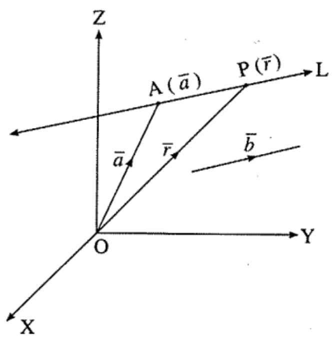
VECTOR EQUATION OF THE LINE PASSING THROUGH A GIVEN POINT AND PARALLEL TO THE GIVEN VECTOR :

Theorem 1 :

The vector equation of a line passing through the point A $(\bar{a})$ and parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar.

Proof :

Let L be a line in space passing through the point A$(\bar{a})$ and parallel to vector $\bar{b}$.



Let P be any point on the line (other than A) whose position vector is $\bar{r}$ w.r.t O.

Since $\overline{AP}$ is parallel to $\bar{b}$

$\therefore$ $\overline{AP} = \lambda\bar{b}$, where λ is a scalar

$\therefore$ $\bar{r}-\bar{a}=\lambda\bar{b}$

$\therefore$ $\bar{r}=\bar{a}+\lambda\bar{b}$

This is the *vector form* of the equation of the line.

Notes :

1. The equation $\bar{r}=\bar{a}+\lambda\bar{b}$ is the vector equation of a line in parametric form, where λ is a parameter.
2. Since vector $\overline{AP}$ is parallel to $\bar{b}$, $\overline{AP}\times\bar{b}=\bar{0}$

   $\therefore$ $(\bar{r}-\bar{a})\times\bar{b}=\bar{0}$ **OR**

   $\bar{r}\times\bar{b}=\bar{a}\times\bar{b}$

   This is called vector equation of a line in non-parametric form.
3. Let A(x_1, y_1, z_1) be the given point and a_1, b_1, c_1 be the direction ratios of the line.

   Let P(x, y, z) be any point on the line.

   Then, $\bar{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$, $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$

   and $\bar{b}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$.

   Putting these values in $\bar{r}=\bar{a}+\lambda\bar{b}$, we get,

   $x\hat{i}+y\hat{j}+z\hat{k}=(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+\lambda(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})$

   $=(x_1+\lambda a_1)\hat{i}+(y_1+\lambda b_1)\hat{j}+(z_1+\lambda c_1)\hat{k}$

   Comparing the coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$ on both the sides, we get,

   $x=x_1+\lambda a_1$, $y=y_1+\lambda b_1$ and $z=z_1+\lambda c_1$

   These are called the **parametric equations** of the line

   $\therefore$ $\dfrac{x-x_1}{a_1}=\lambda$, $\dfrac{y-y_1}{b_1}=\lambda$ and $\dfrac{z-z_1}{c_1}=\lambda$

   $\therefore$ $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$

   This is the **cartesian form** of the equation of the line passing through the point A(x_1, y_1, z_1) and having direction ratios a_1, b_1, c_1.

   This is also called the **symmetric form** of the equation of the line.
4. The equation of the line whose direction cosines are l, m, n and passing through the point (x_1, y_1, z_1) is $\dfrac{x-x_1}{l}=\dfrac{y-y_1}{m}=\dfrac{z-z_1}{n}$.

   The coordinates of any point on the line $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}=\lambda$ are $(x_1+a_1\lambda, y_1+b_1\lambda, z_1+c_1\lambda)$.

   Remark :

   We write the equation of the line in symmetric form, if none of a_1, b_1, c_1 is zero. If any one of a_1, b_1, c_1 is zero, then corresponding to that we use the parametric form.

   For example : Equation of the line passing through the point (x_1, y_1, z_1) and having direction ratios $a_1(=0)$, b_1, c_1 is written as follows :

Consider the symmetric form of the equation of line which is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$.

Dropping $\frac{x-x_1}{a_1}$, we get,

$$\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$$

Now, consider $\frac{x-x_1}{a_1}$.

The corresponding parametric form is

$x=x_1+a_1\lambda$, i.e., $x=x_1+0\cdot\lambda=x_1$

Hence, the equation of the line is

$$x=x_1,\ \frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}.$$

5. The vector equation of a line passing through the origin and parallel to the vector $\bar{b}$ is

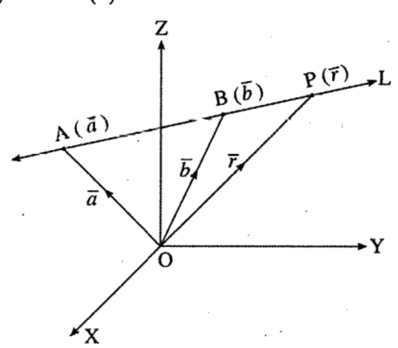
$\bar{r}=\lambda\bar{b}$ ... $[\because \bar{a}=\bar{0}]$

VECTOR EQUATION OF THE LINE PASSING THROUGH TWO GIVEN POINTS :

Theorem 2 :

The vector equation of the line passing through the points A $(\bar{a})$ and B $(\bar{b})$ is $\bar{r}=(1-\lambda)\bar{a}+\lambda\bar{b}$, where λ is a scalar.

Proof : Let L be the line in space passing through the points A $(\bar{a})$ and B $(\bar{b})$.



