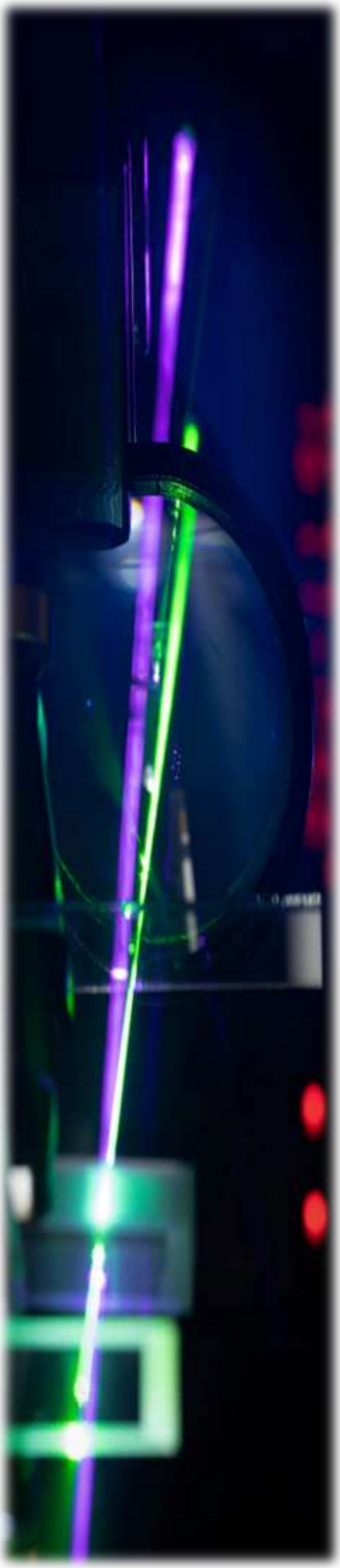
PH447
OPTICS AND SPECTROSCOPY LAB

FOURIER OPTICS



Optical Spatial Filtering – Applied Fourier Optics

Aim:

To understand fundamental optics principles of Babinet's Principle, Abbe's Principle, Kohler illumination, Fourier transformation and its applications.

Components:

LED and its controller, filter, irir, lenses, target with different objects, masks slide, beam splitter, tube lens, camera, viewing screen, computer (for image analysis).

Precautions:

1. Handle lens and slide components at their edges after wearing clean gloves.
2. DO NOT CHANGE the LED controller current settings.
3. DO NOT SATURATE the camera detector by keeping high exposure time or keeping LED light illuminating the camera for more than 20 min continuously.
4. DO NOT RUB the filters or the masks etching, handle at their edges and use the XY mount knob to gently move the object to different positions.
5. DO NOT ALTER the setup without prior permission.
6. DO NOT LEAN or place unnecessary weight on the working table.
7. Carefully cover the lenses and other components with covers before and after the experiment.
8. Please save the images in the screenshot folder by selecting the ⊞+PrtScr to save the image in the screenshot folder. Copy the photos later to your folder and save it accordingly.

Theory:

Source and detector form the polar ends of any optical setup but to connect them together to obtain meaningful results requires the understanding of the various fundamental mechanisms of optics and in this experiment, we shall see the various image transmission techniques and enhancing the structures to obtain desirable features.

The fundamental takeaway is **a lens is used as a Fourier transformer and how this relates to optical filtering and image editing**.

A two-dimensional Fourier transformation F of function f is presented as:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \cdot e^{-i2\pi(ux+vy)}\, dx\, dy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) \cdot e^{i2\pi(ux+vy)}\, du\, dv$$

When the Fourier transformation is applied again to $F(u, v)$, this results in the initial function $f(-x, -y)$ with reversed leading signs.

the coefficients in $F(u, v)$ are generally complex numbers. Accordingly, they contain a real and an imaginary component, or amplitude and phase information, that is

$F(u, v) = Fi(u, v) + i \cdot Fi(u, v) = |F(u, v,)| \cdot e^{i \cdot \varphi(u,v)}$

with the amplitude $|F(u, v,)|$ and the phase $\varphi(u, v) = \tan^{-1}(F_i(u, v)/F_i(u, v))$.

When we examine the Fourier plane behind the objective lens, the human eye perceives the amplitude but not the phase. This means that the brightness distribution alone does not contain the complete information of the diffracting structure

Applying the Fourier transformation twice results in the original image. This often leads to the misconception that a mask in the shape of the brightness distribution could be used as an object which would then project an image of the original object into the Fourier plane.

This assumption, however, is incorrect because the brightness distribution does not contain the necessary phase information. When an image of the diffraction pattern of a single slit is placed in the object plane, it does not produce an image of a slit in the Fourier plane.

The process of image formation (with or without magnification) in terms of a Fourier transform can be understood in following steps:

1. Fraunhofer diffraction as Fourier transform.
2. Image formation as two Fraunhofer diffraction processes occurring sequentially

Let us consider each of the steps in detail: -

1. Fraunhofer diffraction as Fourier transform

Consider a plane wave travelling along the z-axis and incident at z = 0 on an object (transparency) with amplitude transmission function f(x,y). The diffracted light is collected by a lens of focal length F situated at z=U.

