Chapter 3

Simplification Of Boolean Functions.

The Map Method

① Two variable Map

$y \rightarrow$

x		$\bar{y}$ 0	y 1
$\bar{x}$	0	00	01
x	1	10	11

$\rightarrow 0 = \bar{x}$

$1 = y$

$1 = x$

binary Code	Gray Code
000	000
001	001
010	011
011	010

m_0	m_1
m_2	m_3

⑪ Three variable Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

$yz \longrightarrow$

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	000 $\bar{x}\bar{y}\bar{z}$	001 $\bar{x}\bar{y}z$	011 $\bar{x}yz$	010 $\bar{x}y\bar{z}$
x 1	100 $x\bar{y}\bar{z}$	101 $x\bar{y}z$	111 xyz	110 $xy\bar{z}$

(iii) Four Variable Map

$yz \rightarrow$

$wx \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	0000 $\bar{w}\bar{x}\bar{y}\bar{z}$ m_0	0001 $\bar{w}\bar{x}\bar{y}z$ m_1	0011 $\bar{w}\bar{x}yz$ m_3	0010 $\bar{w}\bar{x}y\bar{z}$ m_2
$\bar{w}x$ 01	0100 $\bar{w}x\bar{y}\bar{z}$ m_4	0101 $\bar{w}x\bar{y}z$ m_5	0111 $\bar{w}xyz$ m_7	0110 $\bar{w}xy\bar{z}$ m_6
wx 11	1100 $wx\bar{y}\bar{z}$ m_{12}	1101 $wx\bar{y}z$ m_{13}	1111 $wxyz$ m_{15}	1110 $wxy\bar{z}$ m_{14}
$w\bar{x}$ 10	1000 $w\bar{x}\bar{y}\bar{z}$ m_8	1001 $w\bar{x}\bar{y}z$ m_9	1011 $w\bar{x}yz$ m_{11}	1010 $w\bar{x}y\bar{z}$ m_{10}

1 ko pair garda [Min term ma garda real function Auxa
Max term ma garda Complement Auxa

0 ko pair garda [Max term ma real Auxa.
Min term ma Complement Auxa.

Q.N1. Simply the following boolean functions using k-map.

(i) $F = \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$

Soln:

Since there are variable we draw the k-map of eight cells.

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	000	001	011 1	010 1
x 1	100 1	101 1	111	110

We can make a pair of adjacent 1 trying to accomodate numbers of 1's in a pair of 16, 8, 4, 2.

In this, case we have 2 pairs as follows:

1st pair: ~~$x\bar{z}$~~ $x\bar{y}$
2nd pair: $\bar{x}y$

Hence the simplified function is

$F = x\bar{y} + \bar{x}y$

(ii) $F = \bar{x}yz + y\bar{y}\bar{z} + xyz + xy\bar{z}$

Soln:

Since there are variables, we need to draw a k-map.

$yz \longrightarrow$

	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	0	0	1	0
x 1	1	0	1	1

In min terms,

Here, we can make a pair of adjacent 1 trying to accomodate the numbers of 1's in pair of 16, 8, 4, 2.

In this case we have 2 pairs as follows,

1st pair: yz
2nd pair: $x\bar{z}$

Hence simplified Function is

SOP = $F = yz + x\bar{z}$.

In max terms,

Here we make a pair of adjacent 0 trying to accomodate the numbers of 0's in pair of 16, 8, 4, 2.

Min term Σ 1 normal
0 Complement

max term Π 1 Complement
0 normal

In this case we have 2 pairs as follows,

$F = (y+\bar{z}).(x+z)$

$= xy + x\bar{z} + yz + \bar{z}.z$

$= xy(z+\bar{z}) + x\bar{z} + yz$

$= xyz + yz + xy\bar{z} + x\bar{z}$

$= yz(x+1) + x\bar{z}(y+1)$

$= yz + x\bar{z}$ #

$pos = (y+\bar{z}).(x+z)$

(iii) $F(A,B,C,D) = \Sigma(0,1,2,5,8,9,10)$

Simply it in pos and Sop.

	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	1	1	0	1
$\bar{w}x$ 01	0	1	0	0
wx 11	0	0	0	0
$w\bar{x}$ 10	1	1	0	1

(1) pos = $(\bar{x}+\bar{w})(\bar{y}+\bar{z})(\bar{x}+z)$

(1) Sop = $\bar{x}\bar{z} + \bar{w}\bar{y}z + \bar{x}\bar{y}$

Pos   product of Sum

(max term ma hunx)

Sop   Sum of product

(min term ma hunx)

Solved:

pos = $(x+y)(x+z)$
Sop = $x+yz$

(i) Converting it into standard form:

Sop = $x+yz$

min term
$= (x(y+\bar{y})) + yz(x+\bar{x})$
$= xy + x\bar{y} + xyz + \bar{x}yz$

1-normal
0-Complement
$= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + xyz + \bar{x}yz$
111 110 101 100 111 011

$= \Sigma(3,4,5,6,7)$

max term
1-Complement
0-normal

pos = $(x+y)(x+z)$
$= (x+y+z\cdot\bar{z})(x+z+y\cdot\bar{y})$
$= (x+y+z)(x+y+\bar{z})(x+y+z)(x+\bar{y}+z)$
= 000 001 000 010
$= \Pi(0,1,2)$

Complement:

pos = $(x+y)(x+z)$
$\bar{F} = \overline{(x+y)} \cdot \overline{(x+z)}$ - using demorgan law
$= \bar{x}\cdot\bar{y} + \bar{x}\cdot\bar{z}$
$= \bar{x}(\bar{y}+\bar{z})$

Sop = $(x+yz)$
$\bar{F} = \overline{x+yz}$ - using demorgan law
$= \bar{x}\cdot(\bar{y}+\bar{z})$
$= \bar{x}\bar{y}+\bar{x}\bar{z}$

(ii) Simply the following Function using K-map.

$F = \Sigma(3,4,5,6,7)$

		$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$	0	0	0	1	0
x	1	1	1	1	1

(i) simplied in sop
we make pair of 1,

$F = x+yz$

(ii) Complement in sop
we make pair of 0,
$\bar{F} = \bar{x}\bar{y} + \bar{x}\bar{z}$

(iii) Simplied in pos
we make pair of 0,

$F = \bar{x}\cdot(\bar{y}+\bar{z})$

(iv) Complement in pos
we make pair of 1,
$F = (x+y)(x+z)$

Clear

Date
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①

$F = \Sigma(0,2,4,5,6)$

		$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$	0	1	0	0	1
x	1	1	1	0	1

① Simplified in Sop

Here we make pair of 1.
In this case we have 2 pairs.

$F = \bar{z} + x\bar{y}$ Normal

(iii) Complemented Simplified in pos

Here we make pair of 1.
In this case we have 2 pairs.

$F = z\cdot(\bar{x} + y)$ Complement

(ii) Complement in sop

Here we make pair of 0.
In this case we have 2 pairs.

$F = yz + \bar{x}z$ Complement

(iv) Simplified in pos

Here we make pair of 0.
In this case we have 2 pairs.

$F = (x + \bar{z})\cdot(\bar{y} + \bar{z})$

Normal

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Don't Care Conditions

Q Simplify using don't care Conditions.

(i) $F = \Sigma(0,2,5,6,8,10)$
$d = \Sigma(7,14,15)$

		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	00	1			1
$\bar{A}B$	01		1	X	1
AB	11			X	X
$A\bar{B}$	10	1			1

$F = C\bar{D} + \bar{A}BD + \bar{B}\bar{D}$

(ii)

$F(w,x,y,z) = \Sigma(1,3,7,11,15)$
$d = \Sigma(0,2,5)$

		$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$	00	X	1	1	X
$\bar{w}x$	01	0	X	1	0
wx	11	0	0	1	0
$w\bar{x}$	10	0	0	1	0

① Simplified in Sop

$F = \bar{w}\bar{x} + yz$ $\boxed{\text{Normal}}$

③ Simplified in Pos

$F = (\bar{y}+\bar{z})(w+x)$ $\boxed{\text{Complement}}$

② Complement in Sop

$F = \bar{z} + w\bar{y}$ $\boxed{\text{Complement}}$

④ Complement in pos

$F = z(\bar{w}+y)$ $\boxed{\text{Normal}}$

Sop ko normal ma jni Aauxa → pos ma lagidine tyo pos ko Complement huny

i.e. Sop Normal → pos ko Complement

Sop Complement → pos ko normal

Date: May-24
Tuesday

Five Variables Map [Eg]

AB \ CDE	$\bar{C}\bar{D}\bar{E}$ 000	$\bar{C}\bar{D}E$ 001	$\bar{C}DE$ 011	$\bar{C}D\bar{E}$ 010	$CD\bar{E}$ 110	CDE 111	$C\bar{D}E$ 101	$C\bar{D}\bar{E}$ 100
$\bar{A}\bar{B}$ 00	0	1	3	2	6	7	5	4
$\bar{A}B$ 01	8	9	11	10	14	15	13	12
AB 11	24	25	27	26	30	31	29	28
$A\bar{B}$ 10	16	17	19	18	22	23	21	20

|← 0 ko plane →| |← 1 ko plane →

a) Simplify the Bolean Function :

$$F(A,B,C,D,E) = \Sigma(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$$

AB \ CDE	$\bar{C}\bar{D}\bar{E}$ 000	$\bar{C}\bar{D}E$ 001	$\bar{C}DE$ 011	$\bar{C}D\bar{E}$ 010	$CD\bar{E}$ 110	CDE 111	$C\bar{D}E$ 101	$C\bar{D}\bar{E}$ 100
$\bar{A}\bar{B}$ 00	1			1	1			1
$\bar{A}B$ 01		1	1			1	1	
AB 11		1	1			1	1	
$A\bar{B}$ 10		1					1	

$$F = BE + \bar{A}\bar{B}\bar{E} + A\bar{D}E$$

b)

$$F = A\bar{C} + ACE + AC\bar{E} + \bar{A}C\bar{D} + \bar{A}\bar{D}\bar{E}$$

AB \ CDE	$\bar{C}\bar{D}\bar{E}$ 000	$\bar{C}\bar{D}E$ 001	$\bar{C}DE$ 011	$\bar{C}D\bar{E}$ 010	$CD\bar{E}$ 110	CDE 111	$C\bar{D}E$ 101	$C\bar{D}\bar{E}$ 100
00 $\bar{B}$	1						1	1
1 B	1						1	1
1 B	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1

