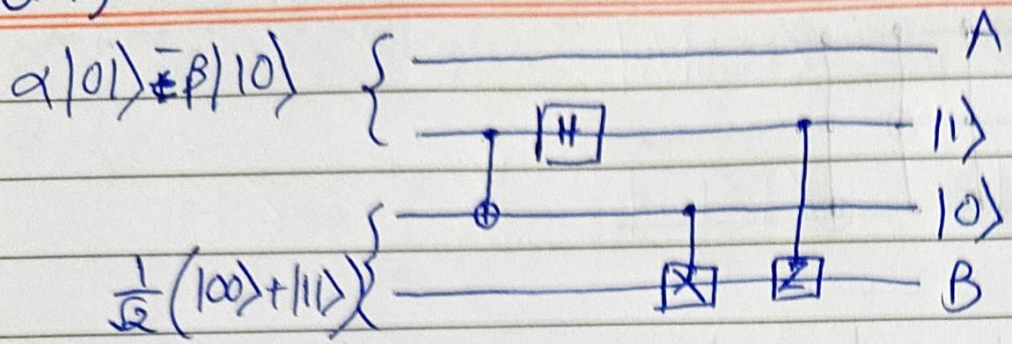
SOC

Assignment 1

Q 1)



$\alpha^2 + \beta^2 = 1$

$$|\psi\rangle = \left(\alpha|01\rangle - \beta|10\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|01\rangle(|00\rangle + |11\rangle) - \beta|10\rangle(|00\rangle + |11\rangle)\right]$$

= CNOT gate (2, 3)

$$= \frac{1}{\sqrt{2}}\left[\alpha|01\rangle(|10\rangle + |01\rangle) - \beta|10\rangle(|00\rangle + |11\rangle)\right]$$

= H gate

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)(|10\rangle + |01\rangle) - \beta|1\rangle\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)(|00\rangle + |11\rangle)\right]$$

$$= \frac{1}{2}\left[\alpha(|00\rangle - |01\rangle)(|10\rangle + |01\rangle)\right.$$
$$\left. - \beta(|10\rangle + |11\rangle)(|00\rangle + |11\rangle)\right]$$

CNOT gate (3, 4)

$$= \frac{1}{2}\left[\alpha(|00\rangle - |01\rangle)(|11\rangle + |01\rangle)\right.$$
$$\left. - \beta(|10\rangle + |11\rangle)(|00\rangle + |10\rangle)\right]$$

CZ gate (2, 4)

$$= \frac{1}{2}\left[\alpha(|0011\rangle + |0001\rangle - |0111\rangle - |0101\rangle)\right.$$
$$\left. - \beta(|1000\rangle + |1010\rangle + |1100\rangle + |1110\rangle)\right]$$

$$= \frac{1}{2}\left[\alpha(|0011\rangle + |0001\rangle + |0111\rangle + |0101\rangle)\right.$$
$$\left. - \beta(|1000\rangle + |1010\rangle + |1100\rangle + |1110\rangle)\right]$$

Final state → $|A|0B\rangle$

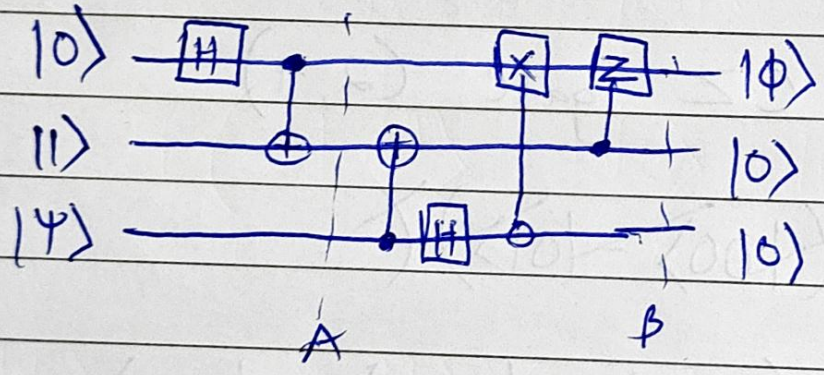
→ $|\Psi\rangle = \frac{1}{2}[\alpha|0101\rangle - \beta|1100\rangle]$

$\frac{1}{2}\sqrt{\alpha^2+\beta^2}$

$= \alpha|0101\rangle - \beta|1100\rangle$

→ $\boxed{|AB\rangle = \alpha|01\rangle - \beta|10\rangle}$

Q2)



$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\quad \alpha^2+\beta^2=1$

$|\Psi\rangle = |0\rangle \otimes |1\rangle \otimes |\Psi\rangle$

$= \alpha|010\rangle + \beta|011\rangle$

Applying H gate,

$$|\Psi\rangle = \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|10\rangle + \beta\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|11\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\alpha(|010\rangle+|110\rangle) + \beta(|011\rangle+|111\rangle)\right]$$

CNOT gate,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha(|010\rangle+|100\rangle) + \beta(|011\rangle+|101\rangle)\right]$$

∴ At A,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\alpha(|010\rangle+|100\rangle) + \beta(|011\rangle+|101\rangle\right.$$

CNOT gate

$$= \frac{1}{\sqrt{2}}\left[\alpha(|010\rangle+|100\rangle) + \beta(|001\rangle+|111\rangle)\right]$$