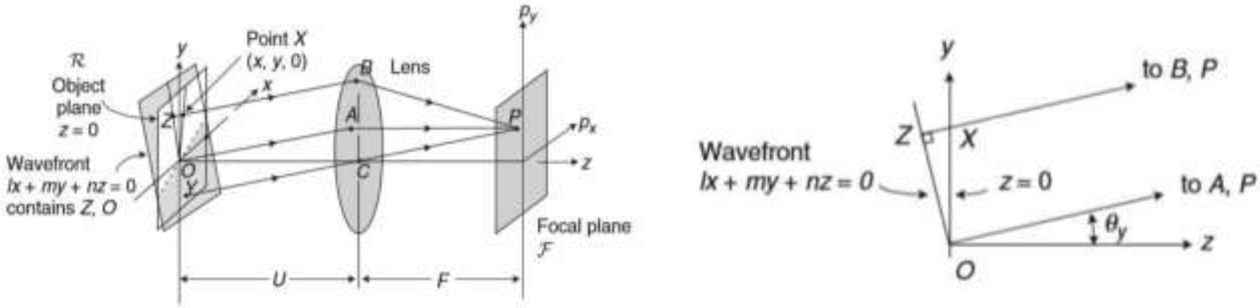
All light waves leaving the object in a particular angle are focused by the lens to a single point in the focal plane. For example, in the figure, *XB*, *OA*, *YC* are all focused at P. The amplitude of the light at P, *ΨP*, is therefore the sum of the amplitudes at X, O, Y, etc., each multiplied with their phase exp(ik_0*XBP*), etc.

Now the amplitude at X(x,y,0), is the amplitude of the incident wave, assumed unity, multiplied by the transmission function *f(x,y)*.

To calculate the optical path *XBP*, consider the wavefront normal to *XB*, *OA*;

$$\tilde{l}x + \tilde{m}y + \tilde{n}z = 0$$

Where, $\tilde{l}, \tilde{m}, \tilde{n}$ are the direction cosines



From Fermat's principle, the optical paths from the various points on a wavefront to its focus are all equal, i.e.

$$\text{OAP} = \text{ZBP}$$

And

$$\text{XBP} = \text{OAP} - \text{ZX} = \text{OAP} - (\tilde{l}x + \tilde{m}y)$$

The amplitude at P is therefore obtained by

$$\psi_P = e^{ik_0 OAP} \iint f(x,y) e^{-ik_0(\tilde{l}x+\tilde{m}y)}\, dxdy$$

$$\psi(u,v) = e^{ik_0 OAP} \iint f(x,y) e^{-i(ux+vy)}\, dxdy$$

Where u ≡ $\tilde{l}k_o$; v ≡ $\tilde{m}k_o$

Therefore, we see that Fraunhofer diffraction pattern amplitude is given by the two-dimensional Fourier transform of the object transmission function f(x,y).

In experiments, we usually measure the intensity, i.e.

$$|\psi(u,v)|^2 = \iint \left|f(x,y) e^{-i(ux+vy)}\, dx\, dy\right|^2$$

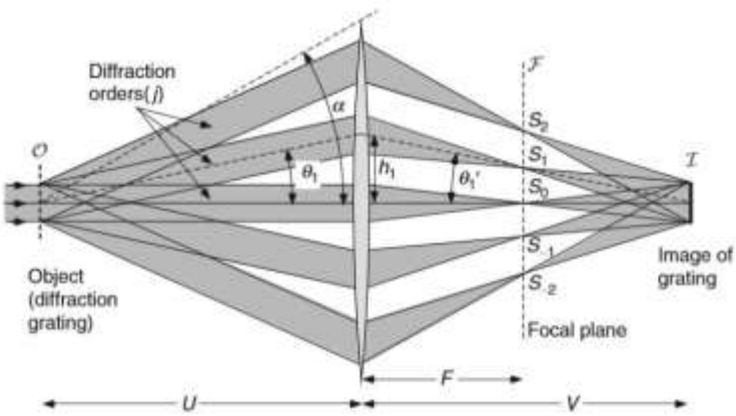
From the above equation, we see that the information about phase is lost. There are ways to retrieve it. However, when the object is at the focal plane before the lens, we obtain a true image.

2. Image formation as two Fraunhofer diffraction processes occurring sequentially

Consider the object to be a diffraction grating. As we have seen above, at focal plane F, we get the diffraction pattern which can be represented as a Fourier transform. From the focal plane, F the waves overlap again at image plane, I forming the image.

The diffraction orders S_2, S_1,...,S_{-2} behave like a set of equally spaced point sources and the image is their interference pattern. Thus, the process of **image formation appears to consist of two diffraction processes, applied sequentially**.*

(Diffraction is nothing but a set of interference phenomena that occur when you have a continuous region of emitters, generally forming a connected set, emitting in a way that causes nontrivial interference patterns in the far field).



The finest detail observable in the image is determined by the highest order of diffraction that is transmitted by the lens. In other words, the greater the extent of the Fraunhofer diffraction accessed by the imaging lens, the better the resolution. This is achieved by using as large an angular aperture as possible.

