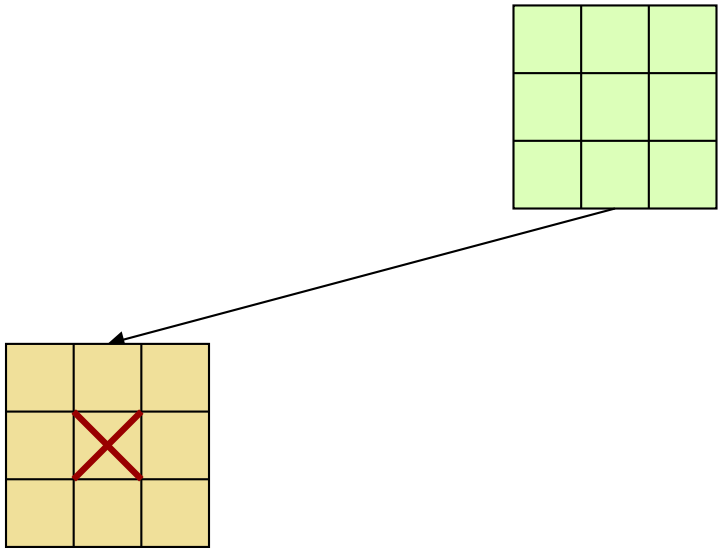
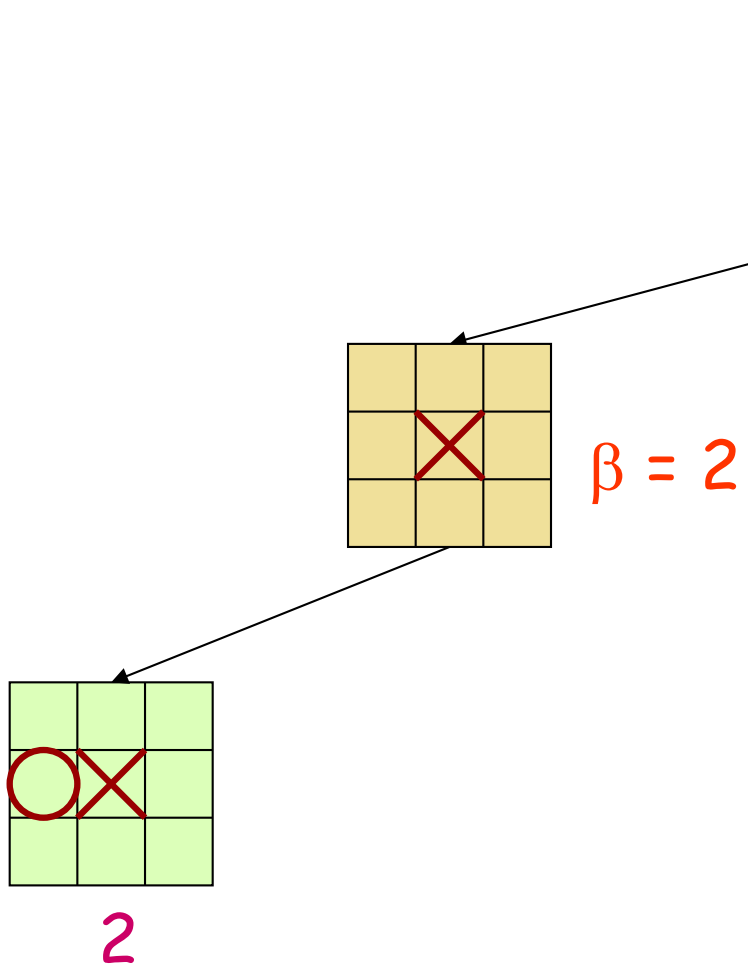
Idea of Alpha-Beta Pruning

- α: Best already explored option along path to the root for maximizer.
- β: Best already explored option along path to the root for minimizer.
- Explore the game tree to depth h in depth-first manner
- Back up α and β values whenever possible
- Prune branches that can’t lead to changing the final decision

Example

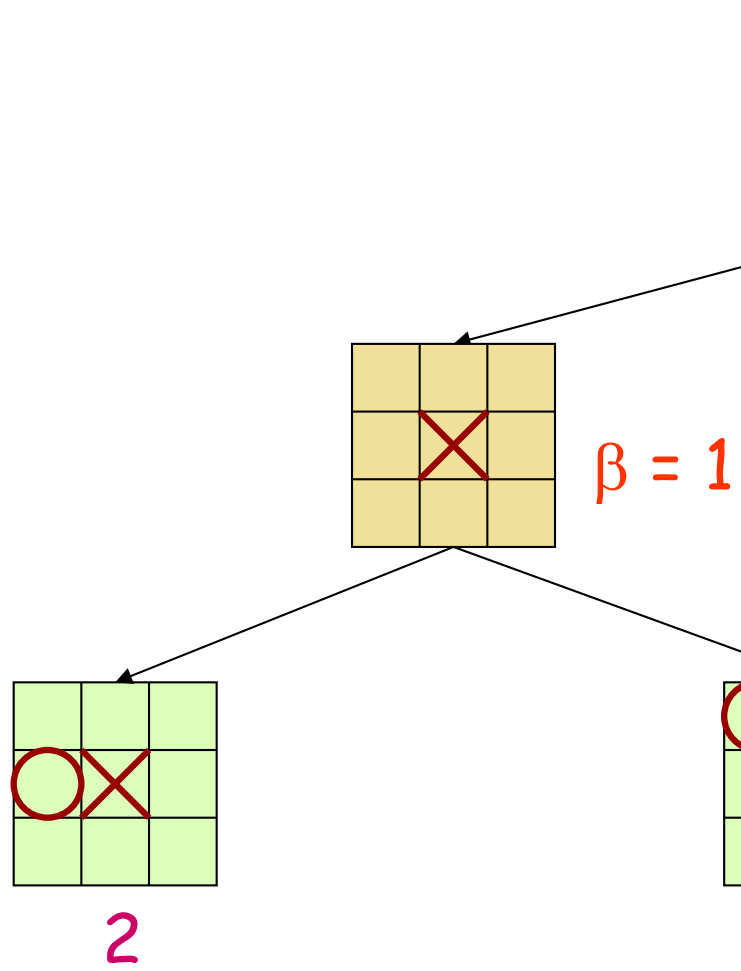


Example



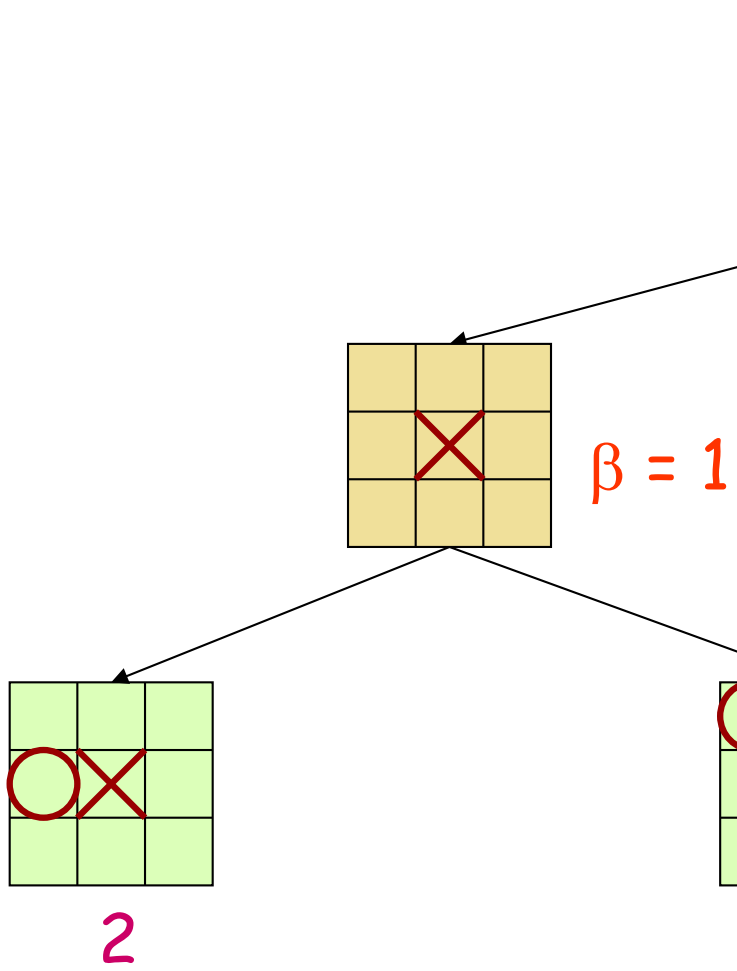
The beta value of a MIN node is an **upper** bound on the final backed-up value. It can never increase