Let P be any point on the line (other than A and B) whose position vector is $\bar{r}$ w.r.t. O.

Since A, P, B are collinear,

$\therefore$ there exist scalar λ such that

$\overline{AP}=\lambda\overline{AB}$

$\therefore\ \bar{r}-\bar{a}=\lambda(\bar{b}-\bar{a})$

$\therefore\ \bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$

$\therefore\ \bar{r}=\bar{a}+\lambda\bar{b}-\lambda\bar{a}$

Notes :

1. The equation $\bar{r}=(1-\lambda)\bar{a}+\lambda\bar{b}$ is called vector equation of a line in parametric form passing through two points, where λ is a parameter.
2. Since $\overline{AP}$ and $\overline{AB}$ are collinear vectors, $\overline{AP}\times\overline{AB}=\overline{O}$

   i.e., $(\bar{r}-\bar{a})\times(\bar{b}-\bar{a})=\bar{0}$.

   This is called the vector equation of the line in non-parametric form.
3. Let $A\equiv(x_1, y_1, z_1)$ and $B\equiv(x_2, y_2, z_2)$ be the given points and $P\equiv(x, y, z)$ be any point on the line.

   $\therefore\ \bar{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$, $\bar{b}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ and

   $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$

   Putting these values in $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$, we get,

   $$x\hat{i}+y\hat{j}+z\hat{k}=(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+\lambda[(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})]$$

   $$\therefore\ (x\hat{i}+y\hat{j}+z\hat{k})-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})=\lambda[(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}]$$

   $$\therefore\ (x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k}=\lambda(x_2-x_1)\hat{i}+\lambda(y_2-y_1)\hat{j}+\lambda(z_2-z_1)\hat{k}$$

   $\because\ \hat{i}, \hat{j}, \hat{k}$ are non-zero and non-coplanar.

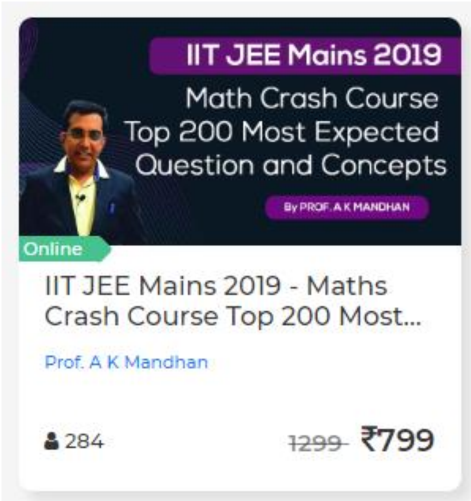
   $\therefore$ by comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$ on both the sides, we get,

   $x-x_1=\lambda(x_2-x_1)$, $y-y_1=\lambda(y_2-y_1)$ and $z-z_1=\lambda(z_2-z_1)$

   $$\therefore\ \frac{x-x_1}{x_2-x_1}=\lambda,\ \frac{y-y_1}{y_2-y_1}=\lambda \text{ and } \frac{z-z_1}{z_2-z_1}=\lambda$$

   $$\therefore\ \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

   This is the **cartesian form** of the equation of the line passing through the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.



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Bernoulli Trials For IIT JEE Mains and Advanced

1. The p.m.f. of binomial distribution is given as :

$$P[X=x]=p(x)={}^nC_x p^x q^{n-x}, \quad x=0,1,2,\ldots,n \quad 0<p<1,\ q=1-p$$
$$=0, \quad \text{otherwise}$$

2. Mean $= E(X) = np$, Variance $= V(X) = npq$
3. Mean $>$ Variance, i.e. $np > npq$
4. The p.m.f. of Bernoulli distribution is given as :

$$P[X=x]=\begin{cases} p^x q^{1-x}, & x=0,\ 1 \\ & 0<p<1,\ q=1-p \\ 0, & \text{otherwise} \end{cases}$$

$E(X)=p$, Var $(X)=pq$

$$P[X=x]={}^nC_x p^x q^{n-x}, \qquad 0\leqslant x\leqslant n$$

This is known as Binomial distribution.

The binomial distribution is used whenever each of the following is satisfied :

1. Trials of experiment are Bernoulli trials.
2. The performance of a Bernoulli trial results in an outcome that can be classified either as a success or a failure.
3. The number of trials (n) is finite.
4. All n trials are independent.
5. The probability of success is constant from trial to trial, i.e. the trials are repeated under identical conditions. Further, if $P(\text{success})=p$ and $P(\text{failure})=q$, then $p+q=1,\ 0<p<1$.
6. The random variable X denotes the number of successes x in n independent Bernoulli trials.

Notations :

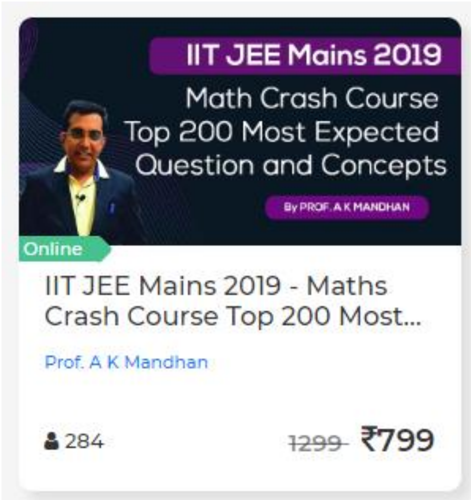
1. $X \sim B(n,p)$ denotes that X follows Binomial Distribution with parameters n and p.
2. If $X \sim B(n,p)$, then $Y=(n-X)\sim B(n,q)$.