Optical Spatial Filtering:

Optical filtering makes use of the Fourier transformation that takes place between the two focal planes of a lens as illustrated above. The spatial period of the light is divided into segments and represented as intensity peaks in the Fourier plane. Similar to the frequencies in the temporal Fourier transformation:

$$v_x = \frac{x'}{\lambda \cdot z_0}$$

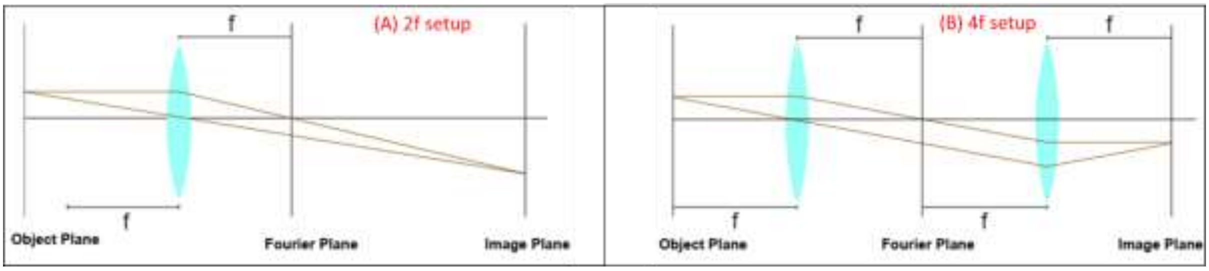
$$v_y = \frac{y'}{\lambda \cdot z_0}$$

are defined as spatial frequencies. The spatial frequencies correspond to the transformed coordinates in position space. Spatial frequencies are not frequencies in the conventional sense. Rather they designate the number of periods per units of length. Familiar examples of spatial frequencies are the wave number $k = 2\pi/\lambda$ of a mechanical or electromagnetic wave and the lattice constant g of an optical lattice.

When representing a structure or an image with the help of a lens**, high spatial frequencies (far away from the optical axis) can be assigned to fine structures and low spatial frequencies (close to the optical axis) to coarse structures**.

Optical filtering takes advantage of the Fourier transformation that takes place in the focal plane of a lens by manipulating the *Fourier transform of an image* rather than the image itself.

A simple lens setup demonstrates this principle. An object placed near the focal plane of a lens generates a real image on a screen, and an optical filter placed at the focal plane on the other side of the lens manipulates the Fourier transform of the image



Optical filters are usually diaphragms intended to block specific information in the Fourier plane to generate a desired effect in the image, for example to **soft-focus or sharpen an image**. Information changes in the Fourier plane appear directly on the screen. Such a setup with a single lens is called a 2f setup and is represented schematically in Figure A.
The reverse transformation from the Fourier plane to the image in the 2f setup takes place through the propagation of light from the focal plane of the lens to the screen.

To avoid large distances between the object plane and image plane, we can expand the 2f setup into a 4f setup which results in a shorter optical path. As shown in Figure B, the object plane in the 4f setup is equivalent to the front focal plane of the first lens. The rear focal plane of the first lens and the front focal plane of the second lens form the Fourier plane. The image plane is in the rear focal plane of the second lens. The reverse transformation from the Fourier plane is carried out by the second lens.

Kohler illumination:

If one wants to examine an object with a microscope, one generally has the problem that the illuminating light source itself has a certain extent and structure (e.g. coiled filament, LED chip), which leads to uneven illumination of the object, may expose the object to heat, and produces scattered light. Köhler illumination, named after its developer August Köhler, allows one to avoid these issues and is common practice in microscopy today.
The lighting system essentially consists of a collector lens for the light source, a condenser lens, and two diaphragms, the aperture diaphragm, and the field diaphragm.

Method:

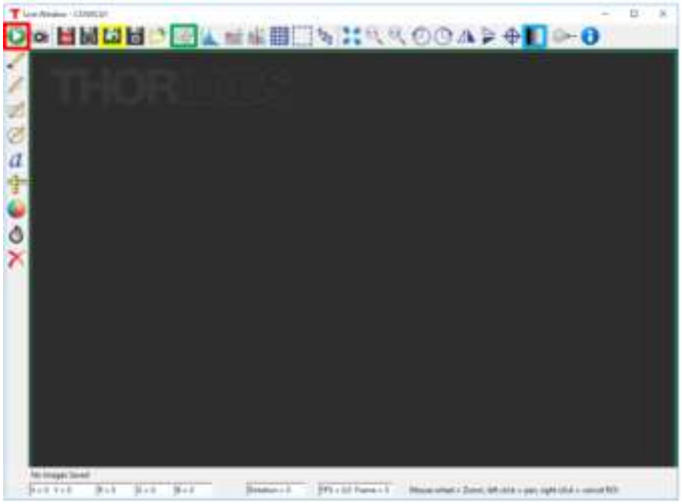
For the sake of convenience, the initial steps have been added in the appendix part and not be stressed upon, we will focus on the alignment, measurement and data analysis part as mentioned below