$A\bar{C}$ → $AB\bar{C}$, $A\bar{B}\bar{C}$

$AB\bar{C}$ → $AB\bar{C}D$, $AB\bar{C}\bar{D}$

$AB\bar{C}D$ → $AB\bar{C}DE$, $AB\bar{C}D\bar{E}$

$AB\bar{C}\bar{D}$ → $AB\bar{C}\bar{D}E$, $AB\bar{C}\bar{D}\bar{E}$

$A\bar{B}\bar{C}$ → $A\bar{B}\bar{C}D$, $A\bar{B}\bar{C}\bar{D}$

$A\bar{B}\bar{C}D$ → $A\bar{B}\bar{C}DE$, $A\bar{B}\bar{C}D\bar{E}$

$A\bar{B}\bar{C}\bar{D}$ → $A\bar{B}\bar{C}\bar{D}E$, $A\bar{B}\bar{C}\bar{D}\bar{E}$

$$F = A + C\bar{D} + \bar{D}\bar{E} \quad \#$$

c)

$$F = A\bar{B} + A\bar{B}C + \bar{A}BD + AB\bar{E} + \bar{A}B\bar{E} + \bar{A}D$$

AB \ CDE	$\bar{C}\bar{D}\bar{E}$ 000	$\bar{C}\bar{D}E$ 001	$\bar{C}DE$ 011	$\bar{C}D\bar{E}$ 010	$CD\bar{E}$ 110	CDE 111	$C\bar{D}E$ 101	$C\bar{D}\bar{E}$ 100
00 $\bar{A}\bar{B}$			1	1	1	1		
01 $\bar{A}B$	1		1	1	1	1		1
11 AB	1			1	1			1
10 $A\bar{B}$	1	1	1	1	1	1	1	1

$$F = A\bar{B} + \bar{A}D + B\bar{E} \quad \#$$

a) Simplify the Boolean Function :

$F(A,B,C,D,E) = \Sigma(0,2,4,6,9,11,13,15,17,21,23,27,29,31)$

AB \ CDE	$\bar C\bar D\bar E$ 000	$\bar C\bar D E$ 001	$\bar C D E$ 011	$\bar C D\bar E$ 010	$C D\bar E$ 110	CDE 111	$C\bar D E$ 101	$C\bar D\bar E$ 100
$\bar A\bar B$ 00	1			1	1			1
$\bar A B$ 01		1	1			1	1	
AB 11		1	1			1	1	
$A\bar B$ 10		1					1	

$F = BE + \bar A\bar B\bar E + A\bar D E$

b) $F = A\bar C + ACE + AC\bar E + \bar A C\bar D + \bar A\bar D E$

AB \ CDE	$\bar C\bar D\bar E$ 000	$\bar C\bar D E$ 001	$\bar C D E$ 011	$\bar C D\bar E$ 010	$C D\bar E$ 110	CDE 111	$C\bar D E$ 101	$C\bar D\bar E$ 100
00 $\bar A\bar B$	1						1	1
01 $\bar A B$	1						1	1
11 AB	1	1	1	1	1	1	1	1
10 $A\bar B$	1	1	1	1	1	1	1	1

$A\bar C$ < $AB\bar C$, $A\bar B\bar C$

$AB\bar C$ < $AB\bar C D$, $AB\bar C\bar D$

$AB\bar C D$ < $\bar A B\bar C D E$, $AB\bar C D E$

$AB\bar C\bar D$ < $AB\bar C\bar D E$, $AB\bar C\bar D\bar E$

$A\bar B\bar C$ < $A\bar B\bar C D$, $A\bar B\bar C\bar D$

$A\bar B\bar C D$ < $A\bar B\bar C D E$, $A\bar B\bar C D\bar E$

$A\bar B\bar C\bar D$ < $A\bar B\bar C\bar D E$, $A\bar B\bar C\bar D\bar E$

$F = A + C\bar D + \bar D\bar E$ #

c) $F = A\bar B + A\bar B C + \bar A B D + AB\bar E + \bar A B\bar E + \bar A D$

AB \ CDE	$\bar C\bar D\bar E$ 000	$\bar C\bar D E$ 001	$\bar C D E$ 011	$\bar C D\bar E$ 0100	$C D\bar E$ 110	CDE 111	$C\bar D E$ 101	$C\bar D\bar E$ 100
00 $\bar A\bar B$			1	1	1	1		
01 $\bar A B$	1		1	1	1	1		1
11 AB	1			1	1			1
10 $A\bar B$	1	1	1	1	1	1	1	1

$F = A\bar B + \bar A D + B\bar E$ #

Chapter 4

Combinational Logic

Design Procedure Of Combinational Circuit:

- The Problem is Stated.
- The number of available input variables and required output variables is determine.
- The input and output variables are assign letter or symbol.
- The truthtable that defines the required relation-ships between inputs and outputs is derived.
- The simplified boolean Function For each output is obtained.
- The logic diagram is draw.

Adders :

↳ Design of a half adders circuit.
↳ No. of input = 2 and output = 2.
↳ let 2 inputs be (x,y) and outputs be (s,c) where s is sum and c is carry.
↳ Truth table of half adders:

Inputs		Outputs	
x	y	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

↳ From the truth table, we can generate the boolean function as well.

$S = \bar{x}y + \bar{y}x$
$= x \oplus y$ [SOP Form]

$C = xy$ [C ma 1 herne]

↳

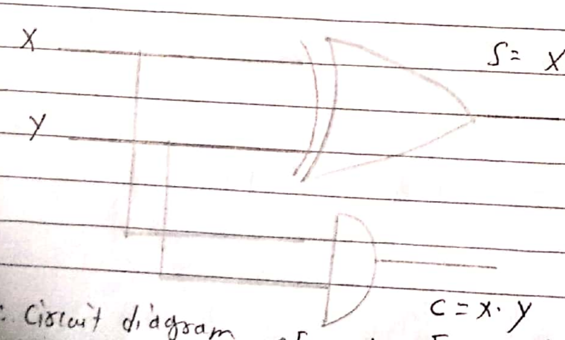


Fig: Circuit diagram of half adders.

Design of a Full adders circuit.
↳ No. of input = 3 and output = 2.
↳ let 3 inputs be (x,y,z) and outputs be (s,c), where s is sum and c is carry.
↳ Truth table of Full adders:

Inputs			Outputs	
x	y	z	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

↳ From the truth table, we can generate boolean equation:

$S = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$
$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$
$= \bar{x}(y \oplus z) + x(y \odot z)$
$= \bar{x}(y \oplus z) + x(y \odot z)$
$= (x \odot (y \odot z))$

[$\bar{x}y + x\bar{y} = x \oplus y$
$xy + \bar{x}\bar{y} = x \odot y$]

$\bar{x}(y \oplus z) + x(y \odot z)$
$\bar{x}(y \odot z) + x(\overline{y \odot z})$
$[x \oplus (y \oplus z)]$

3 bit ko XOR rw XNOR same hunx.