The mean of r.v. X denoted as $E(X)$ is given by $E(X)=np$

The Variance of r.v. X denoted by Var (X) or $\sigma_x{}^2$ is given by Var $(X)=npq$

The standard deviation of r.v. X is defined as the positive square root of Var (X). Hence, the standard deviation of r.v. X denoted by SD(X) or σ_x is given by

$$SD(X)=\sqrt{\text{Var}(X)}=\sqrt{npq}.$$



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PLANE

IMPORTANT FORMULAE

1. The normal form of the equation of plane is

(i) $\bar{r}\cdot\hat{n}=p$ (vector form), where $\hat{n}$ is the unit vector along the normal and p is the distance of the plane from the origin.

(ii) $lx+my+nz=p$ (cartesian form), where l, m, n are the direction cosines of normal to the plane and p is the length of the normal from the origin.

2. Equation of the plane in vector form passing through the point A $(\bar{a})$ and perpendicular to the vector $\bar{n}$ is
$(\bar{r}-\bar{a})\cdot\bar{n}=0$ **OR** $\bar{r}\cdot\bar{n}=\bar{a}\cdot\bar{n}$.

3. Every equation of the form $\bar{r}\cdot\bar{t}=q$ represents a plane.

4. Any linear equation in x, y, z, i.e., $ax+by+cz+d=0$ represents a plane.

5. The length of perpendicular from the origin to the plane $ax+by+cz+d=0$ is
$\left|\dfrac{d}{\sqrt{a^2+b^2+c^2}}\right|$ and the direction cosines of the normal to the plane are

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{b}{\sqrt{a^2+b^2+c^2}},\ \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

6. Equation of the plane passing through the point (x_1, y_1, z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where a, b, c are the direction ratios of the normal to the plane.

7. The vector equation of the plane passing through the point A$(\bar{a})$ and parallel to the two non-zero vectors $\bar{b}$ and $\bar{c}$ is
$\bar{r}\cdot(\bar{b}\times\bar{c})=\bar{a}\cdot(\bar{b}\times\bar{c})$ **OR** $[\bar{r}\ \bar{b}\ \bar{c}]=[\bar{a}\ \bar{b}\ \bar{c}]$
The parametric equation of the plane is
$\bar{r}=\bar{a}+\lambda\bar{b}+\mu\bar{c}$, where λ and μ are parameters.

8. The vector equation of the plane passing through the three non-collinear points A $(\bar{a})$, B $(\bar{b})$ and C $(\bar{c})$ is
$\bar{r}\cdot(\overline{AB}\times\overline{AC})=\bar{a}\cdot(\overline{AB}\times\overline{AC})$,
where $\overline{AB}=\bar{b}-\bar{a}$ and $\overline{AC}=\bar{c}-\bar{a}$.

9. The equation of the plane parallel to the plane $\bar{r}\cdot\bar{n}=p$ is $\bar{r}\cdot\bar{n}=p_1$.

10. The angle θ between the planes $\bar{r}\cdot\bar{n}_1=p_1$ and $\bar{r}.\bar{n}_2=p_2$ is given by $\cos\theta=\dfrac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}_1|\cdot|\bar{n}_2|}$.

11. If $\bar{n}_1=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\bar{n}_2=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

$$\cos\theta=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}$$

12. The planes $\bar{r}\cdot\bar{n}_1=p_1$ and $\bar{r}\cdot\bar{n}_2=p_2$, where $\bar{n}_1=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\bar{n}_2=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are

(i) parallel, if $\dfrac{a_1}{b_1}=\dfrac{a_2}{b_2}=\dfrac{a_3}{b_3}$

(ii) perpendicular, if $a_1b_1+a_2b_2+a_3b_3=0$, i.e., $\bar{n}_1\cdot\bar{n}_2=0$.

13. The angle θ between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}\cdot\bar{n}=p$ is given by

$$\sin\theta=\frac{\bar{b}\cdot\bar{n}}{|\bar{b}|\cdot|\bar{n}|}.$$

14. The angle θ between the line $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ and a plane $ax+by+cz+d=0$ is given by

$$\sin\theta=\frac{aa_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{a_1^2+b_1^2+c_1^2}}.$$

15. The length of perpendicular from the point $A(\bar{a})$ to the plane $\bar{r}\cdot\bar{n}=p$ is $\frac{|\bar{a}\cdot\bar{n}-p|}{|\bar{n}|}$.

16. The distance of the plane $\bar{r}\cdot\bar{n}=q$ from the origin is $\frac{q}{|\bar{n}|}$.

17. The length of perpendicular from the point $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is given by $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$.

18. Two lines $\bar{r}=\overline{a_1}+\lambda\overline{b_1}$ and $\bar{r}=\overline{a_2}+\mu\overline{b_2}$ are coplanar, if $\overline{a_1}\cdot(\overline{b_1}\times\overline{b_2})=\overline{a_2}\cdot(\overline{b_1}\times\overline{b_2})$ and the equation of the plane containing them is $\bar{r}\cdot(\overline{b_1}\times\overline{b_2})=\overline{a_1}\cdot(\overline{b_1}\times\overline{b_2})$ **OR** $\bar{r}\cdot(\overline{b_1}\times\overline{b_2})=\overline{a_2}\cdot(\overline{b_1}\times\overline{b_2})$.