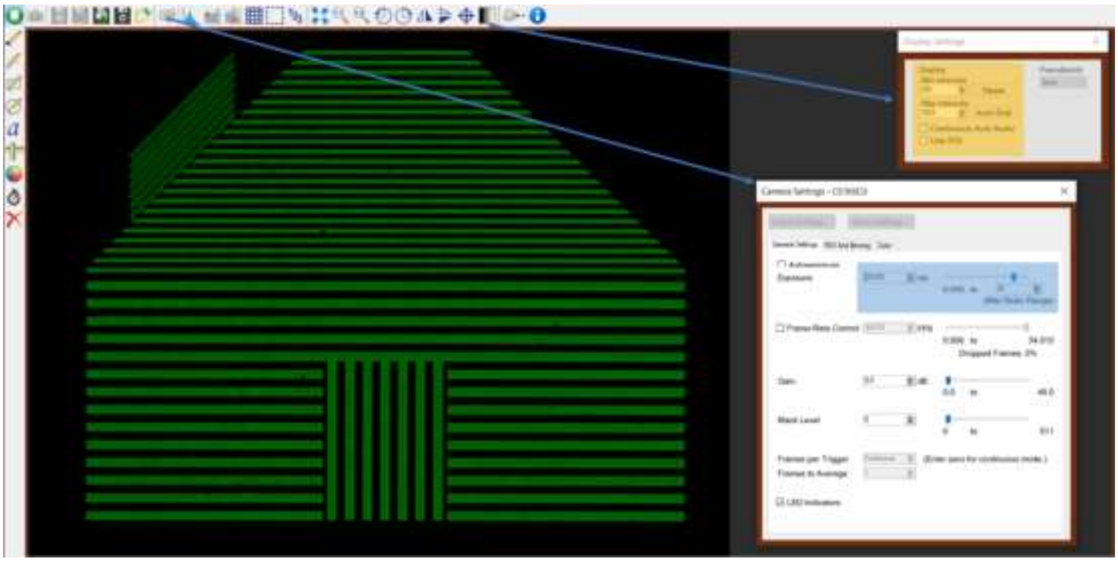
1. Check that all components are at a height of 10 cm using the LMR1AP alignment plate and a scale keeping on the respective element (measured from the center of the optical element to the breadboard).

LMR1AP alignment plate

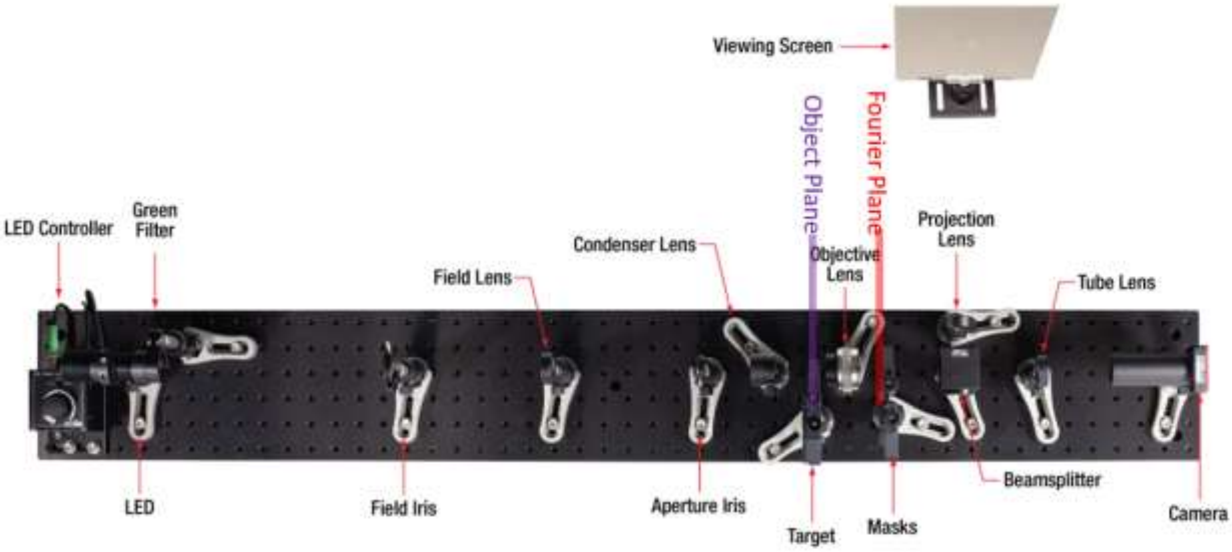
2. Switch on the PC and open the Thorcam software Click the “Open Camera” button and select the camera with the corresponding serial number. The user interface shown in Figure is displayed.

3. Click the “Open Camera” button and select the camera with the corresponding serial number. The user interface shown in Figure above is displayed.
4. Click the Live button (indicated in red in **Figure 29)** to display the live camera image in the window. Click the Save button (indicated in yellow) to open the dialog for saving the camera image to a file. Click the Settings button (indicated in green) and the Display button (indicated in blue) to open the camera menu, as shown in Figure 30, where you can change various settings such as the “Exposure Time”, “Gain” and the color scale.