H gate,

$$= \frac{1}{\sqrt{2}}\left[\alpha(|01\rangle+|10\rangle)\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)\right.$$

$$\left. + \beta(|00\rangle+|11\rangle)\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{2}\left[\alpha\left(|010\rangle + |011\rangle + |100\rangle + |101\rangle\right) + \beta\left(|000\rangle - |001\rangle + |110\rangle - |111\rangle\right)\right]$$

CNOT gate

$$= \frac{1}{2}\left[\alpha\left(|110\rangle + |011\rangle + |000\rangle + |101\rangle\right) + \beta\left(|100\rangle - |001\rangle + |010\rangle - |111\rangle\right)\right]$$

CZ gate

$$= \frac{1}{2}\left[\alpha\left(|000\rangle + |011\rangle + |101\rangle - |110\rangle\right) + \beta\left(-|001\rangle + |010\rangle + |100\rangle + |111\rangle\right)\right]$$

At B,

$$|\Psi\rangle = |\phi\rangle\left(|00\rangle\right)$$

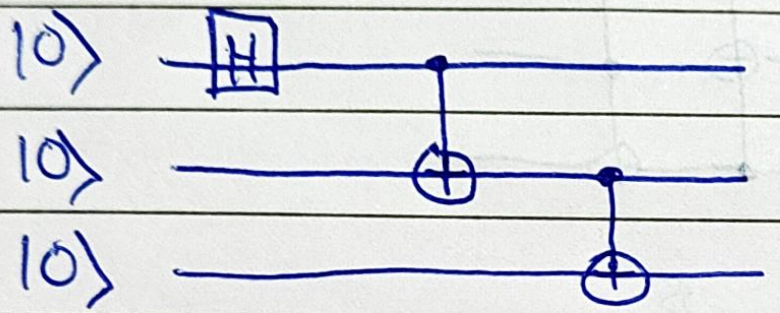
$$= \frac{\left[\alpha|000\rangle + \beta|100\rangle\right]}{\frac{1}{2}\sqrt{\alpha^2+\beta^2}}$$

$$= \left(\alpha|0\rangle + \beta|1\rangle\right)|00\rangle$$

$$\therefore \boxed{|\phi\rangle = \alpha|0\rangle + \beta|1\rangle}$$

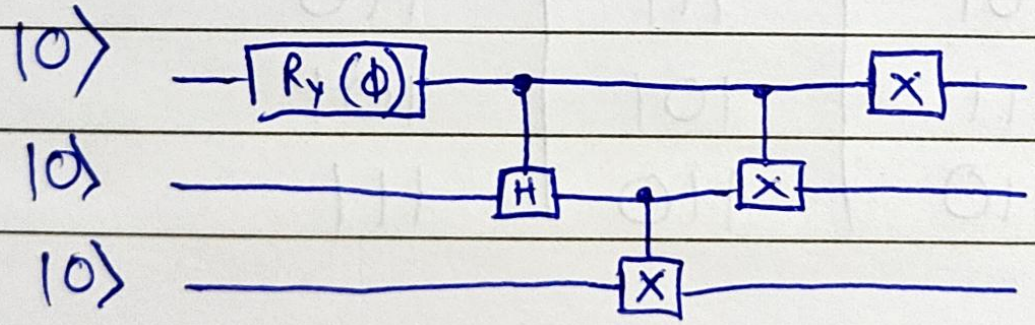
Q3)

$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$



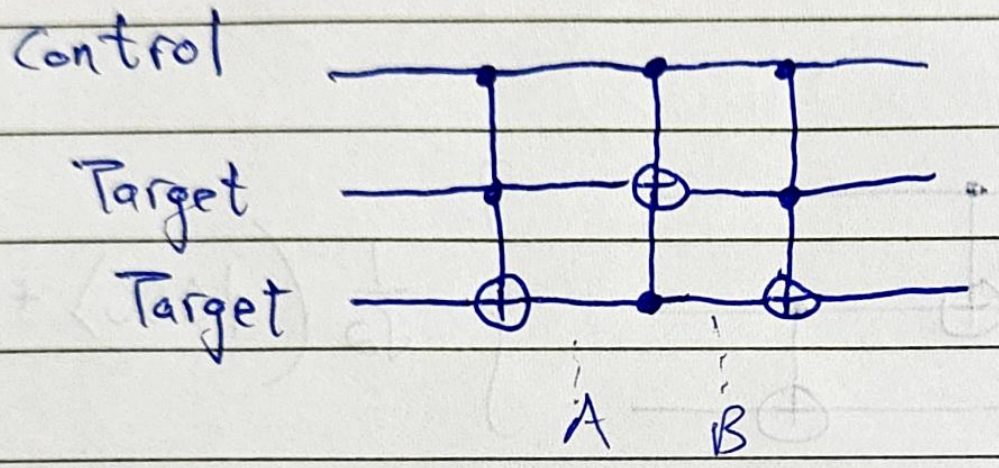
$$\frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|\psi\rangle_{W} = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$



$$\frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)$$

Q4) Controlled Swap gate



When Control is on

Initial	At A	At B	Final
100	100	100	100
101	101	111	110
110	111	101	101
111	110	110	111