Example



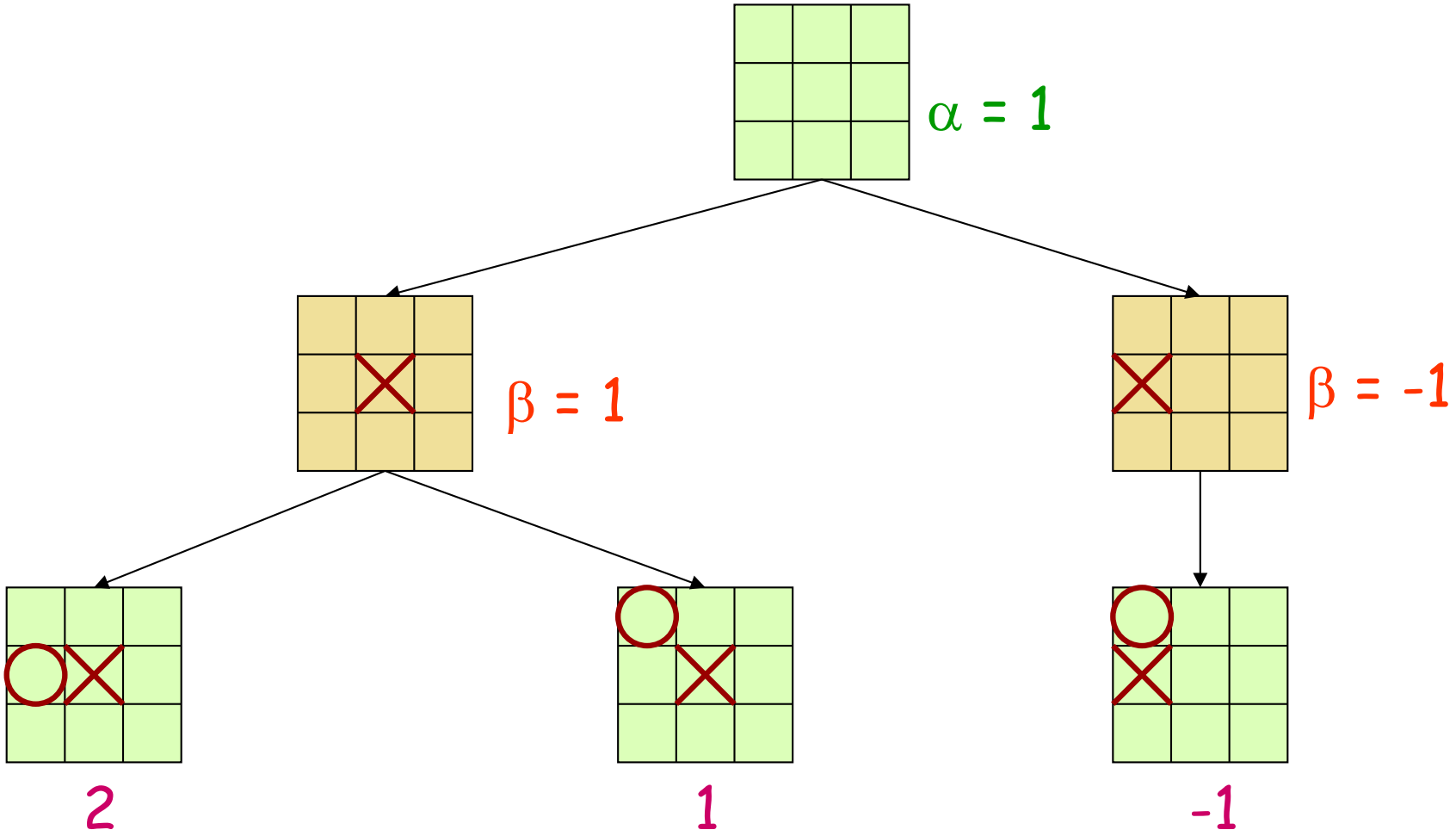
The beta value of a MIN node is an **upper** bound on the final backed-up value. It can never increase

Example

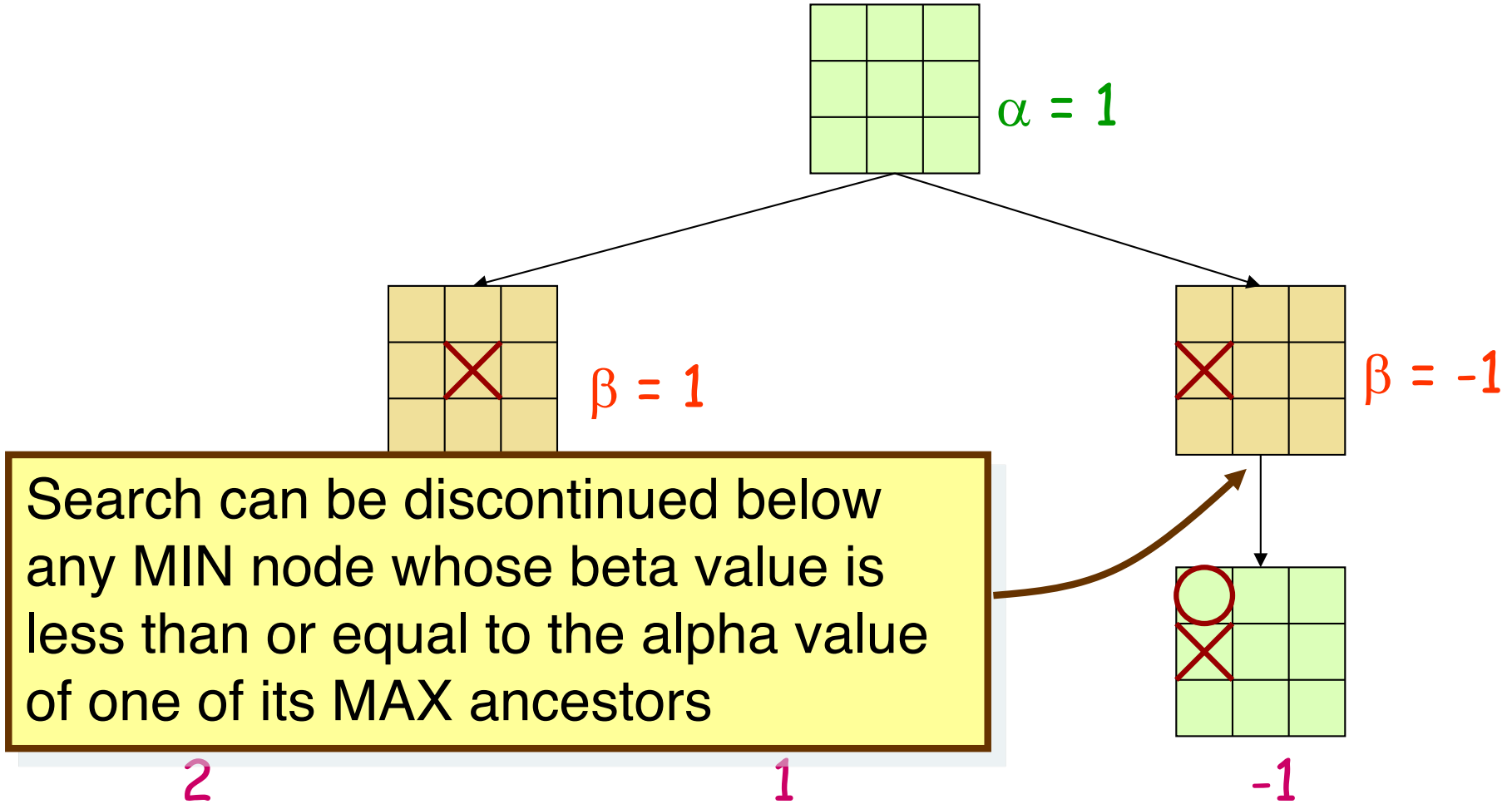


The alpha value of a MAX node is a **lower** bound on the final backed-up value. It can never decrease