$C = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

$= z(\bar{x}y + x\bar{y}) + xy(\bar{z}+z)$

$= z(x \oplus y) + xy$

$yz(\bar{x}+x) + x(\bar{y}z + y\bar{z})$

$x(y \oplus z) + yz$

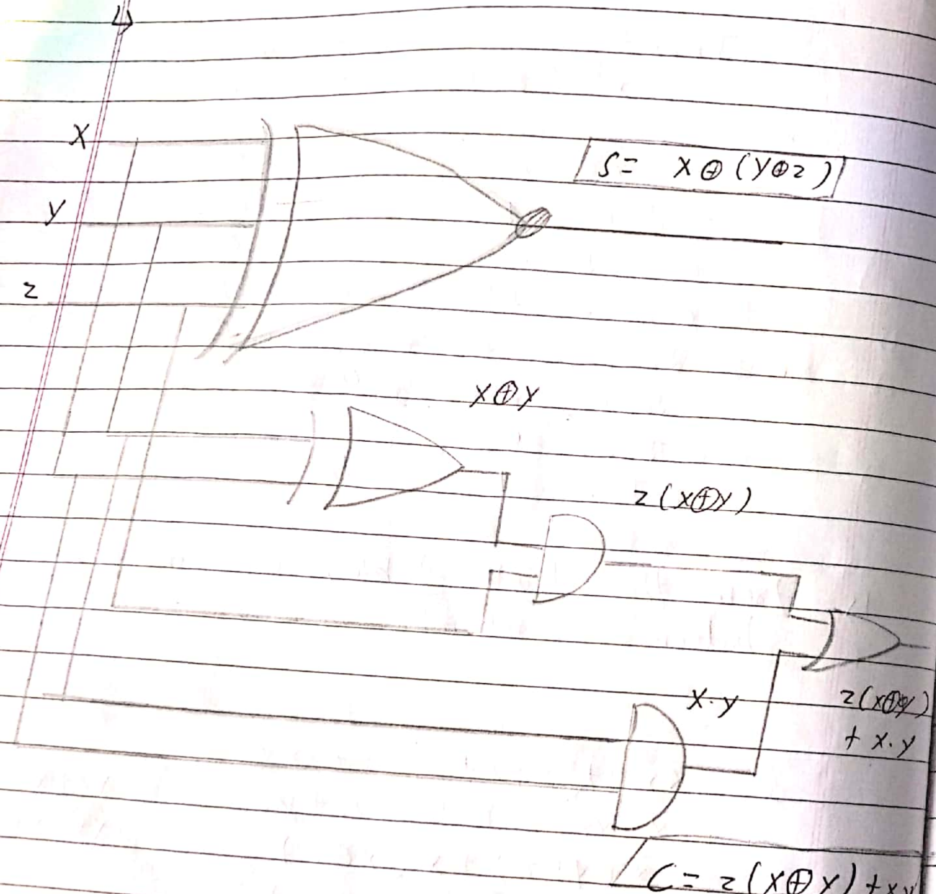


Fig: Circuit diagram of Full adders

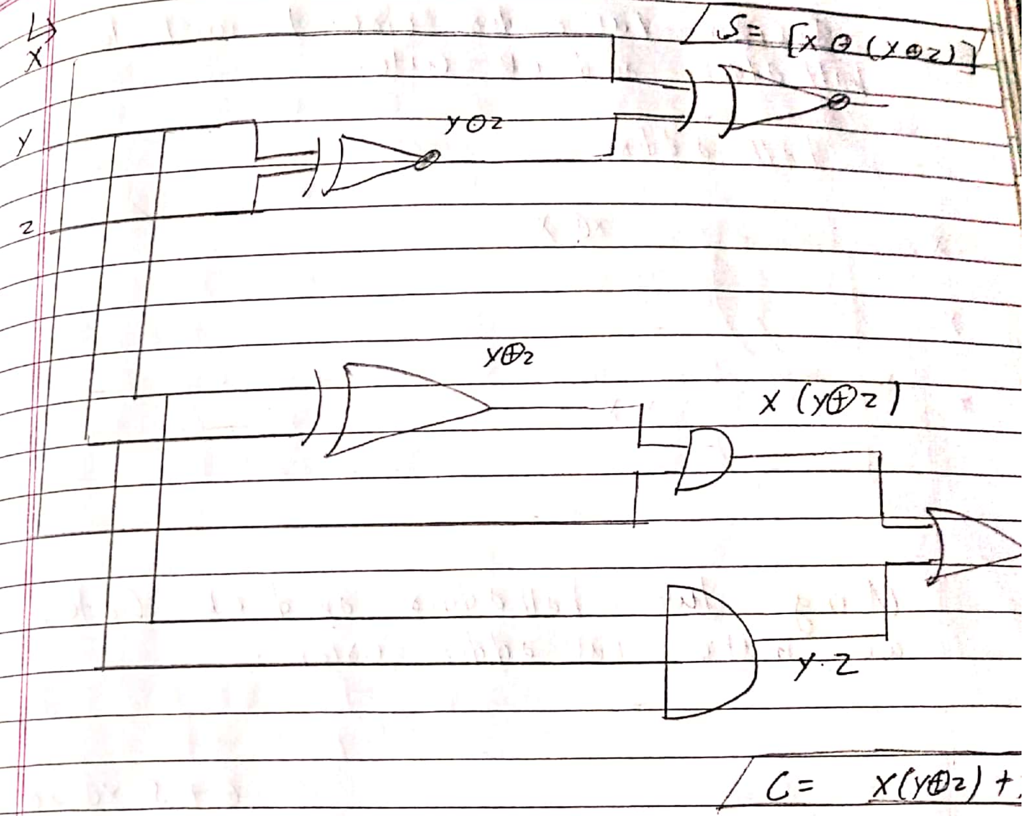
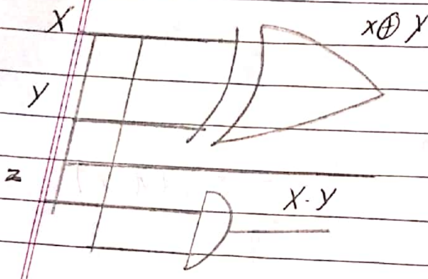


Fig: Circuit diagram of Full adders

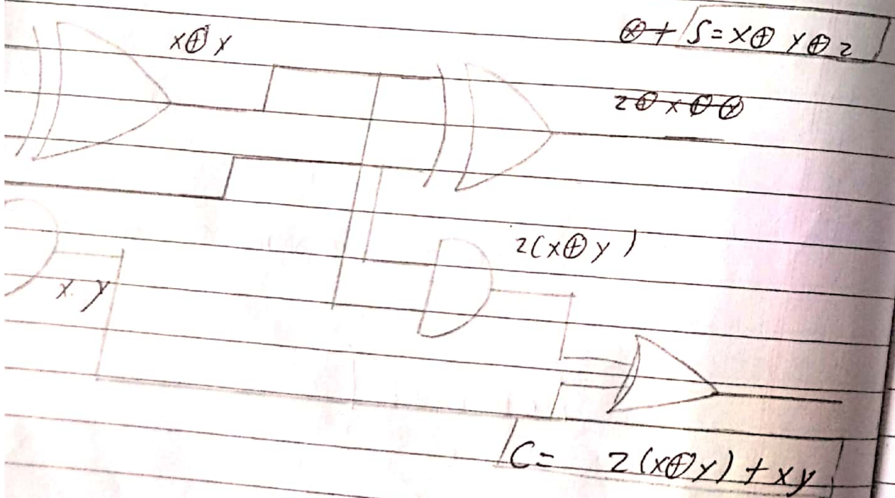
[optional]

Design Full adder circuit with two half adder and OR Gate.

half adders:



Using two half adder and OR Gate, we make Full adder circuit.



Code Conversion Circuit

① Logic diagram of BCD to excess 3 Converter.

↳ design of BCD to excess 3 Converter
No. of inputs = 4 and output = 4

↳ let inputs be A, B, C, D and outputs be w, x, y, z.

↳ Truth table:

Inputs				Outputs			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

↳ Bolean expression using k-map;

For w,

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$				
$\bar{A}B$		1	1	1
AB	X	X	X	X
$A\bar{B}$	1	1	X	X

$W = A + BC + BD$
$= A + B(C + D)$
$= A + B(C + D)$

For x,

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1	1	1
$\bar{A}B$	1			
AB	x	x	x	x
$A\bar{B}$		1	x	x

$x = \cancel{AD} + \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$

$= \bar{B}(C+D) + B\bar{C}\bar{D}$

For y,

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		1	
$\bar{A}B$	1		1	
AB	x	x	x	x
$A\bar{B}$	1		x	x

$y = \bar{C}\bar{D} + CD + \cancel{BC}$

$= \cancel{\bar{C}\bar{D} + C(B+D)}$

$= C \odot D$

For z,

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$	1			1
AB	x	x	x	x
$A\bar{B}$	1		x	x

$x = \bar{B}(C+D) + B(\overline{C+D})$

$= B \oplus (C+D)$

$z = \bar{D}$

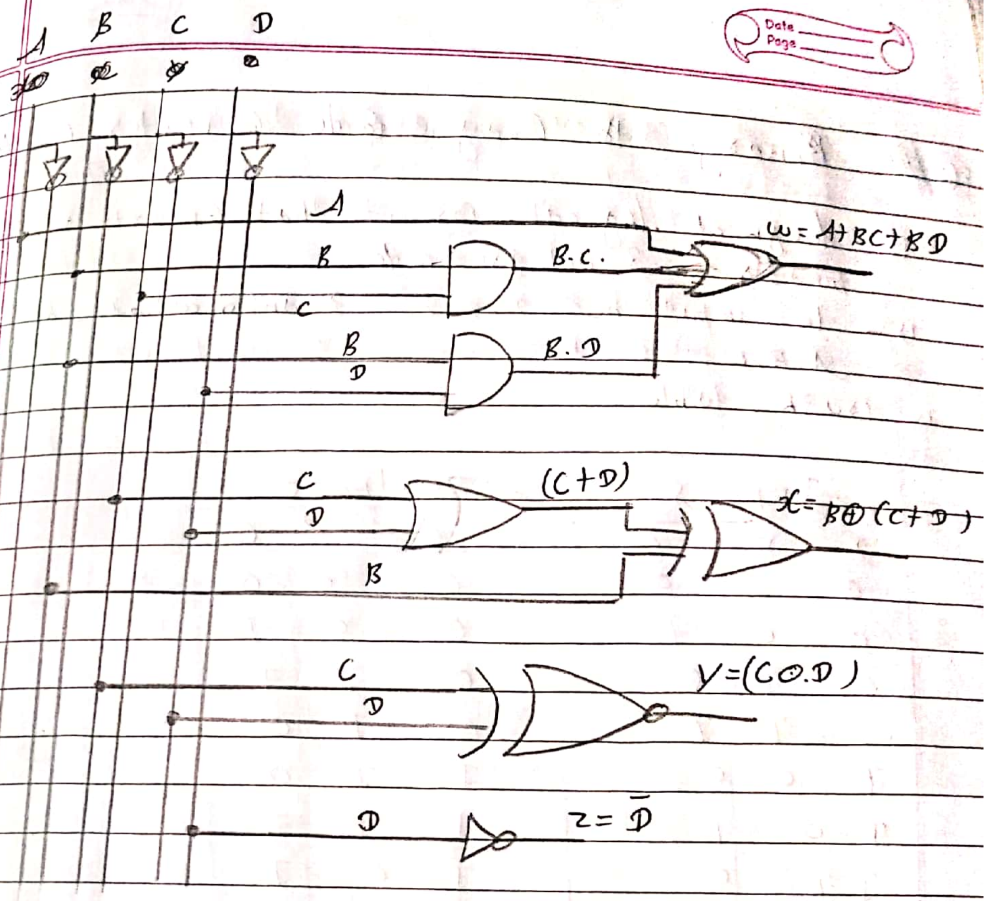


fig: logical diagram of binary to excess 3 Converter.