19. The lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

and the equation of the plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0 \quad \textbf{OR}$$

$$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0.$$

20. The line $\bar{r}=\bar{a}+\lambda\bar{b}$ lies in the plane $\bar{r}\cdot\bar{n}=p$, if $\bar{a}\cdot\bar{n}=p$ and $\bar{b}\cdot\bar{n}=0$.

21. The equation of the plane passing through the intersection of the planes $\bar{r}\cdot\overline{n_1}=p_1$ and $\bar{r}\cdot\overline{n_2}=p_2$ is $\bar{r}\cdot(\overline{n_1}+\lambda\overline{n_2})=p_1+\lambda p_2$.

22. The equation of the plane passing through the intersection of the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is $(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$, where λ is a parameter.

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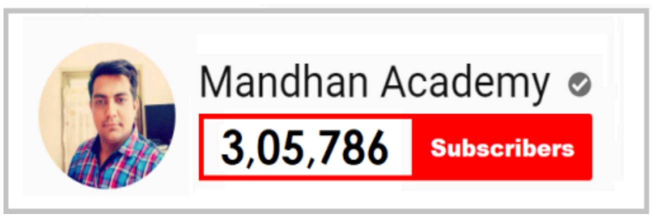


Trigonometry Formulas for Competitive Exams

1. **Principal Solutions:**
The solutions in the interval $[0, 2\pi]$ are called principal solutions.

2. **General Solutions:**

	Trigonometric equation	General solution
i.	$\sin\theta = 0$	$\theta = n\pi, n \in Z$
ii.	$\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}, n \in Z$
iii.	$\tan\theta = 0$	$\theta = n\pi, n \in Z$
iv.	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n\alpha, n \in Z$
v.	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha, n \in Z$
vi.	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha, n \in Z$
vii.	$\left.\begin{array}{l}\sin^2\theta = \sin^2\alpha \\ \cos^2\theta = \cos^2\alpha \\ \tan^2\theta = \tan^2\alpha\end{array}\right\}$	$\theta = n\pi \pm \alpha, n \in Z$

3. **Sine rule:**
In ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

4. **Cosine rule:**
In ΔABC,
i. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
ii. $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$
iii. $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

5. **Projection rule:**
In ΔABC,
i. $a = b\cos C + c\cos B$
ii. $b = c\cos A + a\cos C$
iii. $c = a\cos B + b\cos A$

6. **Half Angled Formulae:**
a. In ΔABC, if $a + b + c = 2s$ then
i. $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
ii. $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$
iii. $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

b. In ΔABC,
i. $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$
ii. $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$
iii. $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

c. In ΔABC,
i. $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
ii. $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$
iii. $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

7. **For ΔABC, its area is given by**
i. $\frac{1}{2}ab\sin C$
ii. $\frac{1}{2}bc\sin A$
iii. $\frac{1}{2}ac\sin B$

8. **Hero's Formulae:**
If a, b, c are the lengths of the sides BC, CA & AB respectively, Then area of ΔABC is
$= \sqrt{s(s-a)(s-b)(s-c)}$
where, $2s = a + b + c$

9. **Napier's Analogies:**
In ΔABC,
i. $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$
ii. $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$
iii. $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$

Inverse Trigonometric Functions

1. If $y = \sin x$, then $x = \sin^{-1}y$.
Similarly, for other T-functions.

2. Domain and Range of Inverse T-functions:

	y	Domain	Range
i.	$\sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
ii.	$\text{cosec}^{-1}x$	$x \le -1$, $x \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
iii.	$\cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$
iv.	$\sec^{-1}x$	$x \le -1$, $x \ge 1$	$0 \le y \le \pi, y \ne \frac{\pi}{2}$
v.	$\tan^{-1}x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y \le \frac{\pi}{2}$
vi.	$\cot^{-1}x$	$-\infty < x < \infty$	$0 < y < \pi$

3. i. $\sin^{-1}(\sin\theta) = \theta$ if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
   and $\sin(\sin^{-1}x) = x$ if $-1 \le x \le 1$
   ii. $\cos^{-1}(\cos\theta) = \theta$ if $0 \le \theta \le \pi$
   and $\cos(\cos^{-1}x) = x$ if $-1 \le x \le 1$
   iii. $\tan^{-1}(\tan\theta) = \theta$ if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
   and $\tan(\tan^{-1}x) = x$ if $-\infty < x < \infty$
   iv. $\cot^{-1}(\cot\theta) = \theta$ if $0 \le \theta < \pi$
   $\cot(\cot^{-1}x) = x$ if $x \in R$

4. i. $\sin^{-1}x = \text{cosec}^{-1}\frac{1}{x}$
   ii. $\cos^{-1}x = \sec^{-1}\frac{1}{x}$
   iii. $\tan^{-1}x = \cot^{-1}\frac{1}{x}$
   iv. $\cot^{-1}x = \tan^{-1}\frac{1}{x}, x > 0$
   $= \tan^{-1}\left(\frac{1}{x}\right) + \pi$ if $x < 0$