Example



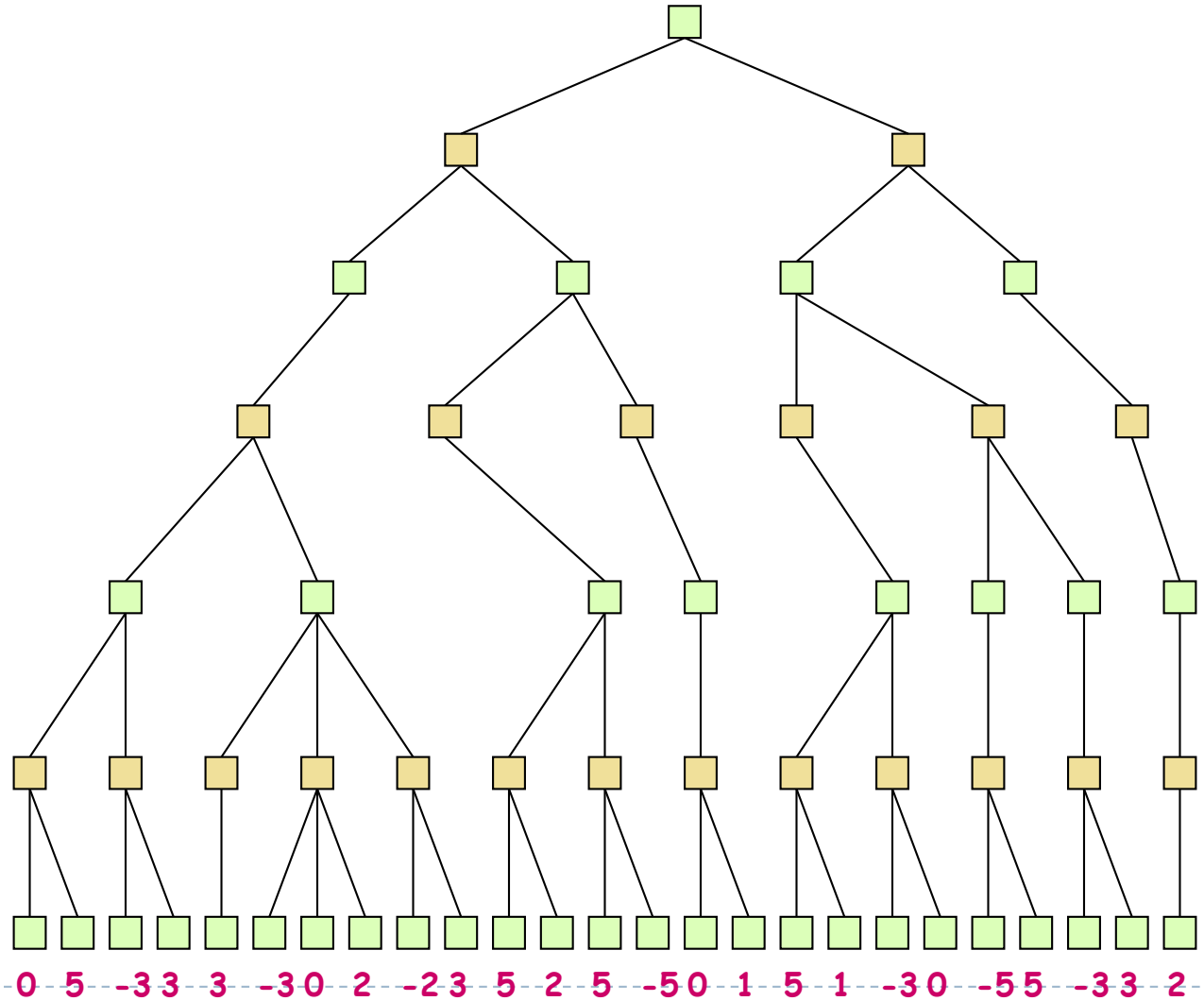
Example


α = 1
β = 1
β = -1
Search can be discontinued below any MIN node whose beta value is less than or equal to the alpha value of one of its MAX ancestors
2
1
-1

Alpha-Beta Algorithm: When to Prune?

- Update the alpha/beta value of the parent of a node N when the search below N has been completed or discontinued
- Discontinue the search below a MAX node N if its alpha value is ≥ the beta value of a MIN ancestor of N
- Discontinue the search below a MIN node N if its beta value is ≤ the alpha value of a MAX ancestor of N

Example



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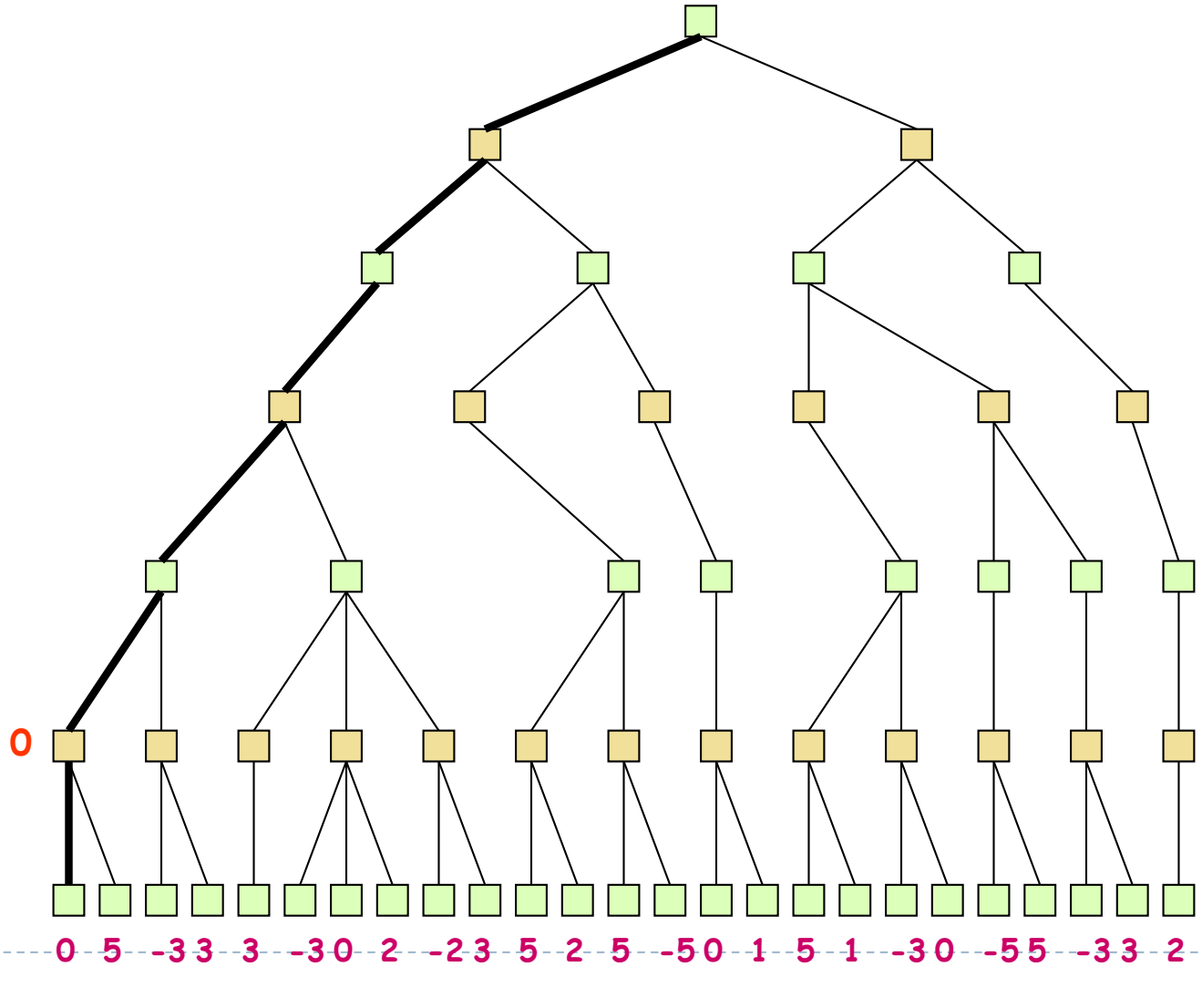

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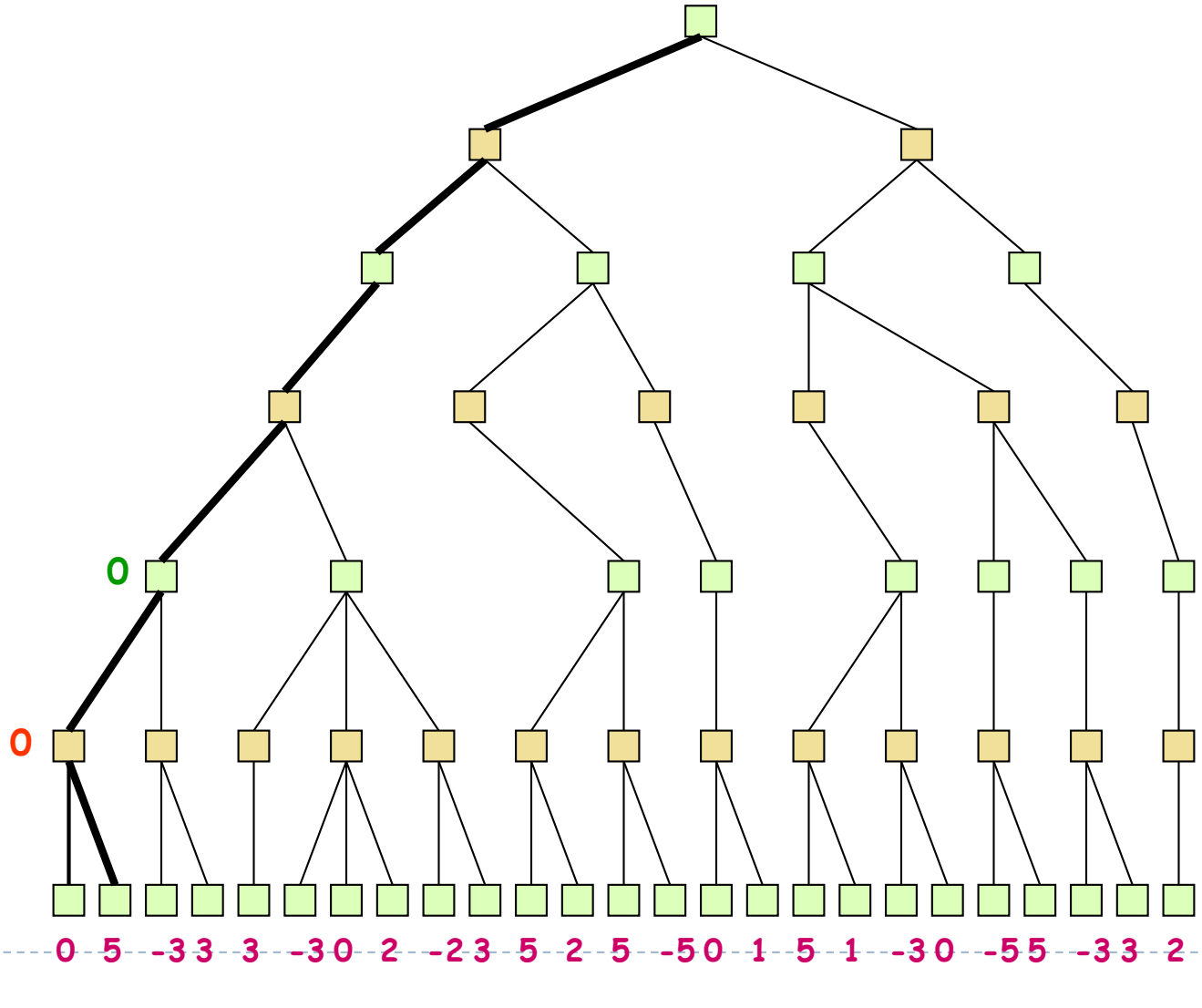