② Binary to Gray Code Converter (3 bit)

- design of logical circuit that converts 3 bit binary to 3 bit grey code.
- let input be $x, y, z = 3$ and output be $A, B, C = 3$.
- Truth table

Inputs			Outputs		
x	y	z	A	B	C
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

Boolean expression:

For A,

$$A = x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$$
$$= x\bar{y}(\bar{z}+z) + xy(\bar{z}+z)$$
$$= x\bar{y} + xy$$
$$= x(\bar{y}+y)$$
$$= x$$

For B,

$$B = \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z$$
$$= z(\bar{x}y + x\bar{y}) + \bar{z}(\bar{x}y + x\bar{y})$$
$$= (x \oplus y)\ (z + \bar{z})$$
$$= (x \oplus y)$$

For C,

$$C = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}z + xy\bar{z}$$
$$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}z + y\bar{z})$$
$$= (\bar{x}+x)\ (\bar{y}z + y\bar{z})$$
$$= (\bar{y}z + y\bar{z})$$
$$= (y \oplus z)$$

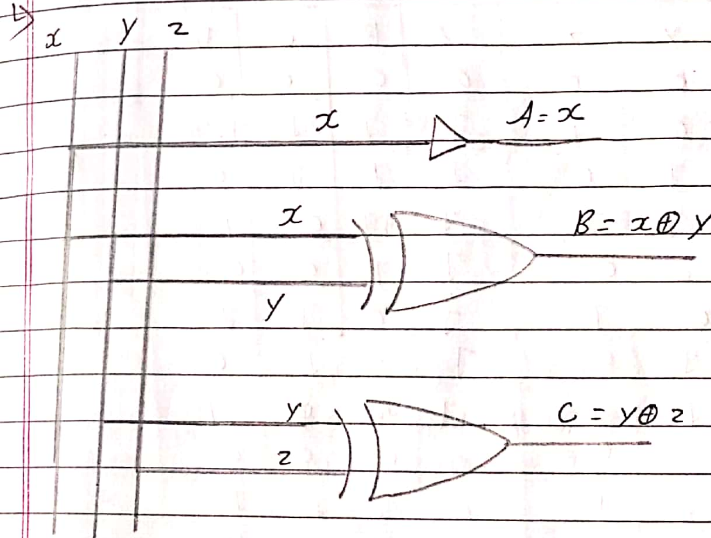


Fig: circuit diagram of 3 bit binary to Gray code converter.

③ # Draw a logical diagram that takes 4 bit binary as an input and provides 2's complement of the input binary number.

↳ design of binary to 2's complement converter
↳ let inputs be $w, x, y, z = 4$ and outputs be $A, B, C, D = 4$.
↳ Truth table of binary to 2's complement converter

Inputs				Outputs			
w	x	y	z	A	B	C	D
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

1) Boolean expression:

For A,

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1	1	1
$\bar{w}x$	1	1	1	1
wx				
$w\bar{x}$	1			

For B,

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1	1	1
$\bar{w}x$	1			
wx	1			
$w\bar{x}$		1	1	1

$A = \bar{w}x + \bar{w}y + \bar{w}z + w\bar{x}\bar{y}\bar{z}$
$= \bar{w}(x+y+z) + w(\overline{x+y+z})$
$= w \oplus (x+y+z)$

$B = \bar{x}y + \bar{x}z + x\bar{y}\bar{z}$
$= \bar{x}(y+z) + x(\overline{y+z})$ ⟵ using demorgan law i.e $(\overline{y+z}) = \bar{y}\cdot\bar{z}$
$= x \oplus (y+z)$

For C,

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1		1
$\bar{w}x$		1		1
wx		1		1
$w\bar{x}$		1		1

For D,

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1	1	
$\bar{w}x$		1	1	
wx		1	1	
$w\bar{x}$		1	1	

$C = \bar{y}z + y\bar{z}$
$= y \oplus z$

$D = z$

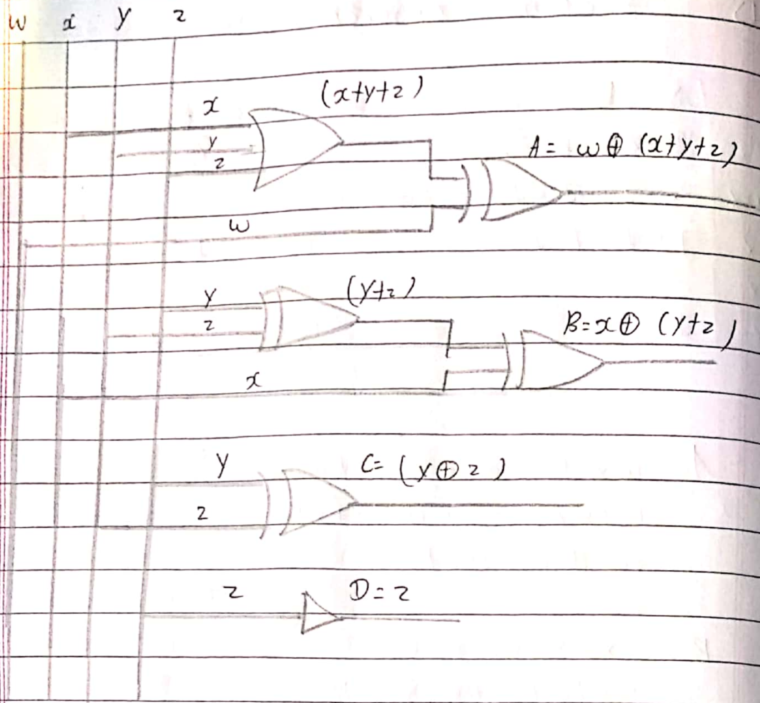


fig: Circuit diagram of binary to 2's Complement Converter

Half Substractor

- design of half substractor circuit.
- Let input be x and y = 2 and output be d and b where d is difference and b is borrow.
- Truth table:

Inputs		Outputs	
x	y	d	b
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$D = \bar{A}B + \bar{B}A$
$= A \oplus B$

- Bolean expression:

$d = \bar{A}B + A\bar{B}$
$= A \oplus B$

$B = \bar{x}y$

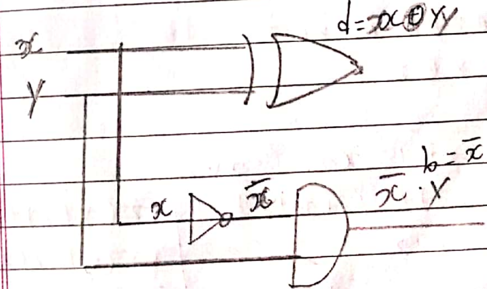


fig: Circiut diagram of half substract

Full Subtractor

↳ design of Full subtractor circuit.
↳ let input be $x, y, z = 3$ and output be d and b
where d is difference and b is borrow.
↳ Truth table

Inputs			Outputs	
x	y	z	d	b
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

↳ Boolean expression:

$$d = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$
$$= \bar{x}(\bar{y}z + y\bar{z}) + x(\bar{y}\bar{z} + yz)$$
$$= \bar{x}(y \oplus z) + x(y \odot z)$$
$$= \bar{x}(y \oplus z) + x(\overline{y \oplus z})$$
$$= x \oplus y \oplus z$$

$$b = \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xyz$$
$$= \bar{x}(\bar{y}z + y\bar{z}) + yz(\bar{x} + x)$$
$$= \bar{x}(y \oplus z) + yz$$

$\cancel{\bar{x}z(\bar{y}+y)+}$
$\bar{x}y(\bar{z}+z) + \bar{x}z(\bar{y}+y)$
$\bar{x}y + \bar{x}z$

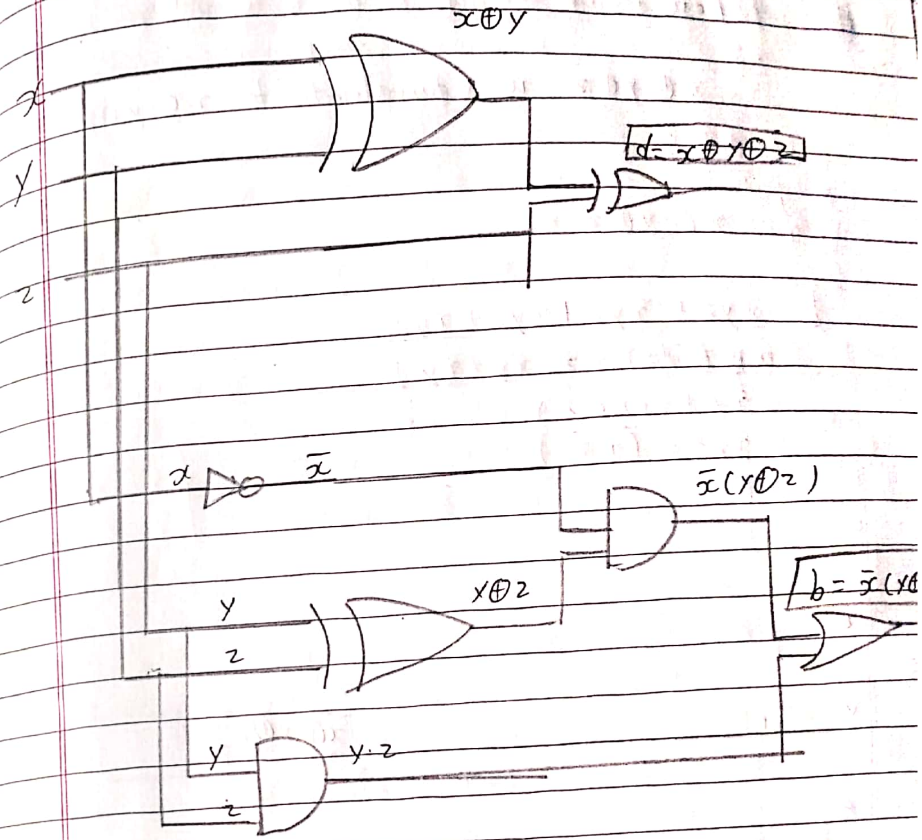


fig: Circuit diagram of Full subtra
circuit.

From K map,

$b =$

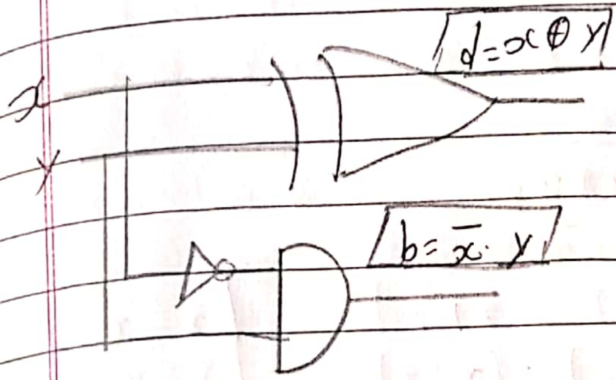
	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0		1	1	1
x 1			1	

$$b = yz + \bar{x}y + \bar{x}z$$

Implement a Full subtractor with two half subtractor and an OR gate.

