Assignment 2: Numerical Modelling of Aerocapture for a Scientific Mission to Venus

(40% Unit Weighting – Main Deadline: 17:00, Wed 2 Dec)

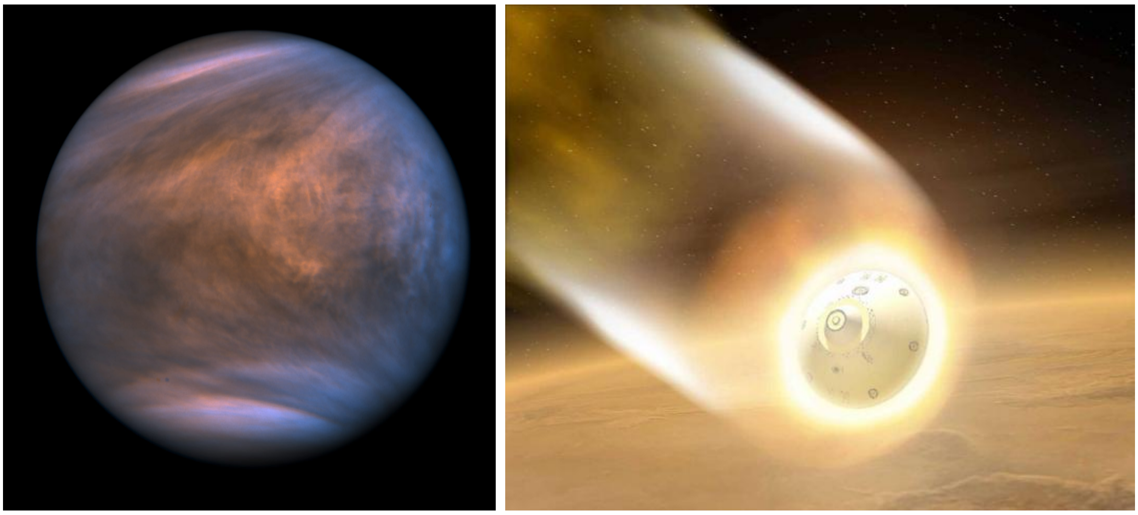

Figure 1: The use of aerocapture in a mission to Venus will allow up to 250% increase in the scientific payload.[†‡]

1 Outline

Aerocapture is a cost-effective technique for inserting a spacecraft into orbit around a planet or moon. It uses drag from the body's atmosphere to reduce the spacecraft's velocity and, thus, transition from a hyperbolic approach trajectory into an elliptical orbit. (Conventionally, propellent would be used instead to generate reactive thrust.) Currently, aerocapture is only a theoretical possibility and not a practical reality. A space science and technology organisation is aiming to use aerocapture to put a scientific satellite into orbit around Venus – Figure 1. **You are part of the engineering team developing the aerocapture system.**

The first phase of the mission consists of a cruise from Earth that puts the spacecraft on a trajectory towards Venus at a velocity of $v_0 = 11$ km/ s. The desired scientific orbit is at an altitude of $H = 1,200$ km above the surface. The mass of the spacecraft is $m = 1,500$ kg and its cross-sectional area is $A = 9$ m^2. Your colleagues in the fluid dynamics team have carried out drag modelling and experimentation to determine that the spacecraft's drag coefficient is $C_d = 1.25$. Colleagues in the astrophysics team have provided an estimate

[†] P. Girija *et al.*, "*Feasibility and Mass-Benefit Analysis of Aerocapture for Missions to Venus*", Journal of Spacecraft and Rockets, Vol. 57, No. 1, Jan 2020

[‡] Image sources:
https://en.wikipedia.org/wiki/Atmosphere_of_Venus/media/File:Global_view_uvi_Venus_(Akatsuki).jpg;
https://spaceflightnow.com/mars/mera/030529landing.htmlflightnow.com/mars/mera/030529landing.html

for the radius of Venus $R = 6,051.8$ km and its mass $M = 4.8675 \times 10^{24}$ kg. They have also produced an atmospheric profile model, which has been implemented in the MATLAB function `profileVenus.m` (supplied). This function gives the atmosphere density in kg/ m^3 with respect to altitude in meters.

2 Tasks

Within the aerocapture team, it is your responsibility to develop the MATLAB software for computing the insertion height α. This will be used within the spacecraft's guidance and control system.

Consider 2-D coordinates centred on Venus and an approach trajectory that puts the spacecraft at an initial distance $x_0 = 10,000$ km and height $\alpha = (y_0 - R)$ at time $t_0 = 0$ s. The geometry and these initial conditions are illustrated in Figure 2. To test and demonstrate your software, you must **compute the value of α that will result in the spacecraft being captured and achieving an apoapsis (semi-major axis in the elliptical orbit) corresponding to the desired scientific orbit with an altitude of** H, as illustrated in Figure 2. **You must also specify the resulting minimum altitude** h for use by the thermodynamics modelling team.

Note that the resulting trajectory will eventually cause the spacecraft to crash into the surface or burn up on re-entry unless a "burn" (reactive thrust) is performed at the apoapsis to transition from an elliptical to circular orbit. However, this aspect of the aerocapture procedure is not of immediate concern and can be ignored for now.