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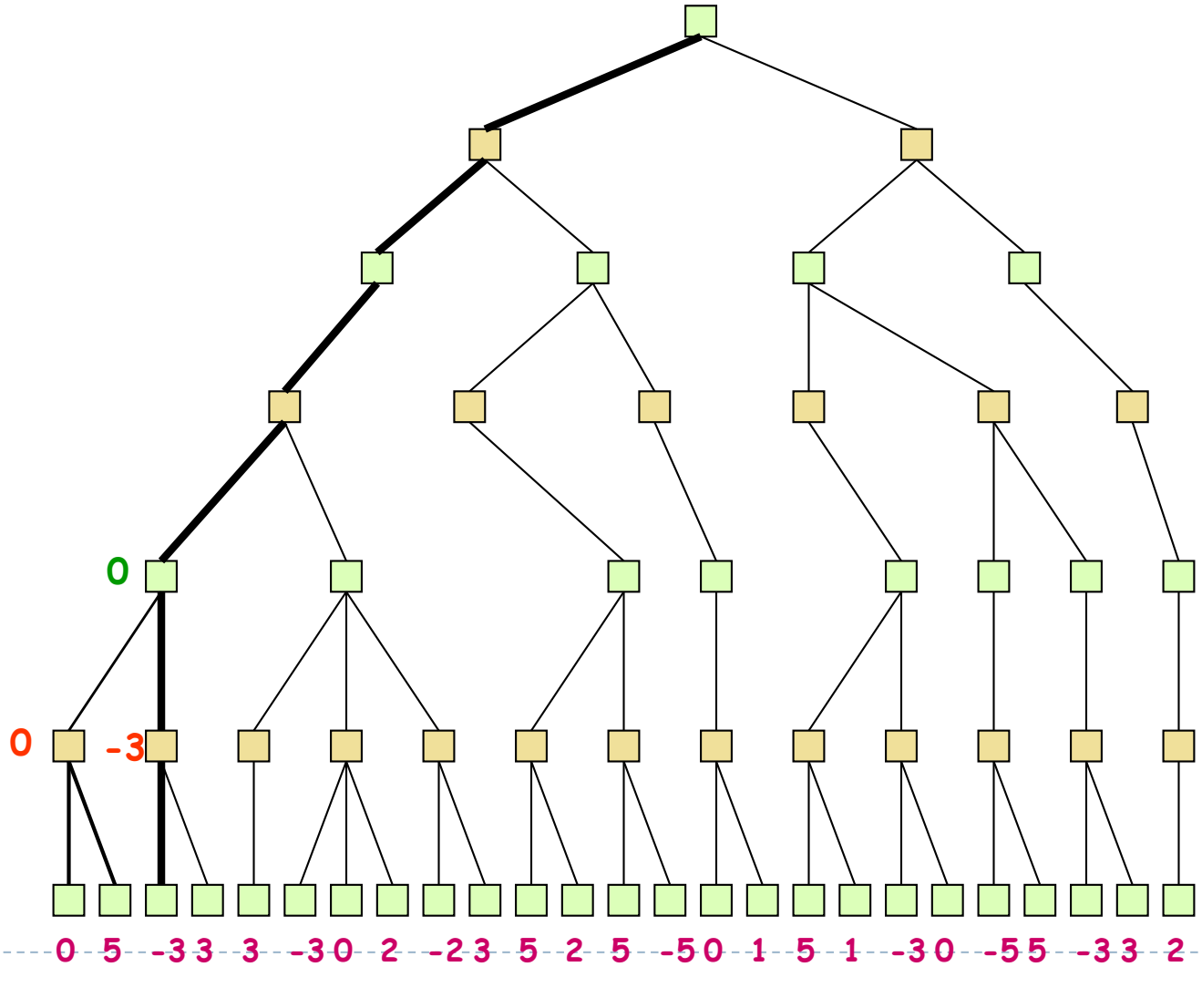

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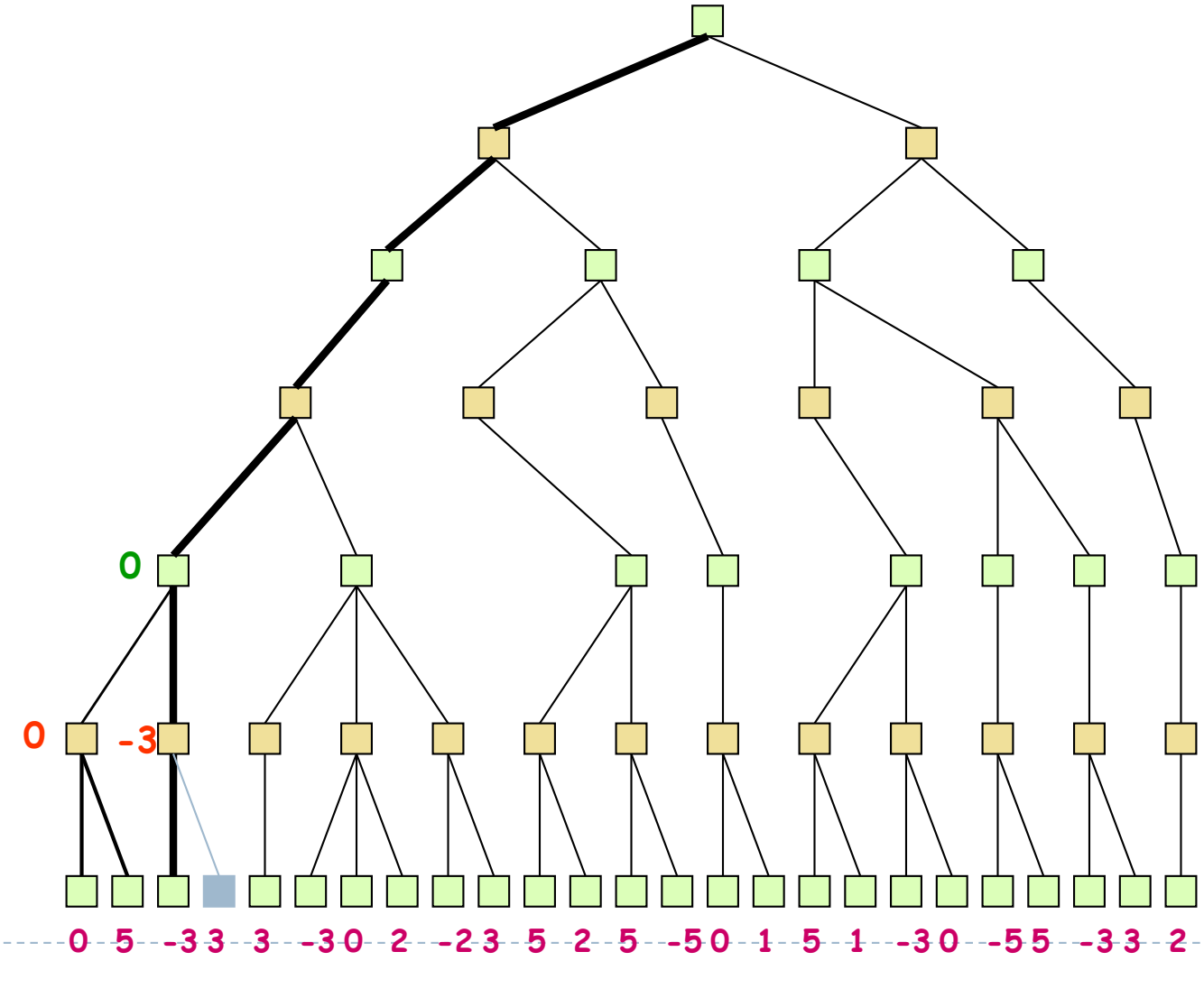