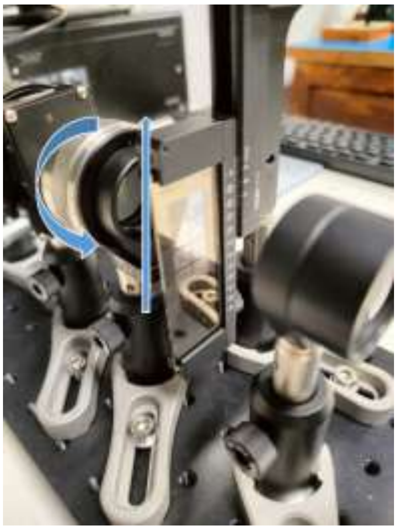


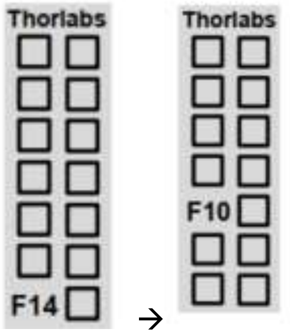


Caution: Do not keep the gain above 30 when the LED light is on. If more intensity is required kindly consult the with the TA.

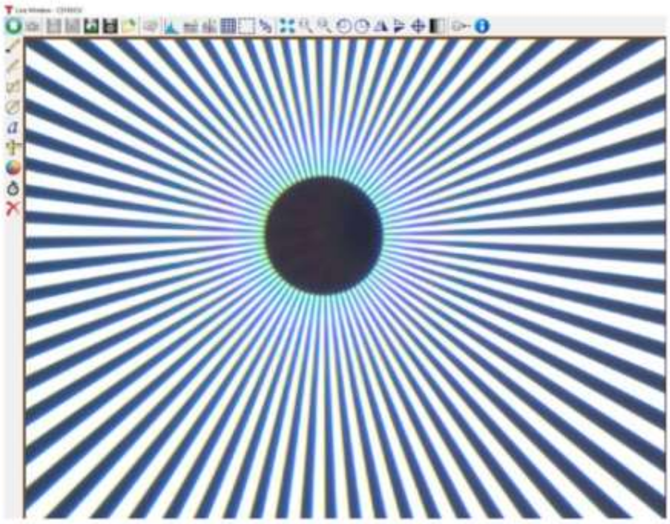


5. The objective lens is placed (f = 30 mm) 18 to 20 cm away from the tube lens, measured from the center of the lenses, the focus can be adjusted by rotating the knob finely by seeing the sharpness of the image on the Thorlabs software.





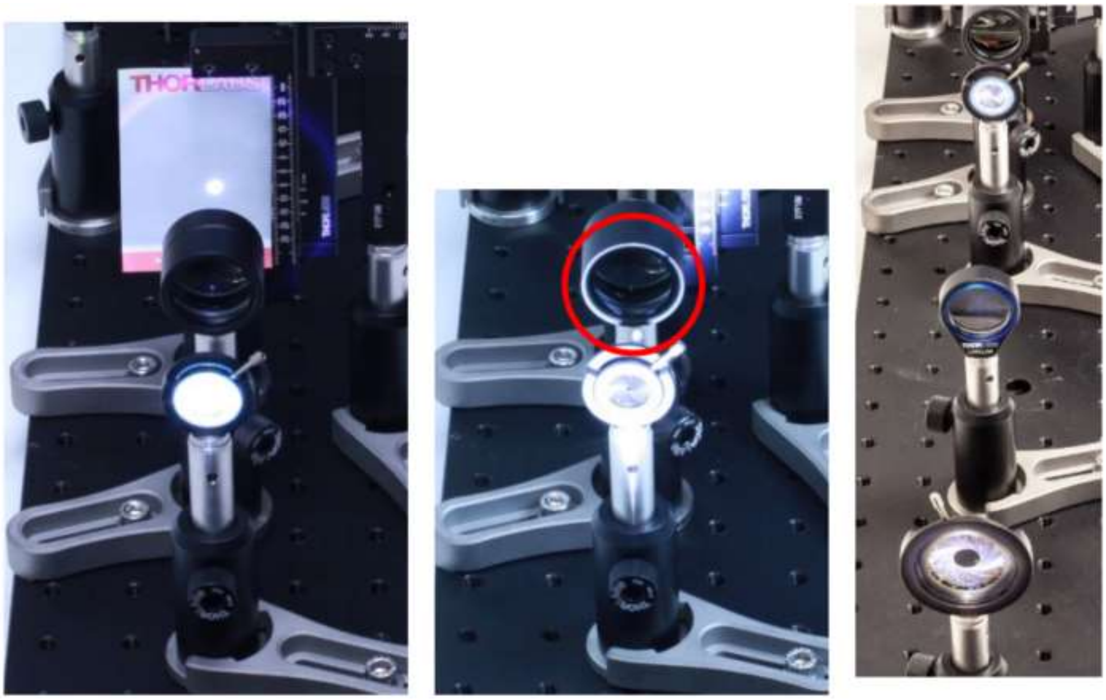
6. Move the target up to the highest possible position by turning the height adjustment screw clockwise. Adjust the post's height so that the sector star (lowest row on the target, field F14 in Figure above) has the same height as your optical axis (10 cm). The target is placed approximately 2 cm away from the objective lens mount. Move the target until you obtain a sharp image of the sector star in the camera (fine adjustment may be done by rotating the objective lens' mount).
   Next move the target downward until you see the center house of the arrangement of nine houses, field F10 in Figure.



7. Adjust the closed aperture iris [the smaller one, ID12(/M)] approximately 50 mm away from the condenser lens. The iris' shadow should be centered on the condenser lens mount.
8. Adjust the almost fully closed field iris at approximately 150 mm distance to the field lens.