5. i. $\sin^{-1}(-x) = -\sin^{-1}x$
   ii. $\tan^{-1}(-x) = -\tan^{-1}x$
   iii. $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$
   iv. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
   v. $\sec^{-1}(-x) = \pi - \sec^{-1}x$
   vi. $\cot^{-1}(-x) = \pi - \cot^{-1}x$

6. i. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, (-1 \le x \le 1)$
   ii. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, (x \in R)$
   iii. $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$ $(x \le -1$ or $x > 1)$

7. i. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$
   ii. $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$
   if $xy > 1$
   iii. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$
   where $x > 0, y > 0$

8. i. $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$
   $= \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$

9. i. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
   ii. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
   iii. $3\tan^{-1}x = \tan^{-1}\frac{3x - x^2}{1 - 3x^2}$

10. i. $2\sin^{-1}x = \sin^{-1}2x\sqrt{1-x^2}$
    ii. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

11. i. $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$
    $= \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$
    $= \text{cosec}^{-1}\frac{1}{x}$ $(0 \le x \le 1)$
    ii. $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$
    $= \text{cosec}^{-1}\frac{1}{\sqrt{1-x^2}} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$
    $= \sec^{-1}\frac{1}{x}$ $(0 \le x \le 1)$
    iii. $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$
    $= \text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x} = \sec^{-1}\sqrt{1+x^2}$
    $= \cot^{-1}\frac{1}{x}$

12. i. $\sin^{-1}x \pm \sin^{-1}y$
    $= \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$
    if $x, y \ge 0$ and $x^2 + y^2 \le 1$
    ii. $\sin^{-1}x \pm \sin^{-1}y$
    $= \pi - \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$
    if $x, y \ge 0$ and $x^2 + y^2 > 1$

13. i. $\cos^{-1}x \pm \cos^{-1}y$
$= \cos^{-1}[xy \pm \sqrt{1-x^2}\ \sqrt{1-y^2}\ ]$,
if $x, y > 0$ and $x^2 + y^2 \le 1$

ii. $\cos^{-1}x \pm \cos^{-1}y$
$= \pi - \cos^{-1}\left[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right]$,
if $x, y > 0$ and $x^2 + y^2 > 1$

14. i. $\cot^{-1}x \pm \cot^{-1}y = \cot^{-1}\left[\dfrac{xy \mp 1}{y \pm x}\right]$

Shortcuts

1. i. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$
$= \tan^{-1}\left[\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right]$

2. i. $\sqrt{f(0)}$ is always +ve
e.g $\sqrt{\cos^2\theta} = |\cos\theta|$ and not $\pm\cos\theta$

3. i. If $\cos A + \cos B + \cos C = \dfrac{3}{2}$, then the triangle is equilateral.
ii. If $\sin A + \sin B + \sin C = \dfrac{3\sqrt{3}}{2}$, then the triangle is equilateral.
iii. If $\tan A + \tan B + \tan C = 3\sqrt{3}$, then the triangle is equilateral.
iv. If $\cot A + \cot B + \cot C = \sqrt{3}$, then the triangle is equilateral.
v. If $\dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$, then the triangle is equilateral.
vi. If $\cos^2 A + \cos^2 B + \cos^2 C = 1$, then the triangle is right angled.

4. i. $\sin A + \sin B + \sin C$ is max, when $A = B = C$
ii. $\cos A + \cos B + \cos C$ is max, when $A = B = C$
iii. $\tan A + \tan B + \tan C$ is min, when $A = B = C$
iv. $\cot A + \cot B + \cot C$ is min, when $A = B = C$

5. i. $\sin\theta = 1 \Rightarrow \theta = (4n+1)\dfrac{\pi}{2}$
ii. $\cos\theta = 1 \Rightarrow \theta = 2n\pi$
iii. $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$
iv. $\cos\theta = 0 \Rightarrow \theta = (2n+1)\dfrac{\pi}{2}$

3.1 Trigonometric equations and the solutions

1. General solution of the equation $\cot\theta - \tan\theta = 2$ is
(A) $n\pi + \dfrac{\pi}{4}$ (B) $\dfrac{n\pi}{2} + \dfrac{\pi}{8}$
(C) $\dfrac{n\pi}{2} \pm \dfrac{\pi}{8}$ (D) $n\pi \pm \dfrac{\pi}{4}$

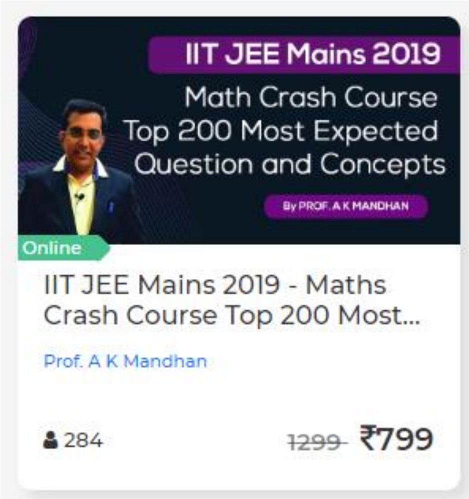
2. If $1 + \cot\theta = \operatorname{cosec}\theta$, then the general value of θ is
(A) $n\pi + \dfrac{\pi}{2}$ (B) $2n\pi - \dfrac{\pi}{2}$
(C) $2n\pi + \dfrac{\pi}{2}$ (D) $2n\pi \pm \dfrac{\pi}{2}$