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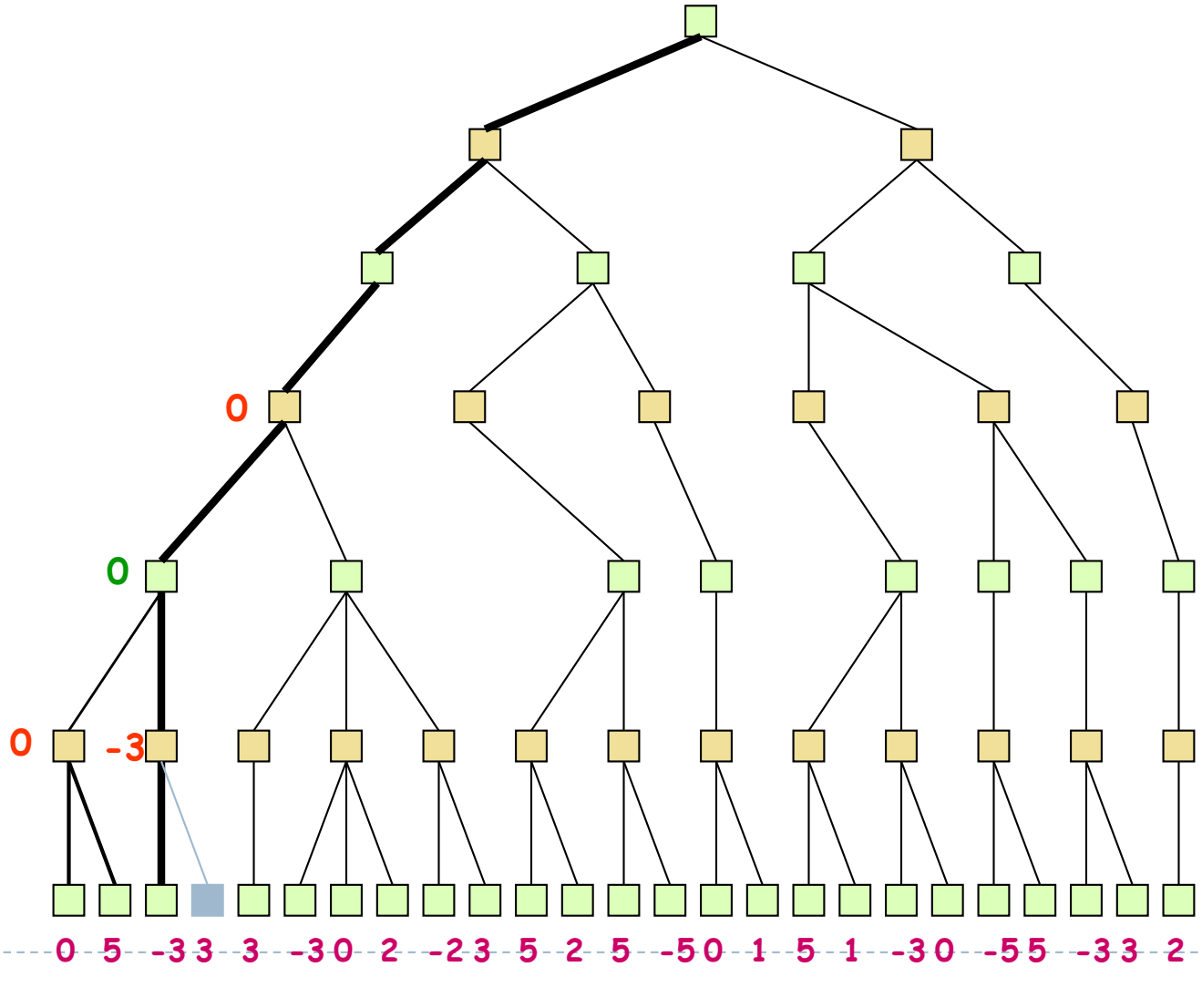

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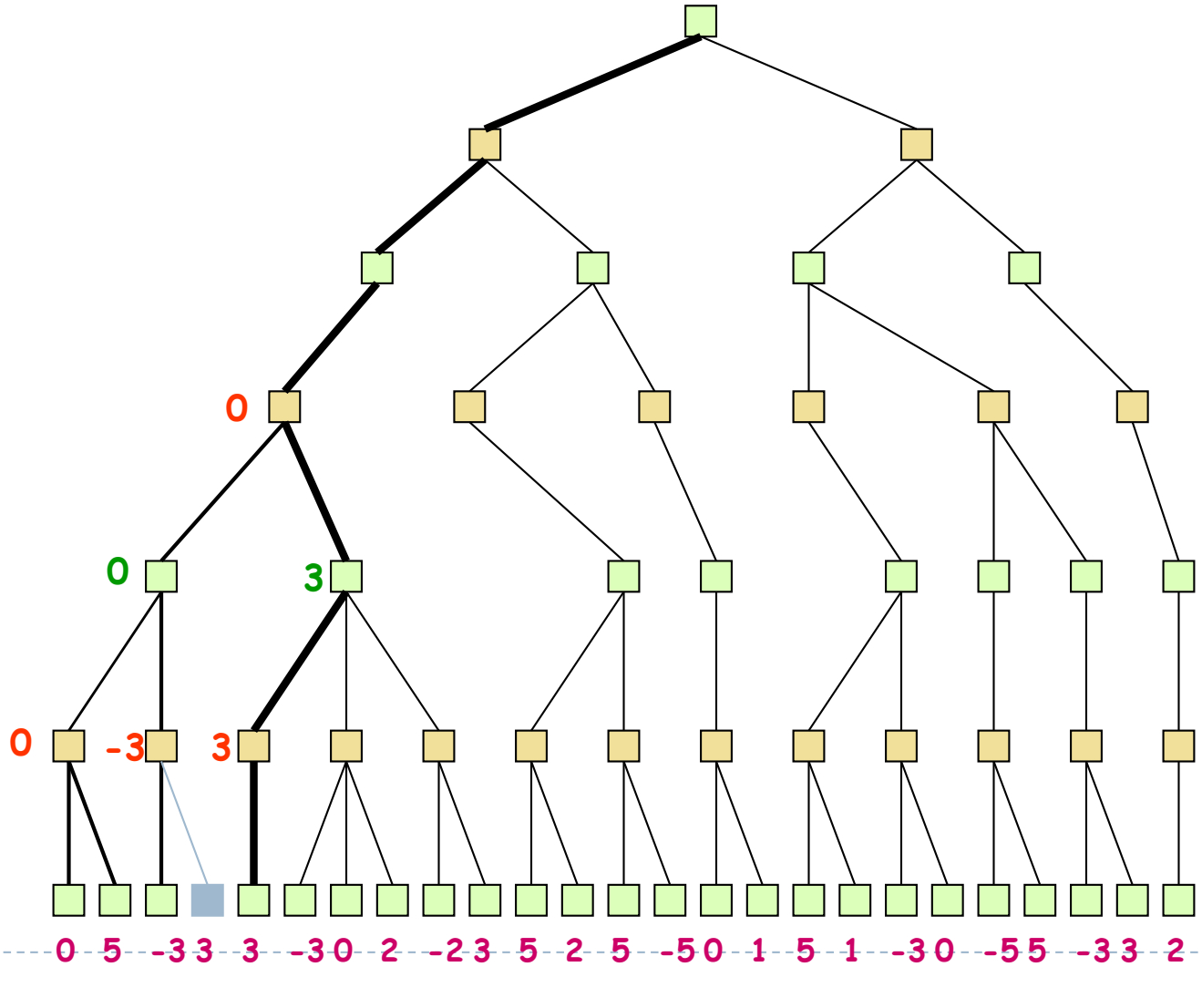

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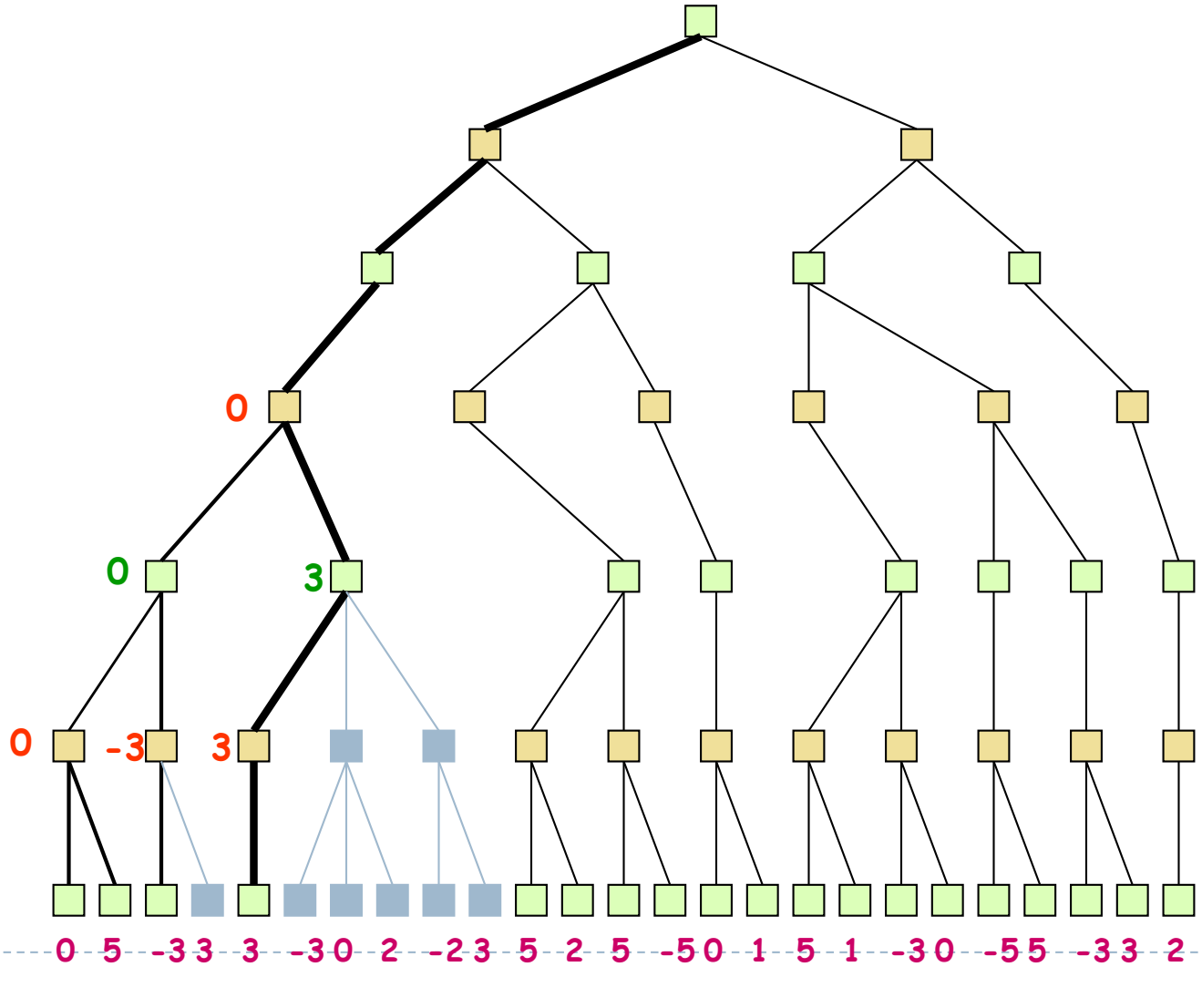