10. Flip the color filter directly in front of the LED

11. Open the field iris (about 3/4). Place the beamsplitter approximately 8 cm away from the objective lens.



12. The projection lens (f = 75 mm) is placed directly behind the beam splitter (at 90° with respect to the main beam path).

13. Adjust the screen to find the distance where you see a sharp image of the Fourier plane.

14. Observe the aperture iris' side that is facing the target. When the target is not perpendicular to the beam, you will get reflections that are off-center.

Observations:

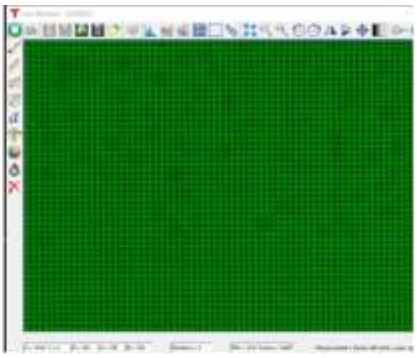
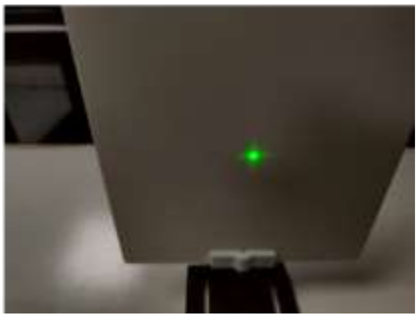
We are now, ready to make note of the observations:

1. Observe the effect that opening either the field iris or the aperture iris has on the image, the Fourier image, and the illumination of the object. Carefully and slowly open the mentioned iris one be one and observe the effects and tabulate your findings.

   Make note of your observations

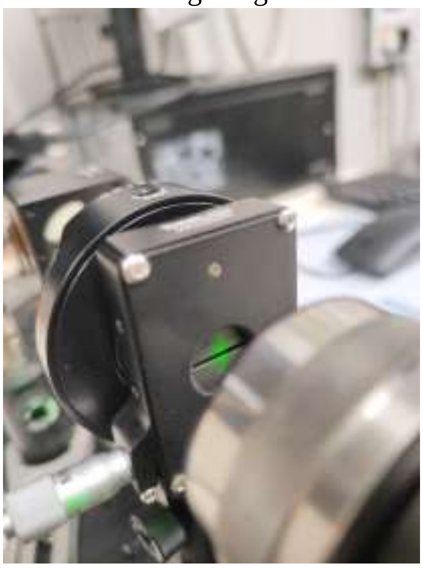
Effect	**Action**	
	Opening the Field Iris	**Opening the Aperture Iris**
Effects on the Camera Image		
Effects on the Illumination of the Target		
Effects on the Fourier Image		

2. Select atleast 2 lattice of your choice in the object plane. Now use a sheet of paper or the screen to find the Fourier plane. The Fourier plane is the camera-side focal plane of the objective lens. It is approx. at a distance ~ 3 cm from the objective lens.
3. View the intensity maxima in the Fourier plane separately and with maximum intensity, the aperture iris should be closed and the field iris opened.
4. Move the Viewing screen from close to the projection lens and away perpendicular to the setup length and observe the sharp image of the Fourier plane.
5. We will observe the effect that filters in the Fourier plane have on the camera image, and the Fourier plane image as well and try to manipulate the image in various ways.
6. Position a cross lattice (Ex: cross lattice with 25 µm lattice constant, field F3) in the object plane and adjust the objective lens if necessary so the camera image is sharp. Position the variable slit in the Fourier plane. Align the central bright spot to the center of the slit in both horizontal and vertical directions. Now adjust it so that light can only pass in a horizontal line along the main maximum.
   One example is show below for your reference:

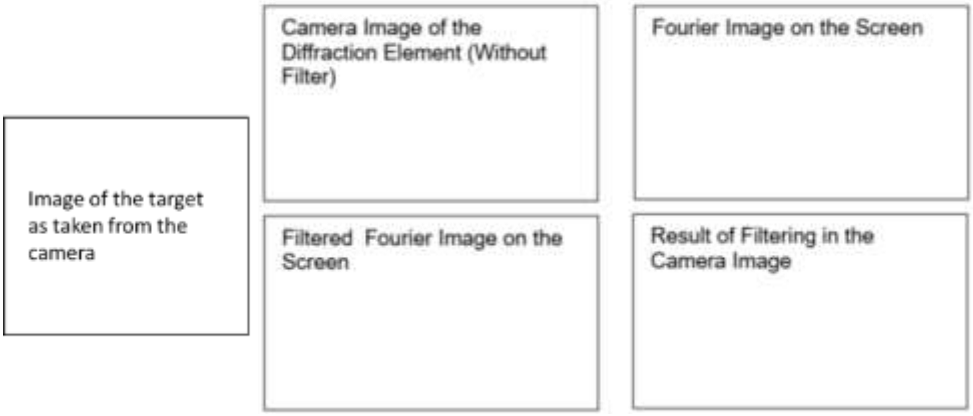




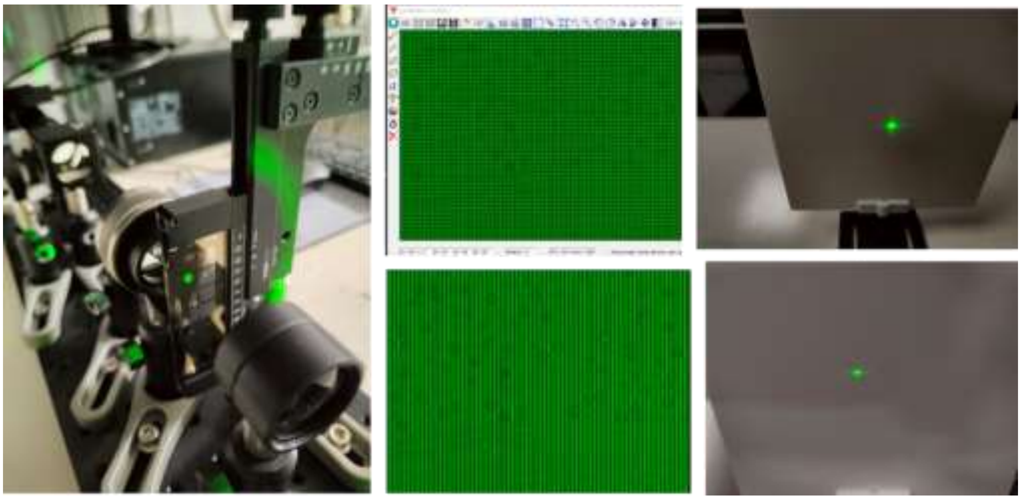
What is the result in the camera/microscope image and how is this image formed?

7. Now place the slit at the Fourier plane and rotate the slit to different angles in steps of 45° angle and only let the light of the main maximum and the corresponding diagonal orders pass. What is the resulting image on the camera and why?



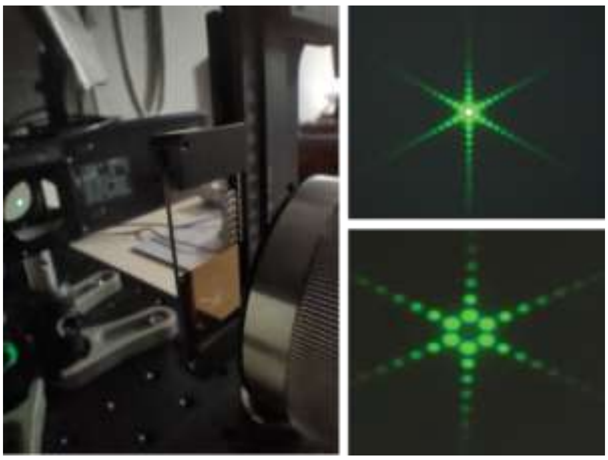

Tabulate the results as shown below:



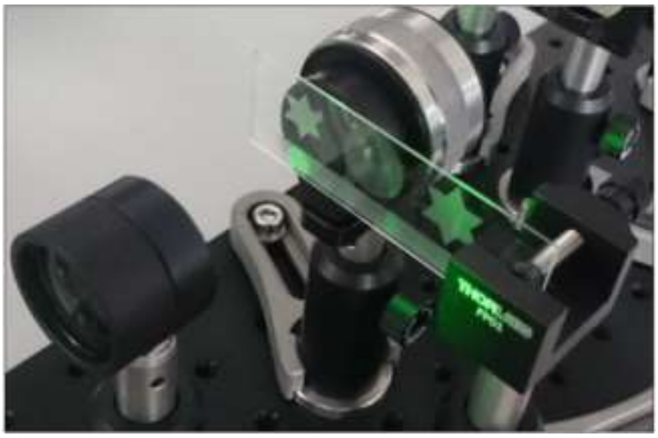


8. Choose either target F9 (Smiley behind bars) or F10 (House with door closed) or F12(Cryptographic message) and try to either free the smiley from behind the bar (F9) or open the house door (F10) or decode the message into characters(F12) and report the images and the necessary manipulation of Fourier plane to obtain the above desired results. Give justifications to the steps and obtained results in detail.

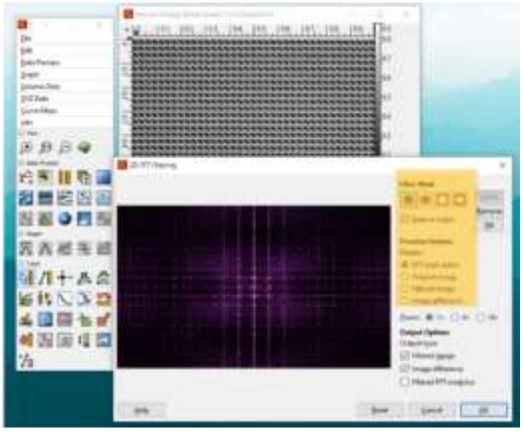
9. Using the filter slide and adjusting the position of spatial filter at the Fourier plane, for the Field F5 , filter the zeroth order (central spot) for the grating of triangles. What happens with the structure? Which fundamental principle is being observed?



10. Place the inverse Fourier target in the object plane (mounted in the FP01 plate holder) after carefully removing the target mask. Observe the Fourier plane image for the base of target.