3. If $3(\sec^2\theta + \tan^2\theta) = 5$, then the general value of θ is
(A) $2n\pi + \dfrac{\pi}{6}$ (B) $2n\pi \pm \dfrac{\pi}{6}$
(C) $n\pi \pm \dfrac{\pi}{6}$ (D) $n\pi \pm \dfrac{\pi}{3}$

4. The general value of θ satisfying the equation $\tan\theta + \tan\left(\dfrac{\pi}{2} - \theta\right) = 2$, is
(A) $n\pi \pm \dfrac{\pi}{4}$ (B) $n\pi + \dfrac{\pi}{4}$
(C) $2n\pi \pm \dfrac{\pi}{4}$ (D) $n\pi + (-1)^n\dfrac{\pi}{4}$

5. The solution of the equation $4\cos^2 x + 6\sin^2 x = 5$, is
(A) $x = n\pi \pm \dfrac{\pi}{2}$ (B) $x = n\pi \pm \dfrac{\pi}{4}$
(C) $x = n\pi \pm \dfrac{3\pi}{2}$ (D) $x = n\pi \pm \dfrac{3\pi}{4}$

6. If $\sin 3\alpha = 4\sin\alpha\ \sin(x+\alpha)\ \sin(x-\alpha)$, then $x =$
(A) $n\pi \pm \dfrac{\pi}{6}$ (B) $n\pi \pm \dfrac{\pi}{3}$
(C) $n\pi \pm \dfrac{\pi}{4}$ (D) $n\pi \pm \dfrac{\pi}{2}$



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Vectors Concepts for Competitive Exams

1. Two or more vectors are said to be collinear if they are parallel to the same line or along the same line irrespective of their magnitudes and directions.

2. If $\bar{a}$ and $\bar{b}$ are collinear, then $\bar{a} = m\bar{b}$, where m is a non-zero scalar.

3. Three points $A(\bar{a})$, $B(\bar{b})$, $C(\bar{c})$ are collinear if and only if $\overline{AB} \times \overline{AC} = \bar{0}$ or $\overline{AB}$ is expressed as the scalar multiple of $\overline{AC}$.

4. A set of vectors are said to be coplanar if they are parallel to the same plane or they lie on the same plane.

5. If $\bar{a}, \bar{b}, \bar{c}$ are coplanar vectors, then any one of them can be expressed as the linear combination of the other two.

6. If $\bar{a}, \bar{b}, \bar{c}$ are position vectors of the points A, B, C respectively, then the position vector of :
   (i) the point P dividing seg AB internally in the ratio $m : n$ is given by $\bar{p} = \dfrac{m\bar{b} + n\bar{a}}{m+n}$
   (ii) the point Q dividing seg AB externally in the ratio $m : n$ is given by $\bar{q} = \dfrac{m\bar{b} - n\bar{a}}{m-n}$
   (iii) midpoint M of seg AB is $\bar{m} = \dfrac{\bar{a}+\bar{b}}{2}$
   (iv) centroid G of ΔABC is $\bar{g} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$.

7. If $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

$$[\bar{a}\ \bar{b}\ \bar{c}] = \bar{a}\cdot(\bar{b}\times\bar{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

8. $[\bar{a}\ \bar{b}\ \bar{c}] = [\bar{b}\ \bar{c}\ \bar{a}] = [\bar{c}\ \bar{a}\ \bar{b}]$.

9. If any one of $\bar{a}, \bar{b}, \bar{c}$ is a zero vector, then $[\bar{a}\ \bar{b}\ \bar{c}] = 0$.

10. If any two of $\bar{a}, \bar{b}, \bar{c}$ are collinear or equal then $[\bar{a}\ \bar{b}\ \bar{c}] = 0$.

11. If $\bar{a}, \bar{b}, \bar{c}$ are the coterminus edges of a parallelopiped, then its volume $= [\bar{a}\ \bar{b}\ \bar{c}]$.

12. The vectors $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar if $[\bar{a}\ \bar{b}\ \bar{c}] = 0$.

13. Volume of tetrahedron A–BCD $= \dfrac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]$.

14. The points A, B, C, D are coplanar if $[\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$.

15. The medians of a triangle are concurrent. The point of concurrence is called the **centroid.**

16. The angle bisectors of a triangle are concurrent. The point of concurrence is called the **incentre.**

17. The altitudes of a triangle are concurrent. The point of concurrence is called the **orthocentre.**

18. The angle subtended on a semi-circle is the right angle.

19. The quadrilateral is a parallelogram if and only if its diagonals bisect each other.

20. The median of a trapezium is parallel to the parallel sides of the trapezium and the length of the median is half of the sum of the lengths of the parallel sides.

Scalars and Vectors :

1. A physical quantity which has magnitude only is called a **scalar** quantity. e.g. distance, mass, volume, temperature, etc.
2. A physical quantity which has magnitude and direction is called a **vector** quantity. e.g. displacement, velocity, acceleration, force, etc.

Vectors are usually denoted by $\overline{P}$, $\overline{F}$, $\overline{a}$, $\overline{AB}$, etc. and their magnitudes by $|\overline{P}|$, $|\overline{F}|$, $|\overline{a}|$, $|\overline{AB}|$ or simply P, F, a, AB respectively.