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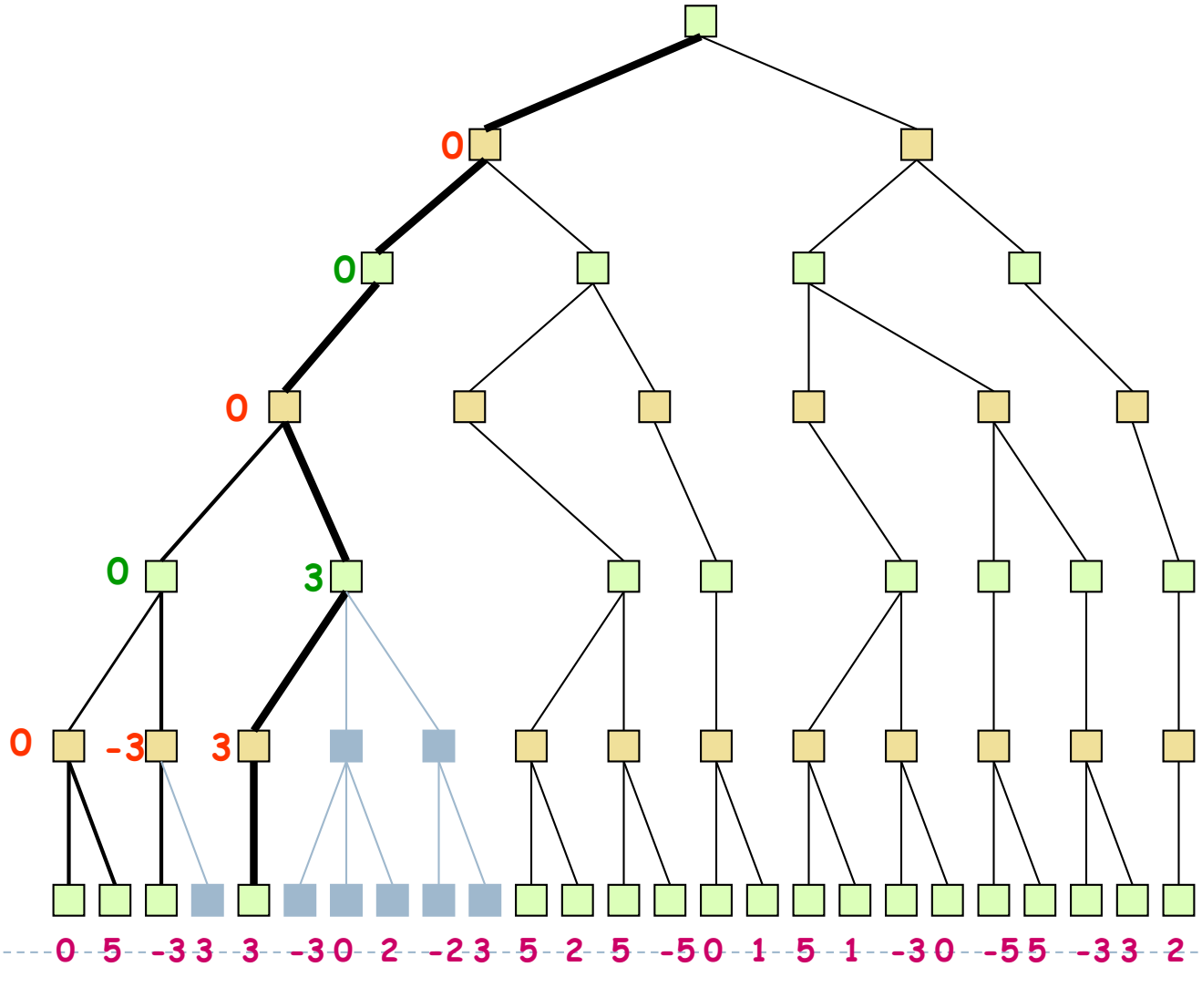
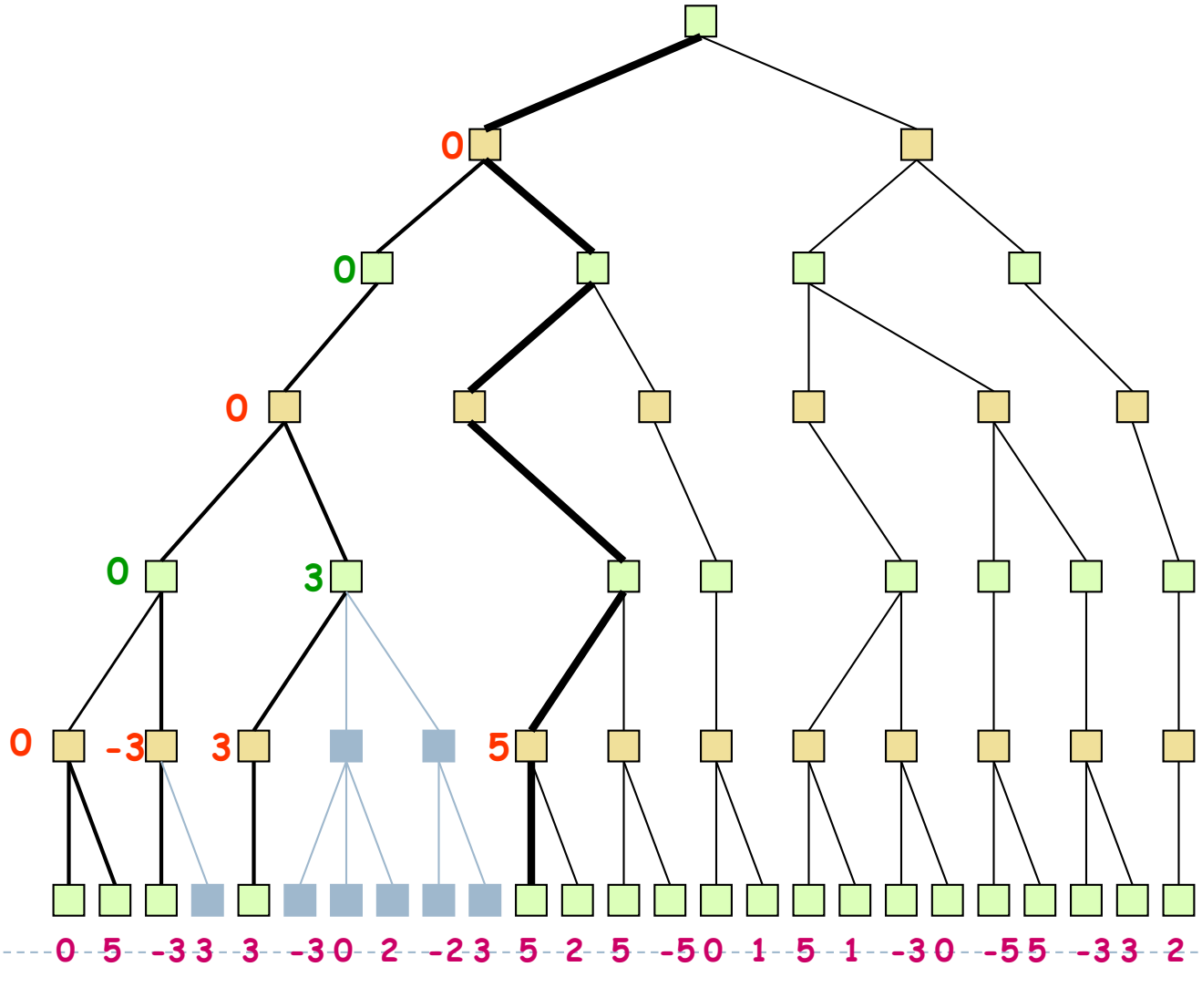