Half subtractor

Full subtractor

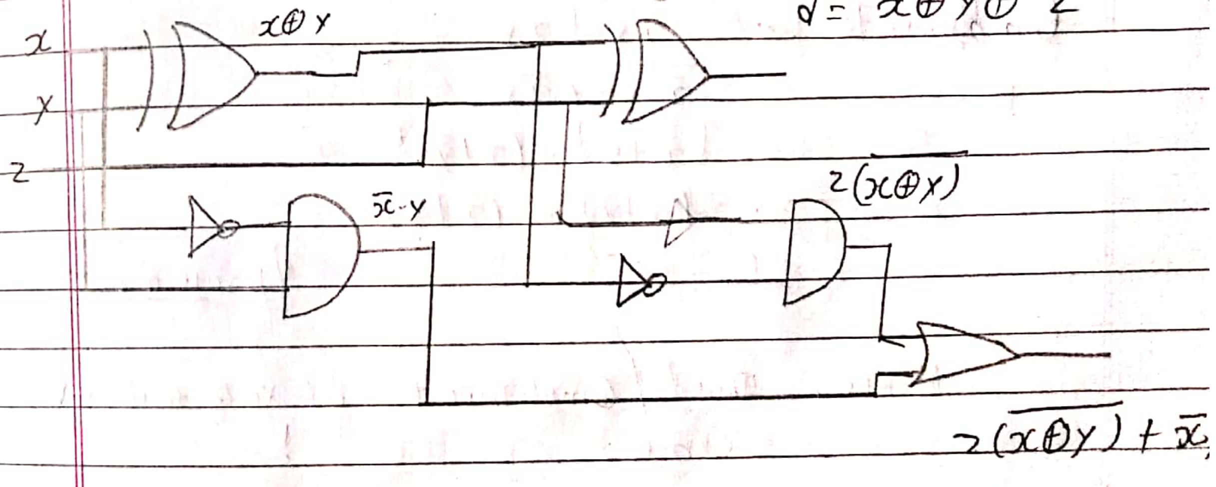


$d = x \oplus y \oplus z$

$b = \bar{x}y + \bar{x}z + yz$

$= \bar{x}y + z(\overline{x \oplus y})$

Implementing Full subtractor with two half subtractor.



Show that the dual and complement of $x \oplus y$ and $x \odot y$ are same.

$x \oplus y$

Dual : $\bar{x}y + x\bar{y}$

$(\bar{x}+y) \cdot (x+\bar{y})$

Complement : $\overline{\bar{x}y + x\bar{y}}$

$= \overline{\bar{x}y} \cdot \overline{x\bar{y}}$

$= (\bar{\bar{x}}+\bar{y}) \cdot (\bar{x}+\bar{\bar{y}})$

$= (x+\bar{y}) \cdot (\bar{x}+y)$

$\left[\begin{array}{l} \bar{\bar{x}} = x \\ \bar{\bar{y}} = y \end{array} \right]$

$x \odot y$

Dual : $\bar{x}\bar{y} + xy$

$(\bar{x}+\bar{y}) \cdot (x+y)$

Complement : $\overline{\bar{x}\bar{y} + xy}$

$\overline{\bar{x}\bar{y}} \cdot \overline{xy}$

$(\bar{\bar{x}}+\bar{\bar{y}}) \cdot (\bar{x}+\bar{y})$

$(x+y) \cdot (\bar{x}+\bar{y})$.

$\boxed{\text{proved}}$

Hence, Dual & Complement of XOR and XNOR are same.

Show that $x \oplus y \oplus z$ is equivalent to $x \odot y \odot z$.

let,

$A = x \oplus y$

$B = z$

$x \oplus y \oplus z$

$A \oplus B$

$\bar{A}B + A\bar{B}$

$\overline{x \oplus y} \cdot z + x \oplus y \cdot \bar{z}$

$x \odot y \cdot z + \overline{x \odot y} \cdot \bar{z}$

$x \odot y \odot z$ ✓

let, $A = x \odot y$

$B = z$

$x \odot y \odot z$

$A \odot B$

$\bar{A}\bar{B} + AB$

$\overline{x \odot y} \cdot \bar{z} + x \odot y \cdot z$

$x \oplus y \cdot \bar{z} + \overline{x \oplus y} \cdot z$

$x \oplus y \oplus z$ ✓

Design a Combinational Circuit which takes a two bit number and generates an output binary numbers equal to the cube of the input numbers.

design of circuit which takes a two bit number and generates an output binary number equal to cube of input number.
Let input be $x, y = 2$ and output be $A, B = 2$.
Truth table:

Inputs		Outputs				
x	y	A	B	C	D	E
0	0	0	0	0	0	0
0	1	0	~~1~~ 0	0	0	1
1	0	~~0~~ 0	~~0~~ 1	0	0	0
1	1	1	1	0	1	1

oolean expression:

$A = x\bar{y}$ $\qquad A = xy$

$B = x\bar{y} + xy$

$= x(\bar{y} + y) = x$ $\qquad B = x$

$D = xy$ $\qquad D = xy$

$E = \bar{x}y + xy$

$= y(\bar{x} + x)$ $\qquad E = y$

$= y$

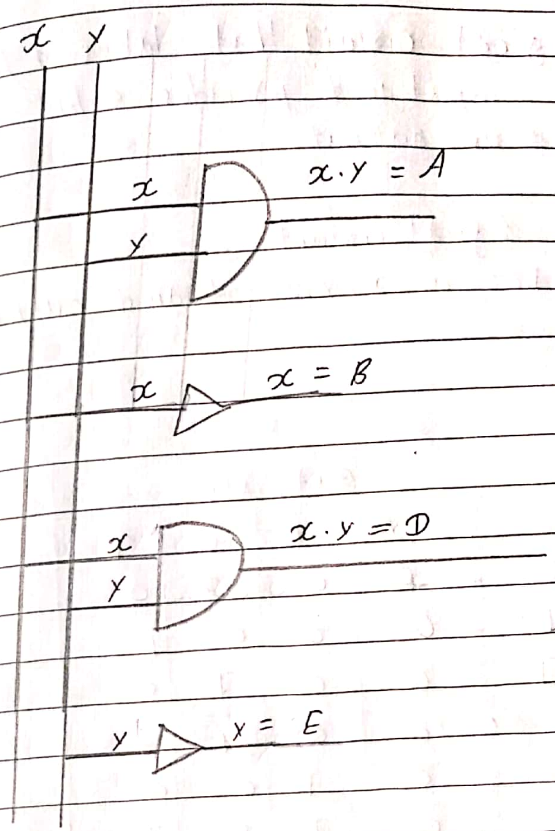


Fig: Circuit diagram of two bit inp takes and generating an output bi number equal to the input numbe

7) Design a logical circuit that takes 4 bit gray code as input and provides 4 bit binary code as an output.

→ Design of logical circuit ...

→ let input be $w, x, y, z = 4$ and output be $A, B, C, D = 4$.

→ Truth table:

Inputs				Outputs			
w	x	y	z	A	B	C	D
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	1
1	1	1	1	1	0	1	0
1	1	1	0	1	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	1	1	0	1
1	0	0	1	1	1	1	0
1	0	0	0	1	1	1	1

Boolean expression

→ $A = wx\bar{y}\bar{z} + wx\bar{y}z + wxyz + wxy\bar{z} + w\bar{x}y\bar{z} + w\bar{x}yz + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z}$

$= wx\bar{y}(\bar{z}+z) + wxy(z+\bar{z}) + w\bar{x}y(z+\bar{z}) + w\bar{x}\bar{y}(z+\bar{z})$

$= wx(\bar{y}+y) + w\bar{x}(y+\bar{y})$

$= wx + w\bar{x}$

$= w(x+\bar{x})$

$= w$

For A,

$wx \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$				
wx	1	1	1	1
$w\bar{x}$	1	1	1	1

$A = w$

For B,

	00	01	11	10
00				
01	1	1	1	1
11				
10	1	1	1	1

$B = x\bar{w}x + w\bar{x}$

$= w \oplus x$

For C,

	00	01	11	10
00			1	1
01	1	1		
11			1	1
10	1	1		

$C = y$

For D,

	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

$D = z$

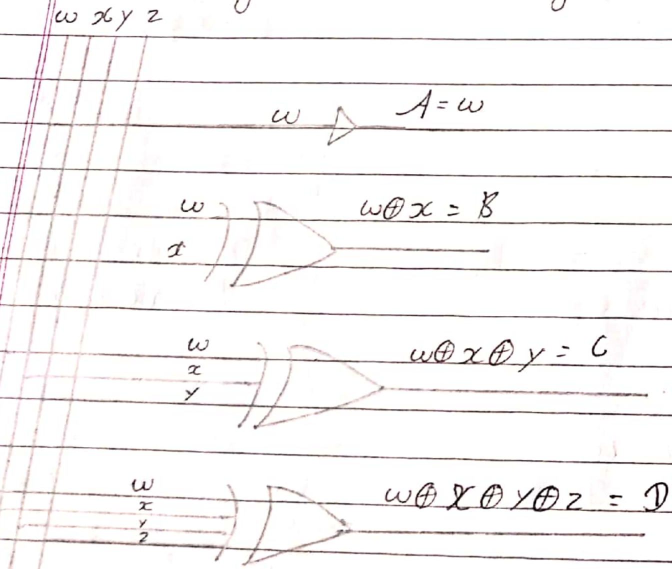
w x y z

$C = \bar{w}\bar{x}y + \bar{w}x\bar{y} + wxy + w\bar{x}\bar{y}$
$= \bar{w}(\bar{x}y + x\bar{y}) + w(xy + \bar{x}\bar{y})$
$= \bar{w}(x \oplus y) + w(x \odot y)$
$= \bar{w}(x \oplus y) + w(\overline{x \oplus y})$
$= w \oplus (x \oplus y)$

~~w = A~~
~~x = B~~
y = C
~~z = D~~

$D = \cancel{A \oplus B \oplus C \oplus D}$ $w \oplus x \oplus y \oplus z$

Fig: Circuit diagram



6) Design a Combinational Circuit that takes 4 bit binary as input and provides 4 bit gray Code as an output.