Types of Vectors :

1. **Zero Vector :** A vector whose magnitude is zero, is called the *zero vector* or the *null vector*. It is denoted by $\overline{0}$.
2. **Unit Vector :** A vector whose magnitude is one is called a *unit vector*.

   If $\hat{e}$ is the unit vector in the direction of the vector $\overline{a}$, then $\hat{e} = \frac{\overline{a}}{a}$, where a is the magnitude of the vector $\overline{a}$.
3. **Equal Vectors :** Two vectors are said to be equal, if they have the same magnitude and they are in the same direction.
4. **Negative of a Vector :** A vector having the same magnitude as that of a given vector $\overline{a}$ and the direction opposite to $\overline{a}$ is called the *negative of a vector* $\overline{a}$ and is denoted by $-\overline{a}$.

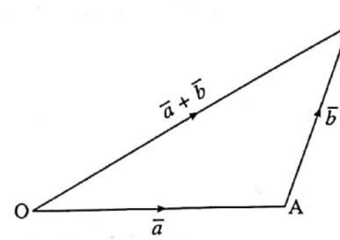
   The negative of the vector $\overline{AB}$ is $\overline{BA}$. Thus, $\overline{BA} = -\overline{AB}$ and $\overline{AB} = -\overline{BA}$ but $|\overline{AB}| = |\overline{BA}|$.
5. **Collinear Vectors :** Two or more vectors are said to be collinear, if their directions are same or opposite, irrespective of their magnitudes.

   Such vectors represent parallel lines, hence they are also called **parallel vectors.**
6. **Like or Unlike Vectors :** The vectors having same direction are called *like vectors* and the vectors having opposite directions are called *unlike vectors.*
7. **Coplanar Vectors :** A set of vectors which lie in the same plane or in parallel planes are called *coplanar vectors.*

The vectors which are not coplanar are called **non-coplanar vectors.**

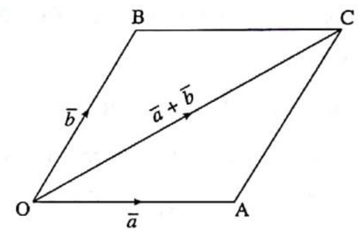
Algebra of Vectors :

1. Addition of Vectors :



Let $\overline{a}$ and $\overline{b}$ be two non-zero vectors represented by $\overline{OA}$ and $\overline{AB}$ respectively. Then the vector $\overline{OB}$ is said to represent the sum of vectors $\overline{a}$ and $\overline{b}$,
i.e., $\overline{OB} = \overline{OA} + \overline{AB} = \overline{a} + \overline{b}$.
This is called *Triangle Law of Vector Addition.*

2. Parallelogram Law of Vectors :



Let $\overline{a}$ and $\overline{b}$ be two non-zero vectors represented by $\overline{OA}$ and $\overline{OB}$ respectively. Complete the parallelogram OACB. Then the diagonal $\overline{OC}$ is said to represent the sum of vectors $\overline{a}$ and $\overline{b}$. If $\overline{OC} = \overline{c}$, then $\overline{OA} + \overline{OB} = \overline{OC}$, i.e., $\overline{a} + \overline{b} = \overline{c}$.
Thus, parallelogram law of vectors can be stated as : "The sum of two vectors $\overline{OA}$ and $\overline{OB}$ is represented in magnitude and direction by the diagonal of the parallelogram through O of which OA and OB are adjacent sides."

3. **Scalar Multiplication :** Let $\overline{a}$ be a vector and m be a real number, then $m\overline{a}$ is a vector which is called scalar multiple of $\overline{a}$.

   If $m > 0$, then $m\overline{a}$ is in the same direction of $\overline{a}$ and has magnitude ma.

   If $m = 0$, then $m\overline{a}$ is a zero vector.

   If $m < 0$, then $m\overline{a}$ is in the direction opposite to $\overline{a}$ and has magnitude ma.

Properties of vector addition and scalar multiplication :

Let $\bar{a}, \bar{b}, \bar{c}$ be the vectors and m, n are scalars. Then

1. $\bar{a}+\bar{b}=\bar{b}+\bar{a}$, *commutative law*
2. $\bar{a}+(\bar{b}+\bar{c})=(\bar{a}+\bar{b})+\bar{c}$, *associative law*
3. $\bar{a}+\bar{0}=\bar{0}+\bar{a}=\bar{a}$, where $\bar{0}$ is a zero vector
4. $\bar{a}+(-\bar{a})=(-\bar{a})+\bar{a}=\bar{0}$
5. $m(\bar{a}+\bar{b})=m\bar{a}+m\bar{b}$
6. $(m+n)\bar{a}=m\bar{a}+n\bar{a}$
7. $m(n\bar{a})=n(m\bar{a})=(mn)\bar{a}$.

Position vector of a point :

Let O be a fixed point and P be any point. Then the vector $\overline{OP}$ is called the position vector of the point P w.r.t. the fixed point O. This is generally denoted by $\bar{p}$.