Example



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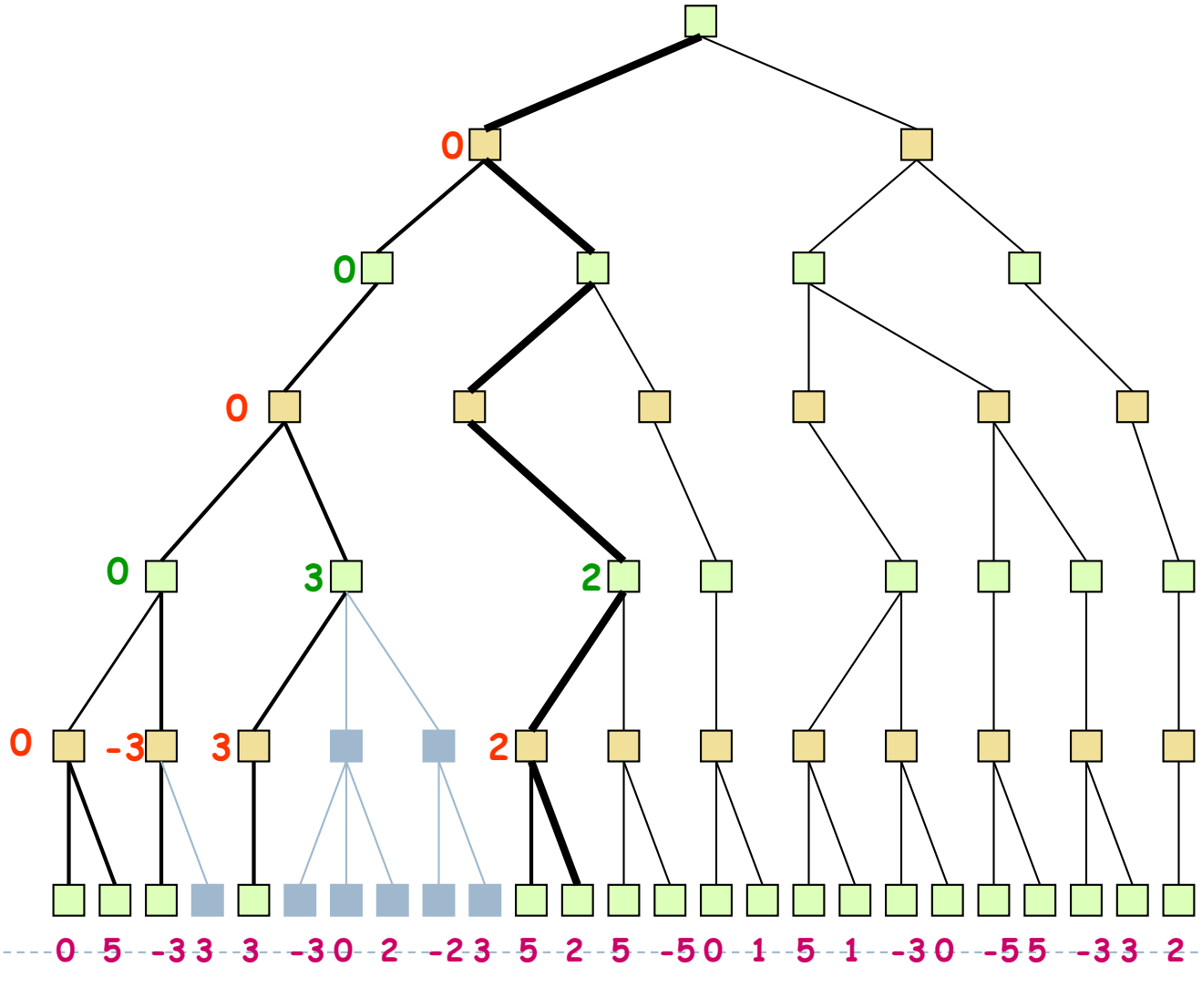

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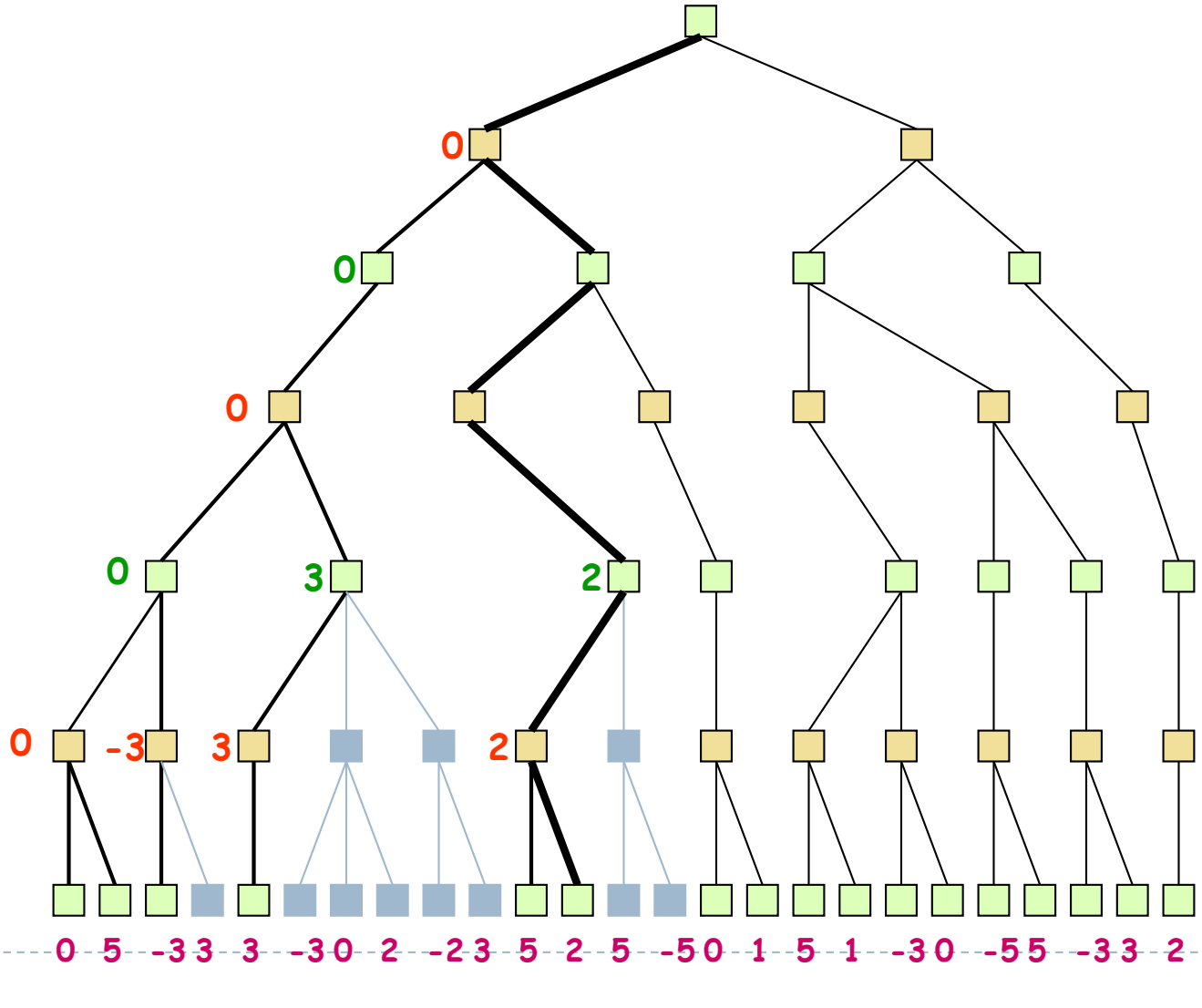

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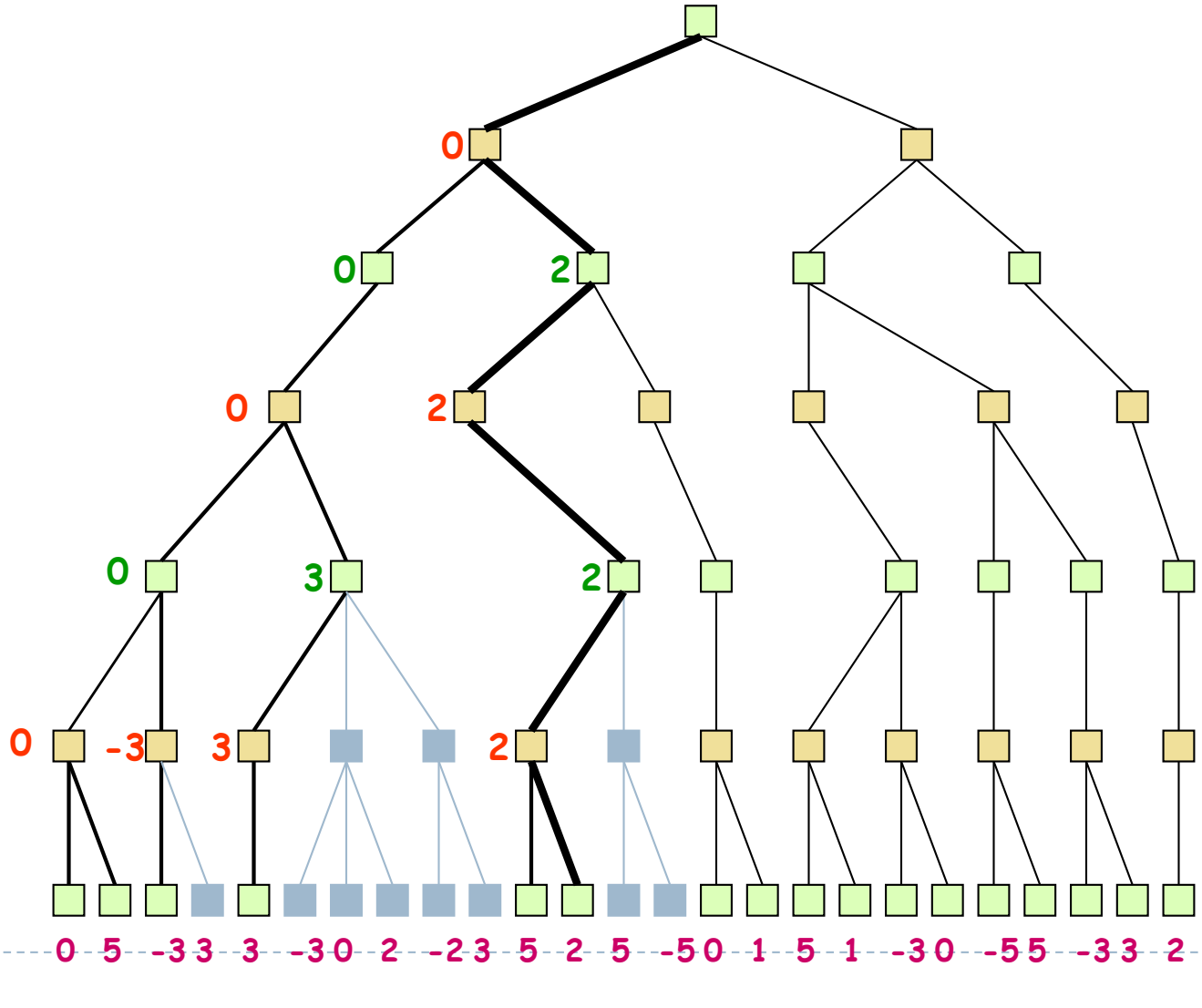