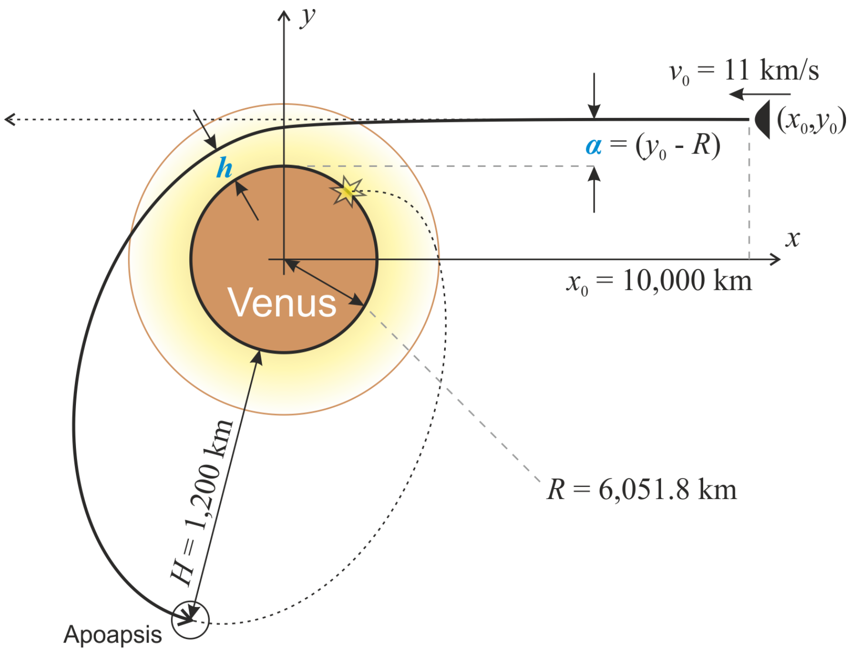

Figure 2: Illustration of the aerocapture geometry (not to scale) and boundary conditions.

You can achieve your goal by following these steps:

1. Draw the free-body diagram for the spacecraft and use this to derive the coupled 1st-order ordinary differential equations (ODEs) governing its motion in x and y.

2. Modify the function `stateDeriv` from Assignment 1 to compute the state derivative vector for these ODEs.

3. Modify the function `ivpSolver` from Assignment 1 to compute and plot the trajectory of the spacecraft for the free variable α.

Test that your solver for the initial value problem is working correctly before moving on to the next step.

4 Create a new function that uses the shooting method to compute the value of α required to achieve the desired orbit. This function should reuse the functions from the previous steps.

It is a good idea to make a plan for implementing your shooting method function before beginning to generate code, e.g., use pseudo-code and / or a flow diagram.

Once you have solved the problem – well done! However, this is the minimum requirement to pass the assignment.

3 Extension Tasks

Like most engineering problems, this one is open-ended. To access a higher grade, you must use your own creativity to develop the problem further. How you choose to do this is completely up to you, but the following thoughts might provide inspiration:

- Consider the assumptions that have been made in arriving at your solution. How can these be improved to provide more realism? e.g.,
  - Can the drag be modelled more accurately?
  - What would be the effect of lift?
  - How would a 3-D geometry combined with rotation of the planet affect the problem?
- Are there other aspects of the mission that could be modelled? e.g.,
  - The subsequent "burn" for circular orbit
  - The launch and cruise phases of the mission
  - Can the effects of uncertainty be modelled to determine a safe operating window?
- Can your code be made more robust, accurate, or efficient?
- Can your code be generalised? e.g., can the parameters be provided as user inputs?
- Would a graphical user interface be useful?
- Perhaps you could create a "game" environment for training ...

4 Deliverables

Report

Summarise your work in a brief report. This must include:

- The derivation of your ODE

- Graph(s) of the spacecraft trajectory, indicating the points corresponding to the apoapsis and minimum altitude.
- Discussion of the simplifying assumptions
- Descriptions and results of the extension tasks
- MATLAB code in an appendix (not included in the page limit)

The report is strictly limited to *8 pages*; this includes title, abstract, figures, and references, but excludes the appendices..

Do not provide a table of contents (this is unnecessary in a brief report). Do not use unreasonably small fonts or margins to satisfy the page limit (this adversely affects readability). Except for the MATLAB code, any other material provided in the appendices will be considered as supplementary and will not be assessed.

MATLAB Code

Put all of your code into a single Zip file.

5 Submission Deadlines

Work-in-Progress (0% Weighting): 17:00, Wed 25 Nov

Submit your MATLAB code (Zip) as "work-in-progress" to the *progress submission portal* on Moodle. It is not expected that your code will be complete. However, a large proportion of it should be functional and nearing completion.

Final Report & MATLAB Code (85% Weighting): *17:00, Wed 2 Dec*

Submit your report (PDF) and final MATLAB code (Zip) to the *main submission portal* on Moodle.

Peer-Review (15% Weighting): 17:00, Wed 9 Dec

Shortly after the main submission deadline, you will be asked to assess the performance of the peers in your group. This will be done via the *peer-review portal* on Moodle.