1. If $\bar{a}$ and $\bar{b}$ be the position vectors of A and B w.r.t. the fixed point O respectively. Then $\overline{AB}=\bar{b}-\bar{a}$.
2. If P is (x, y, z) then position vector of P w.r.t. the origin O is given by
   $\bar{p}=\overline{OP}=x\hat{i}+y\hat{j}+z\hat{k}$ and $|\overline{OP}|=\sqrt{x^2+y^2+z^2}$
3. If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then
   (i) $\bar{a}+\bar{b}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})+(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})$
   $=(a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$
   (ii) $\bar{a}-\bar{b}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})-(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})$
   $=(a_1-b_1)\hat{i}+(a_2-b_2)\hat{j}+(a_3-b_3)\hat{k}$
   and
   (iii) $m\bar{a}=m(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$
   $=(ma_1)\hat{i}+(ma_2)\hat{j}+(ma_3)\hat{k}$.

Scalar product of two vectors :

If $\bar{a}$ and $\bar{b}$ are two vectors inclined at an angle θ, then their scalar product (denoted by $\bar{a}\cdot\bar{b}$) is defined by $\bar{a}\cdot\bar{b}=ab\cos\theta$

If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then

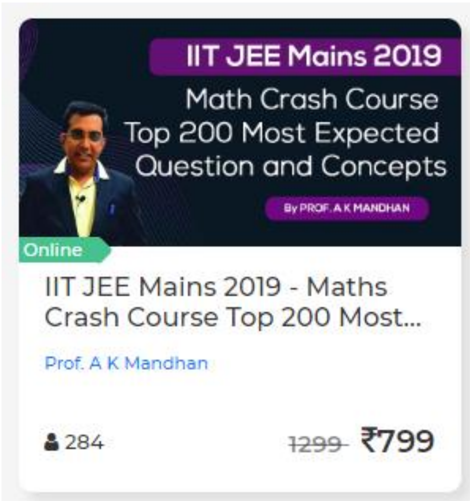
1. $\bar{a}\cdot\bar{b}=a_1b_1+a_2b_2+a_3b_3$
2. $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{a}$
3. $\bar{a}\cdot\bar{a}=a^2 \quad \therefore\ a=\sqrt{\bar{a}\cdot\bar{a}}$
4. Angle θ between $\bar{a}$ and $\bar{b}$ is given by $\cos\theta=\dfrac{\bar{a}\cdot\bar{b}}{ab}$
5. $\bar{a}$ is perpendicular to $\bar{b}$,
   if $\bar{a}\cdot\bar{b}=a_1b_1+a_2b_2+a_3b_3=0$
6. Projection of $\bar{a}$ on $\bar{b}=\dfrac{\bar{a}\cdot\bar{b}}{b}$
   Projection of $\bar{b}$ on $\bar{a}=\dfrac{\bar{a}\cdot\bar{b}}{a}$
7. Scalar product is distributive
   i.e., $\bar{a}\cdot(\bar{b}+\bar{c})=\bar{a}\cdot\bar{b}+\bar{a}\cdot\bar{c}$
8. $\bar{a}\cdot\bar{a}=1$, if $\bar{a}$ is the unit vector.

Vector product of two vectors :

Vector product of two vectors $\bar{a}$ and $\bar{b}$ which are inclined at an angle θ is denoted by $\bar{a}\times\bar{b}$ and is defined as $\bar{a}\times\bar{b}=ab\sin\theta\cdot\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\bar{a}$ and $\bar{b}$ such that $\bar{a}$, $\bar{b}$ and $\hat{n}$ form a right-handed set.

Results :

1. $\bar{a}\times\bar{b}=-\bar{b}\times\bar{a}$ but $|\bar{a}\times\bar{b}|=|\bar{b}\times\bar{a}|$
2. If $\bar{a}$ and $\bar{b}$ are non-zero vectors and $\bar{a}\times\bar{b}=\bar{0}$, then $\bar{a}$ is collinear (or parallel) to $\bar{b}$.
3. $\bar{a}\times\bar{a}=\bar{0}$
4. $\sin\theta=\dfrac{|\bar{a}\times\bar{b}|}{ab}$
5. Unit vectors perpendicular to both $\bar{a}$ and $\bar{b}=\dfrac{\pm(\bar{a}\times\bar{b})}{|\bar{a}\times\bar{b}|}$.
6. $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\bar{0}$
   $\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{i}=-\hat{k}$
   $\hat{j}\times\hat{k}=\hat{i},\ \hat{k}\times\hat{j}=-\hat{i}$
   $\hat{k}\times\hat{i}=\hat{j},\ \hat{i}\times\hat{k}=-\hat{j}$
7. If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then
   $$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$
8. If $\bar{a}$ and $\bar{b}$ represent the adjacent sides of a parallelogram, then the vector area of the parallelogram is $\bar{a}\times\bar{b}$ and its area is $|\bar{a}\times\bar{b}|$.
9. If $\bar{a}$ and $\bar{b}$ represent the diagonals of a parallelogram, then its area is $\frac{1}{2}|\bar{a}\times\bar{b}|$.
10. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are the position vectors of the vertices of ΔABC, then the vector area of ΔABC
    $=\frac{1}{2}(\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a})$.
11. Vector area of $\Delta ABC=\frac{1}{2}(\overline{AB}\times\overline{AC})$
    $\therefore$ area of $\Delta ABC=\frac{1}{2}|\overline{AB}\times\overline{AC}|$.
12. $(\bar{a}\times\bar{b})^2=a^2b^2-(\bar{a}\cdot\bar{b})^2$
    This is called *Lagrange's Identity*.



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