↳ design a logical Circuit that takes 4 bit binary as input and provides 4 bit grey code as an output.

↳ let no. of inputs be w, x, y, z = 4 and no. of outputs be A, B, C, D = 4.

↳ Truth table of logical circuit that takes 4 bit number and provides 4 bit gray Code.

Inputs				outputs			
w	x	y	z	A	B	C	D
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1
1	0	1	0	1	1	1	1
1	0	1	1	1	1	1	0
1	1	0	0	1	0	1	0
1	1	0	1	1	0	1	1
1	1	1	0	1	0	0	1
[illegible]	[illegible]	[illegible]	[illegible]	1	0	0	0

Boolean expression :

for A,

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$				
wx	1	1	1	1
$w\bar{x}$	1	1	1	1

$A = w$

for B,

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$	1	1	1	1
wx				
$w\bar{x}$	1	1	1	1

$B = \bar{w}x + w\bar{x} = w \oplus x$

for C,

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$			1	1
$\bar{w}x$	1	1		
wx	1	1		
$w\bar{x}$	1		1	1

$C = \bar{x}y + x\bar{y}$

$= x \oplus y$

for D,

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$		1		1
$\bar{w}x$		1		1
wx		1		1
$w\bar{x}$		1		1

$D = \bar{y}z + y\bar{z}$

$= y \oplus z$

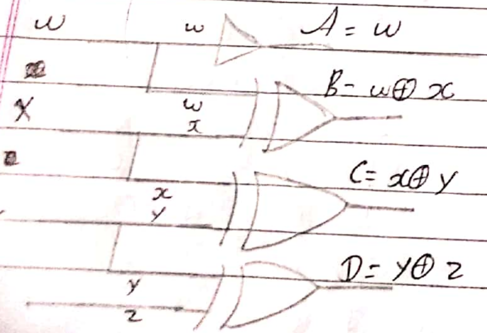


Fig: Logical diagram

(4) Design a Combinational Circuit that takes 3 bit number and provides an output binary number equal to the square of the input number.

→ design a Combinational circuit that takes 3 bit number and provides square of the input number.

→ Let inputs be $x, y, z = 3$ and outputs be a, b, c.

→ Truth table of Combinational Circuit:

Inputs			Outputs					
x	y	z	A	B	C	D	E	F
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	1
0	1	0	0	0	0	1	0	0
0	1	1	0	0	1	0	0	1
1	0	0	0	1	0	0	0	0
1	0	1	0	1	1	0	0	1
1	1	0	1	0	0	1	0	0
1	1	1	1	1	0	0	0	1

→ Bolean expression:

$A = xy\bar{z} + xyz \quad = xy(\bar{z} + z) = xy$

$B = x\bar{y}\bar{z} + x\bar{y}z + xyz \quad = x\bar{y}\bar{z} + xz$

$C = \bar{x}yz + x\bar{y}z \quad = z(\bar{x}y + x\bar{y}) = z(x \oplus y)$

$D = \bar{x}y\bar{z} + xy\bar{z} \quad = y\bar{z}(\bar{x} + x) = y\bar{z}$

$A = xy$

$C = z(x \oplus y)$

$B = x\bar{y}\bar{z} + xyz + x\bar{y}z$

$= x(\bar{y}\bar{z} + yz) + x\bar{y}z$

$= x(y \odot z) + x\bar{y}z$

$D = y\bar{z}$

$E = 0$

$F = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}z + xyz$

$= \bar{y}z(\bar{x} + x) + yz(\bar{x} + x)$

$= \bar{y}z + yz$

$= z(\bar{y} + y)$

$= z$

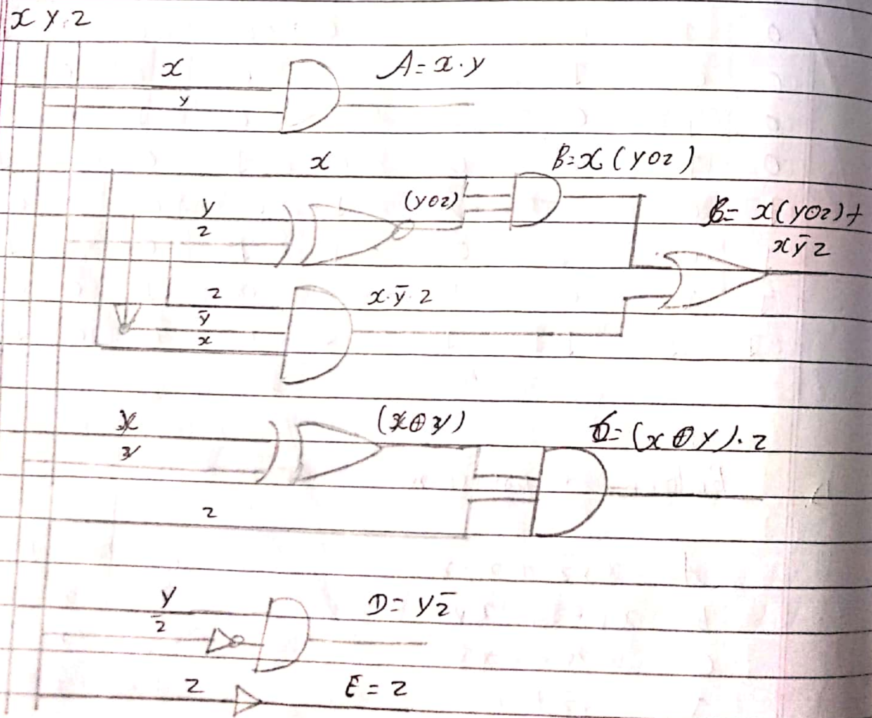


Fig: Logical diagram

(8) Gray Code to binary Converter [3-bit]

↳ design of Combinational Circuit that takes 3 bi~~numb~~ gray Code input and provides 3-bit number output.

↳ Let input be x, y, z and output be A, B, C.

↳ Truth table of Combination Circuit.

Inputs			Outputs		
x	y	z	A	B	C
0	0	0	0	0	0
0	0	1	0	0	1
0	1	1	0	1	0
0	1	0	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1
1	0	1	1	1	0
1	0	0	1	1	1

write orderly

↳ Boolean expression:

$A = xy\bar{z} + xyz + x\bar{y}z + x\bar{y}\bar{z}$

$= xy(\bar{z} + z) + x\bar{y}(z + \bar{z})$

$= xy + x\bar{y}$

$= x(y + \bar{y})$

$= x$

$B = \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}z + xy\bar{z}$ (struck through)

$= \bar{x}(yz + y\bar{z}) + x(y\bar{z} + \bar{y}z)$ (struck through)

$= \bar{x}(y \odot z) + \bar{x}(y \oplus z)$ (struck through)

$= \bar{x}(\overline{y \oplus z}) +$ (struck through)

$\bar{x}yz + \bar{x}y\bar{z} + x\bar{y}z + x\bar{y}\bar{z}$

$\bar{z}(\bar{x}y + x\bar{y}) + z(\bar{x}y + x\bar{y})$

$\bar{z}(x \oplus y) + z(x \oplus y)$

$(x \oplus y)(z + \bar{z})$

$(x \oplus y)$

$C = \bar{x}\bar{y}z + \bar{x}y\bar{z} + xyz + x\bar{y}\bar{z}$

$= \bar{z}(\bar{x}y + x\bar{y}) + z(\bar{x}\bar{y} + xy)$

$= \bar{z}(x \oplus y) + z(x \odot y)$

$= \bar{z}(x \oplus y) + z(\overline{x \oplus y})$

$= z \oplus (x \oplus y)$

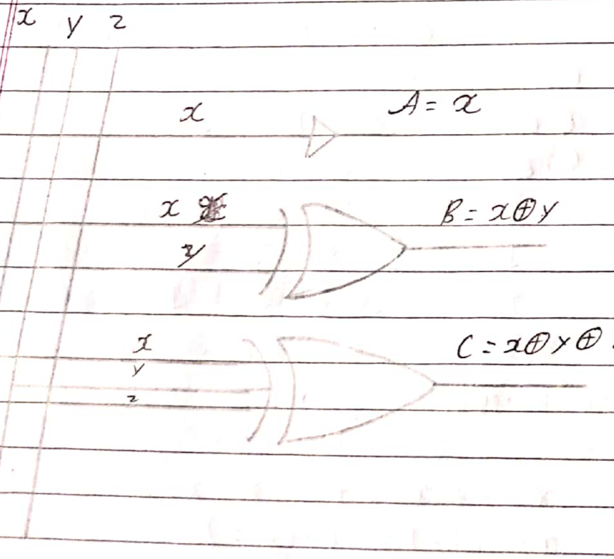


Fig: logical diagram of 3 bit gray code as input and

Date: Falgun 1 sunday

Design of Parity Generator

Design of parity checker

Parity generator are of two types:

(i) Odd parity

(ii) Even parity

Parity is a mechanism to ensure that transmissi[on] is right or not.

It can be checked by odd parity and Even parity.

In odd, Even parity, p bit depends upon the num[ber] of 1 in input.

In odd parity, If there is even number of 1 in input then we need to add more 1 in p bit. And in even parity, if there is odd number of 1 in input then we need to add more 1 in p bit to make it even.

When data is transmitted from one location to another, it is necessary to know at the receiving end whether received data is free of error. A simple form of error detection is achieved by adding an extra bit to the transmitted

Design of 3 bit odd parity generator.

- design of 3 bit odd parity generator,
- Let inputs be x, y, z and output be $p = 1$
- Truth table of 3 bit odd parity generator.