11. After saving the images the theoretical calculation of the Fourier transform is to be done to further understand the basic principles using the Gwyddion software using the saved images and comparing the results.



Appendix:

Complete Instructions for setup:

1. Set all components to a height of 10 cm using the LMR1AP alignment plate (measured from the center of the optical element to the breadboard).

2. Place the camera at the right end of the breadboard (4th hole line) and place the tube lens (f = 150 mm) approximately 150 mm from the camera chip. Get a sharp camera image of an object that is more than 4 meters away.

3. Aim the LED at a wall or the ceiling and use the variable lens tube to position the collector lens so that you can see the LED chip (a pattern of black spots). Place the LED 100 cm away from the camera (post-to-post distance).

4. Place the objective lens (f = 30 mm) 18 to 20 cm away from the tube lens, measured from the center of the lenses.

5. Move the target up to the highest possible position by turning the height adjustment screw clockwise. Adjust the post's height so that the sector star (lowest row on the target, field F14 in Figure 31) has the same height as your optical axis (10 cm). Place the target approximately 2 cm away from the objective lens mount. Move the target until you obtain a sharp image of the sector star in the camera (fine adjustment may be done by rotating the objective lens' mount).

Move the target downward until you see the center house of the arrangement of nine houses, field F10 in Figure 31.

6. Place the condenser lens (f = 50 mm) 50 mm away from the target (measured approx. from the center of the lens). Place the condenser lens to achieve the smallest spot size on the target. Make sure the center house is illuminated (otherwise change condenser lens height).

7. Place the closed aperture iris [the smaller one, ID12(/M)] approximately 50 mm away from the condenser lens. The iris' shadow should be centered on the condenser lens mount.

8. Place the field lens (f = 150 mm) approximately 150 mm away from the aperture iris. Move the field lens until you see the LED chip on the iris.

9. Place the almost fully closed field iris at approximately 150 mm distance to the field lens. Move back and forth until you see an image of the iris' edge on the camera (hard to see, mainly just a deviation from the round shape).

10. Place the color filter directly in front of the LED

11. Open the field iris (about 3/4). Place the beamsplitter approximately 8 cm away from the objective lens.

12. Position the projection lens (f = 75 mm) directly behind the beam splitter (at 90° with respect to the main beam path). Use the screen to find the distance where you see a sharp image of the Fourier plane.

13. Observe the aperture iris' side that's facing the target. When the target is not perpendicular to the beam, you will get reflections that are off-center.

Appendix 2:

Fourier optics has a wide range of applications in various fields. Some of the key applications of Fourier optics include:

1. Image Formation and Processing: Fourier optics is fundamental to understanding image formation in optical systems. It helps explain how lenses and other optical components transform an object's spatial distribution of light intensity into an image. Fourier analysis can be used to enhance or filter images, correct aberrations, and improve image quality.

2. Holography : Holography is a technique that records and reconstructs the complete wavefront of light, capturing both intensity and phase information. Fourier optics plays a crucial role in understanding the principles of holography, allowing the reconstruction of 3D images from the recorded interference patterns.

3. Diffraction and Interference : Fourier optics is used to analyze and predict the diffraction and interference patterns that arise when light waves encounter obstacles or apertures. This is important in designing optical elements such as gratings, beam splitters, and diffraction-limited imaging systems.

4. Spatial Filtering : Spatial filters based on Fourier optics principles are used to selectively filter or modify specific spatial frequency components of an optical wavefront. These filters are used in applications like image sharpening, noise reduction, and edge enhancement.

5. Fourier Transform Spectroscopy : Fourier optics is the basis for Fourier transform spectroscopy techniques, which are widely used in infrared and other spectroscopic applications. These techniques allow the measurement of a material's absorption or emission spectrum with high precision.

6. Optical Information Processing : Fourier optics has been applied to optical information processing tasks such as pattern recognition, data encryption, and optical data storage. It enables the manipulation and analysis of optical signals for various computational tasks.

7. X-ray Crystallography : In X-ray crystallography, Fourier methods are used to analyze the diffraction patterns produced by X-rays passing through a crystalline sample. This helps determine the atomic arrangement within the crystal and has applications in studying molecular structures.

8. Synthetic Aperture Radar (SAR) : SAR systems utilize Fourier-based techniques to process radar data from multiple antenna positions to create high-resolution images of terrain, even in remote or challenging environments.

9. Optical Metrology : Fourier optics is used in various optical metrology techniques to measure surface profiles, detect defects, and analyze the quality of optical components and surfaces.

10. Fiber Optics and Communications : Fourier optics principles are applied in designing and analyzing optical fibers, which are essential components in modern telecommunications systems. It helps in understanding signal propagation, dispersion, and spectral analysis in optical fibers.

These applications highlight the importance of Fourier optics in various fields, enabling researchers and engineers to manipulate and control light for a wide range of practical purposes.

References:

• Eugene Hecht: Optics, 1st Edition (chapter 11 Fourier optics)

•Joseph W. Goodman: Introduction to Fourier Optics

•Directional Denoising Using Fourier Spectrum Cloning - Laurent Navarro and Jérôme Molimar

•THORLABS Fourier Optics Educational Kit – Serial Number -EDU-FOP2/M