Inputs			Output
x	y	z	p
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

- Bolean expression from truth table:

$p = \bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z}$

$p = \bar{x}\,(\bar{y}\bar{z} + zy) + x\,(\bar{y}z + y\bar{z})$

$p = \bar{x}\,(y \odot z) + x\,(y \oplus z)$ | $\bar{x}(y \odot z) + x(y \oplus z)$

$p = \bar{x}\,(\overline{y \oplus z}) + x\,(y \oplus z)$ | $\bar{x}(y \odot z) + x(\overline{y \odot z})$

$p = x \odot (y \oplus z)$ | $x \oplus (y \odot z)$

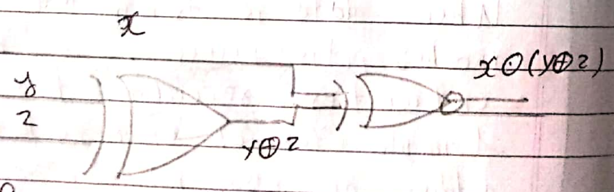


fig: logical diagram of 3 bit odd parity generator

Odd Parity Checker

H.W

- design of 3 bit odd parity checker.
- let inputs be $x, y, z, p = 4$ and outputs
- Truth table :

Inputs				Output
x	y	z	p	C
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Boolean expression:

$xy \backslash zp$	$\bar{z}\bar{p}$	$\bar{z}p$	zp	$z\bar{p}$
$\bar{x}\bar{y}$	1		1	
$\bar{x}y$		1		1
xy	1		1	
$x\bar{y}$		1		1

$$C = \bar{x}\bar{y}\bar{z}\bar{p} + \bar{x}\bar{y}zp + \bar{x}y\bar{z}p + \bar{x}yz\bar{p} + xy\bar{z}\bar{p} + xyzp + x\bar{y}\bar{z}p + x\bar{y}z\bar{p}$$

$$C = \bar{x}\bar{y}(\bar{z}\bar{p}+zp) + \bar{x}y(\bar{z}p+z\bar{p}) + xy(\bar{z}\bar{p}+zp) + x\bar{y}(\bar{z}p+z\bar{p})$$

$$C = \bar{x}\bar{y}(z\odot p) + \bar{x}y(z\oplus p) + xy(z\odot p) + x\bar{y}(z\oplus p)$$

$$C = (\bar{x}\bar{y}+xy)(z\odot p) + (\bar{x}y+x\bar{y})(z\oplus p)$$

$$C = (x\odot y)(z\odot p) + (x\oplus y)(z\oplus p)$$

$$C = (x\odot y)(z\odot p) + \overline{(x\odot y)}\,\overline{(z\odot p)}$$

$$C = x\odot y\odot z\odot p$$

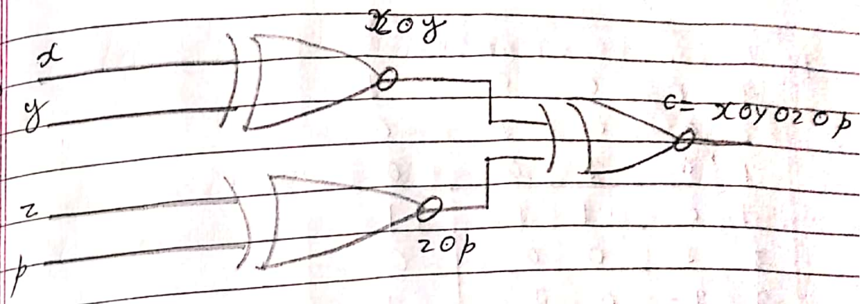


Fig: logical diagram of odd parity checker.

H.W

10) Design of 4 bit even parity gene

design of 4 bit even parity generator.

- let inputs be A, B, C, D and output be
- Total no. of input = 4 and output = 1.

Inputs				Output
A	B	C	D	P
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	01	1	0	0
0	1	1	1	1

1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

↳ Bolean expression:

by using K-Map,

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1		1
$\bar{A}B$	1		1	
AB		1		1
$A\bar{B}$	1		1	

$p = A \oplus B \oplus C \oplus D$

odd ko gada kheri
XNOR ako thyo vanesi
Even ma XOR Aauxa

↳

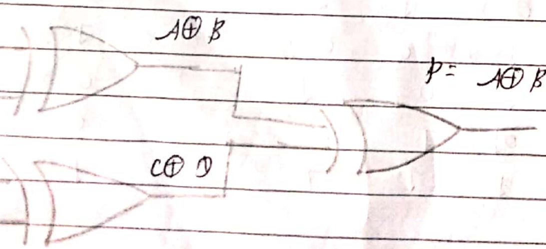


Fig: Logical diagram of 4 bit even

Even Parity Checker

↳ design of 3 bit even polarity checker.

↳ Total no. of input = 4 and output = 1.

↳ let inputs be ~~A, B, C, D~~ x, y, z & p and output be C.

↳ Truth table of 3 bit even polarity checker.

Inputs				Outputs
x	y	z	p	C
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Bolean expression:

xy \ zp	$\bar z\bar p$	$\bar z p$	zp	$z\bar p$
$\bar x\bar y$		1		1
$\bar x y$	1		1	
xy		1		1
$x\bar y$	1		1	

$$C = \bar x\bar y\bar z p + \bar x\bar y z\bar p + \bar x y\bar z\bar p + \bar x y z p + xy\bar z p + xyz\bar p + x\bar y\bar z\bar p + x\bar y z p$$

$$C = \bar z p(\bar x\bar y + xy) + z\bar p(\bar x y + x\bar y) + \bar z\bar p(\bar x y + x\bar y) + z\bar p(\bar x\bar y + xy)$$

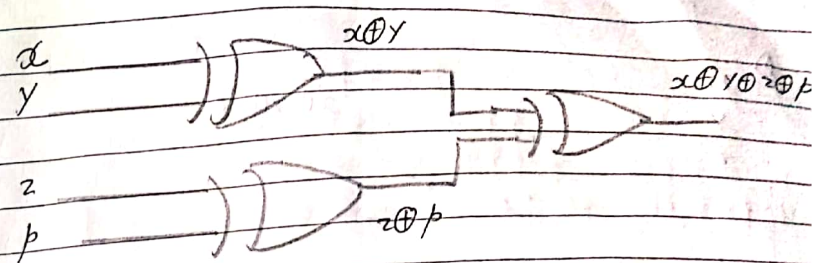
$$C = \bar z p(x\odot y) + zp(x\oplus y) + \bar z\bar p(x\oplus y) + z\bar p(x\odot y)$$

$$C = (\bar z p + z\bar p)(x\odot y) + (zp + \bar z\bar p)(x\oplus y)$$

$$C = (z\oplus p)(x\odot y) + \cancel{(z\oplus p)(x} \; (z\odot p)(x\oplus y)$$

$$C = \underbrace{(z\oplus p)}_{B}\underbrace{\overline{(x\oplus y)}}_{\bar A} + \underbrace{\overline{(z\oplus p)}}_{\bar B}\underbrace{(x\oplus y)}_{A}$$

$$C = x\oplus y\oplus z\oplus p$$



design of 3 bit even parity generato

design of 3 bit even parity generator.

- design of 3 bit even parity generator.
- no. of input = 3 and output = 1.
- let input be x, y, z and output be p.
- Truth table:

Inputs			Outputs
x	y	z	p
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- Boolean expression:

$$\bar x\bar y z + \bar x y\bar z + x\bar y\bar z + xyz$$

$$z(\bar x\bar y + xy) + \bar z(\bar x y + x\bar y)$$

$$z(x\odot y) + \bar z(x\oplus y)$$

$$z\overline{(x\oplus y)} + \bar z(x\oplus y) \Rightarrow x\oplus y\oplus z$$

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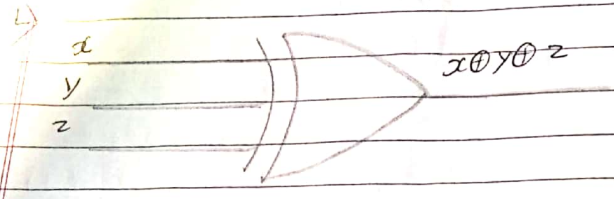

Fig: logical diagram of 3 bit even parity generator

Odd Parity :

[3 bit] odd parity generator - $x \odot (y \oplus z)$
[3 bit] odd parity checker - $x \odot y \odot z \odot p$
[4 bit] odd parity generator - $x \odot y \odot z \odot w$

Even Parity :

[3 bit] Even parity generator = $x \oplus y \oplus z$
[3 bit] Even parity checker = = $x \oplus y \oplus z \oplus p$
[4 bit] Even parity generator = $x \oplus y \oplus z \oplus w$

Odd	Even
P.G - $x \odot (y \oplus z)$	P.G. - $x \oplus y \oplus z$
P.C - $x \odot y \odot z \odot p$	P.C. - $x \oplus y \oplus z \oplus p$
P.G - $w \oplus x \odot y \odot z$	P.G. - $x \oplus y \oplus z \oplus w$

falgun-13
monday
1st class

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Carry Propagation

There are two types of carry:

(I) Input Carry $[C_i]$
(II) Output Carry $[C_{i+1}]$

let $P_1 = A_1 \oplus B_1$
$G_1 = A_1 B_1$

For full adder circuit,

$$Sum_i = A_1 \oplus B_1 \oplus C_1 = P_1 \oplus C_1$$

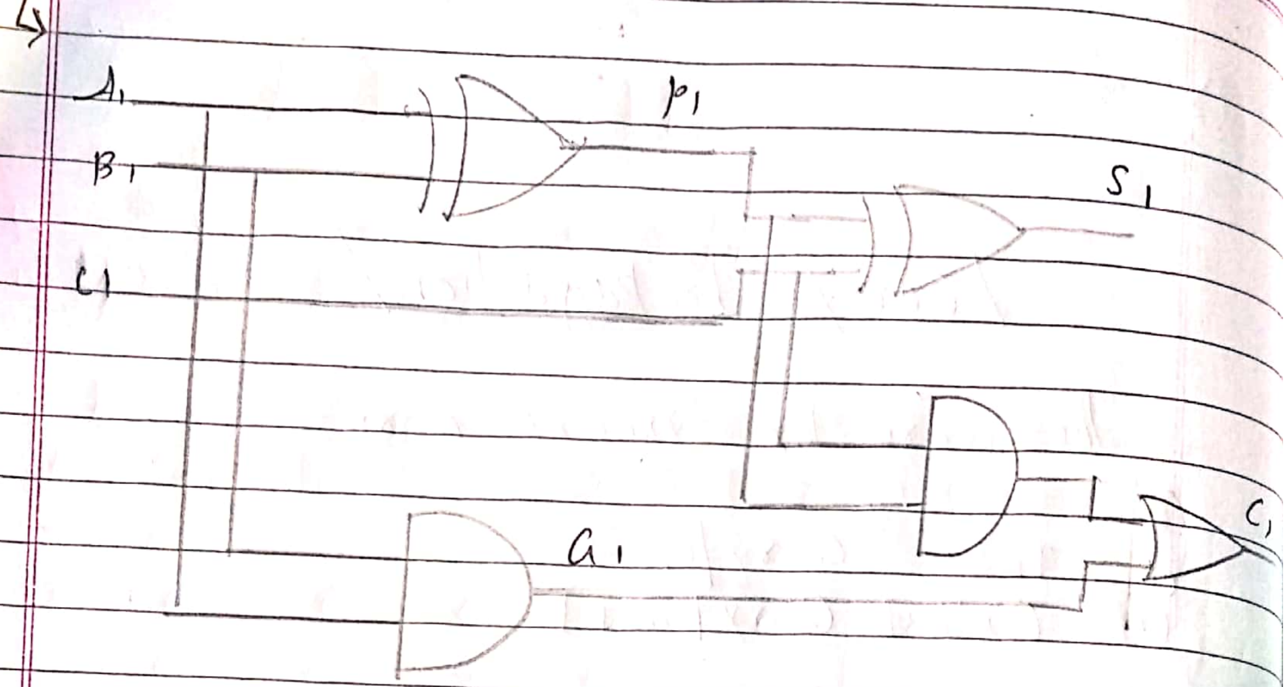
$$Carry_{i+1} = (A_1 \oplus B_1) C_1 + A_1 B_1 = P_1 C_1 + G_1 \quad -(1)$$

$$C_1 = P_0 C_0 + G_0$$

when $i = 1$, $C_2 = P_1 C_1 + G_1 = G_1 + P_1 G_0 + P_1 P_0 C_0$

when $i = 2$, $C_3 = P_2 C_2 + G_2$
$= \cancel{P_2 (P_1 C_1 + G_1)}$ $P_2 (P_1 C_1 + G_1) + G_2$
$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 G_0 P_0$

when $i = 3$, $C_4 = P_3 C_3 + G_3$
$= \cancel{P_3 (P_1 C_1 + G_1)}$



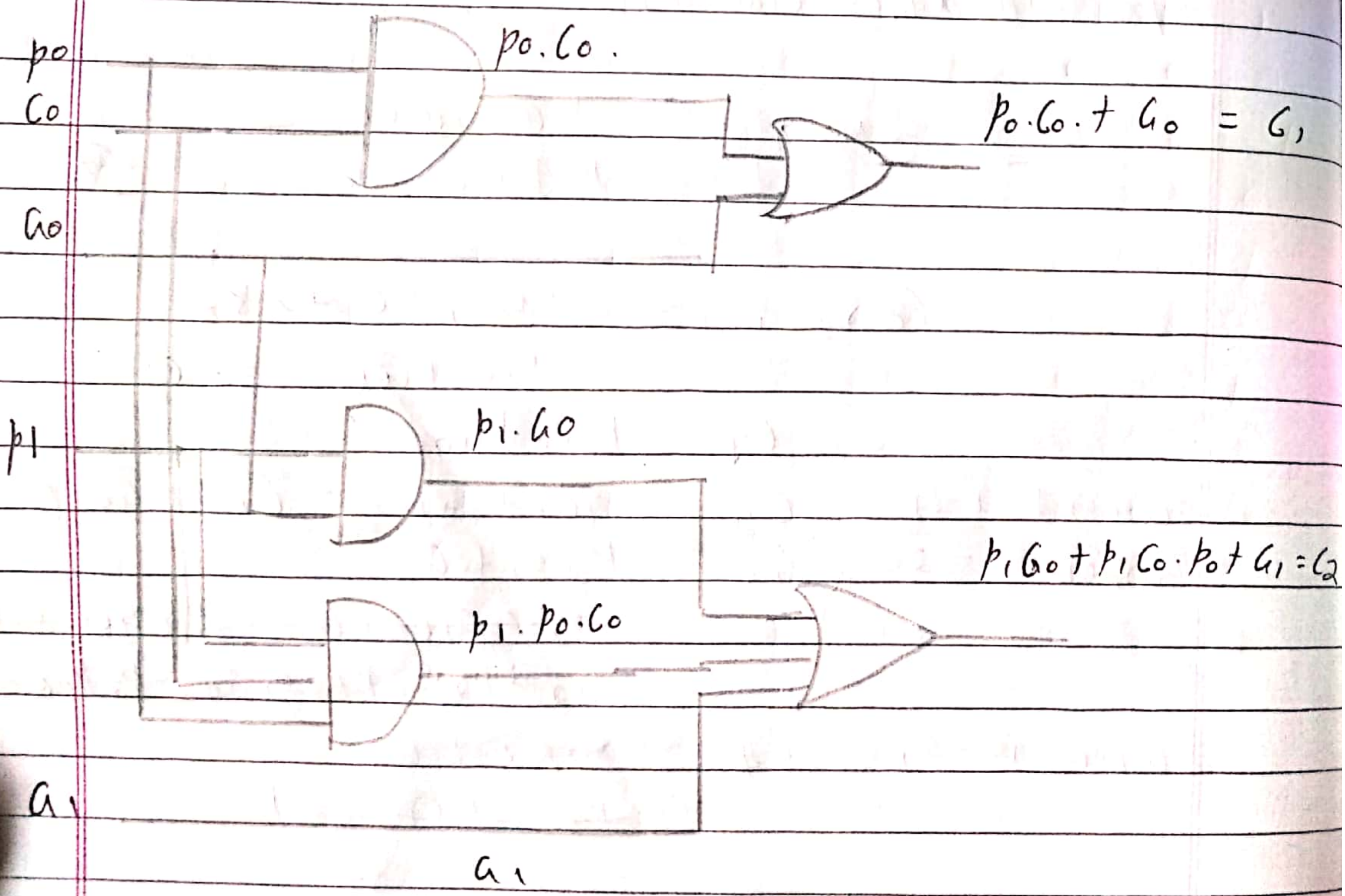


fig: logical diagram

(11) design of Bcd adder.

BCD ADDER

↳ Design of BCD Adder,
↳ Truth table:

Decimal	Binary Sum					BCD Sum				
	K	Z_8	Z_4	Z_2	Z_1	C	S_8	S_4	S_2	S_1
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	1
2	0	0	0	1	0	0	0	0	1	0
3	0	0	0	1	1	0	0	0	1	1
4	0	0	1	0	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	1
6	0	0	1	1	0	0	0	1	1	0
7	0	0	1	1	1	0	0	1	1	1
8	0	1	0	0	0	0	1	0	0	0
9	0	1	0	0	1	0	01	0	0	1
10	0	1	0	1	0	1	0	0	0	0
11	0	1	0	1	1	1	0	0	0	1
12	0	1	1	0	0	1	0	0	1	0
13	0	1	1	0	1	1	0	0	1	1
14	0	1	1	1	0	1	0	1	0	0
15	0	1	1	1	1	1	0	1	0	1
16	1	0	0	0	0	1	0	1	1	0
17	1	0	0	0	1	1	0	1	1	1
18	1	0	0	1	0	1	1	0	0	0
19	1	0	0	1	1	1	1	0	0	1
	1	0	1	0	0					
	1	0	1	0	1					

↳ ~~Boolean expression: by using K-map~~

~~$Z_4.Z_2.Z_1$~~

~~$K Z_8$ | $\bar{Z}_4\bar{Z}_2\bar{Z}_2$ | $Z_4 Z_2$ | $Z_4.Z_2$~~

~~$\bar{K}\bar{Z}_8$~~

~~$\bar{K}Z_8$~~

~~KZ_8~~

~~$K\bar{Z}_8$~~

↳ Bolean expression by using K-map,

$K.Z_8$ \ $Z_4.Z_2.Z_1$	$\bar{Z}_4\bar{Z}_2\bar{Z}_1$	$\bar{Z}_4\bar{Z}_2Z_1$	$\bar{Z}_4Z_2Z_1$	$\bar{Z}_4Z_2\bar{Z}_1$	$Z_4Z_2\bar{Z}_1$	$Z_4Z_2Z_1$	$Z_4\bar{Z}_2Z_1$	$Z_4\bar{Z}_2\bar{Z}_1$
$\bar{K}.\bar{Z}_8$	0	1	2	3	6	7	5	4
$\bar{K}.Z_8$	8	9	1 11	1 10	1 14	1 15	1 13	1 12
$K.Z_8$	X 24	X 25	X 27	X 26	X 30	X 31	X 29	X 28
$K.\bar{Z}_8$	1 16	1 17	1 19	1 18	X 22	X 23	X 21	X 20

$C = K + Z_8 Z_4 + Z_8.Z_2$

↳

$z_8.z_4$	$\bar{z}_2\bar{z}_1$	$\bar{z}_2.z_1$	$z_2.z_1$	$z_2.\bar{z}_1$
$\bar{z}_8\bar{z}_4$				
$\bar{z}_8 z_4$				
$z_8.z_4$	1	1	1	1
$z_8.\bar{z}_4$			1	1

$$C = z_8.z_4 + z_8.z_2$$

↳