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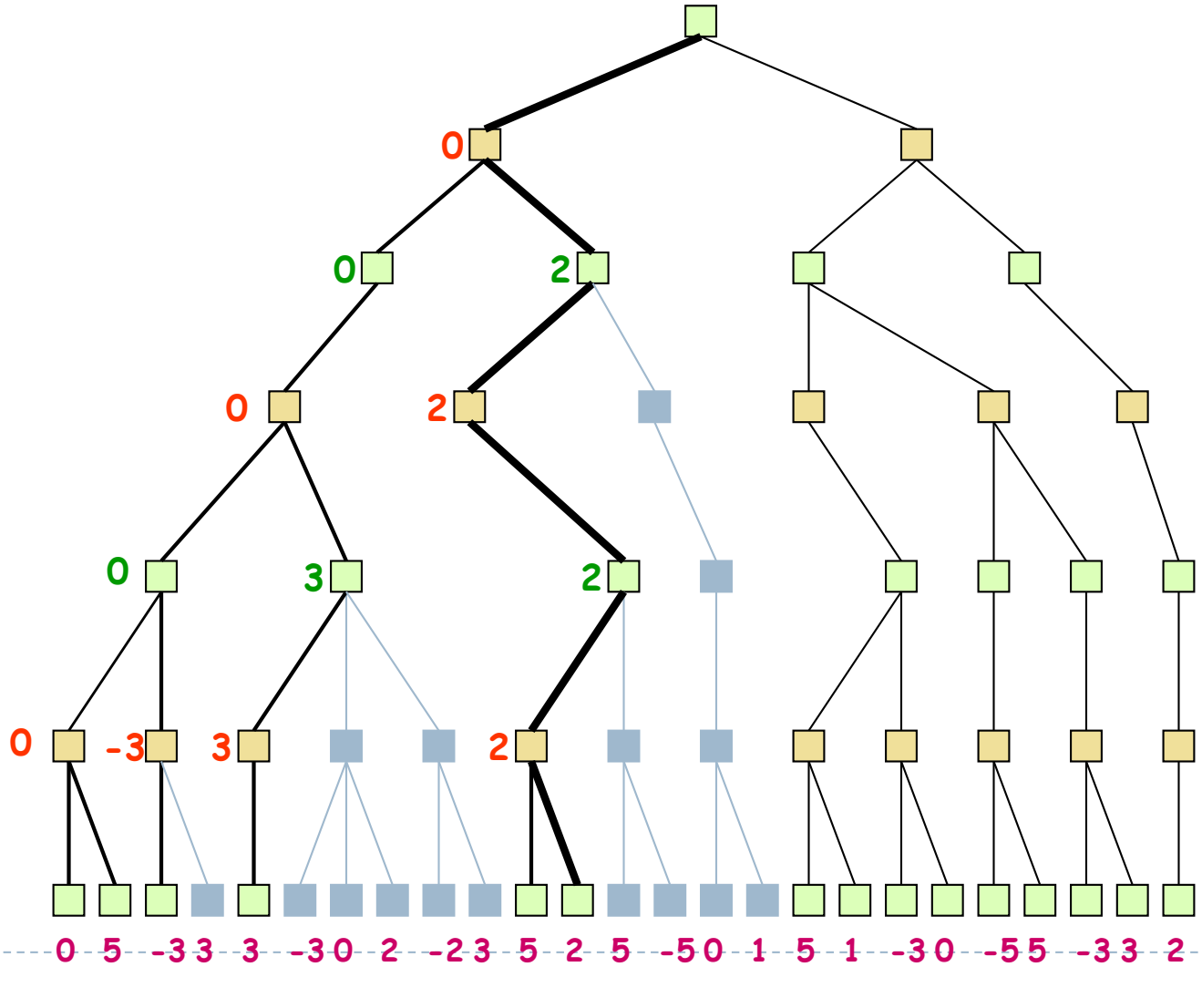

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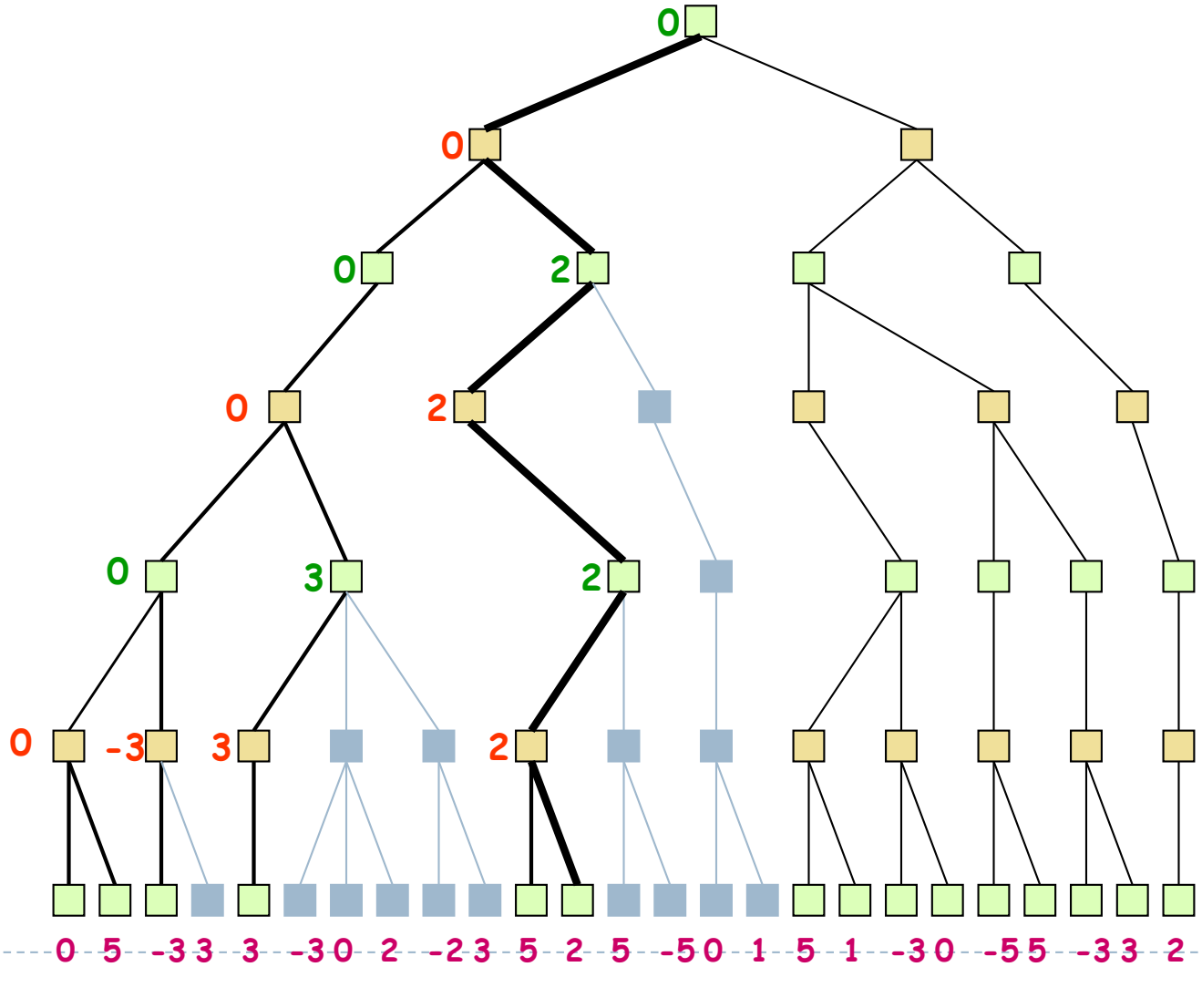

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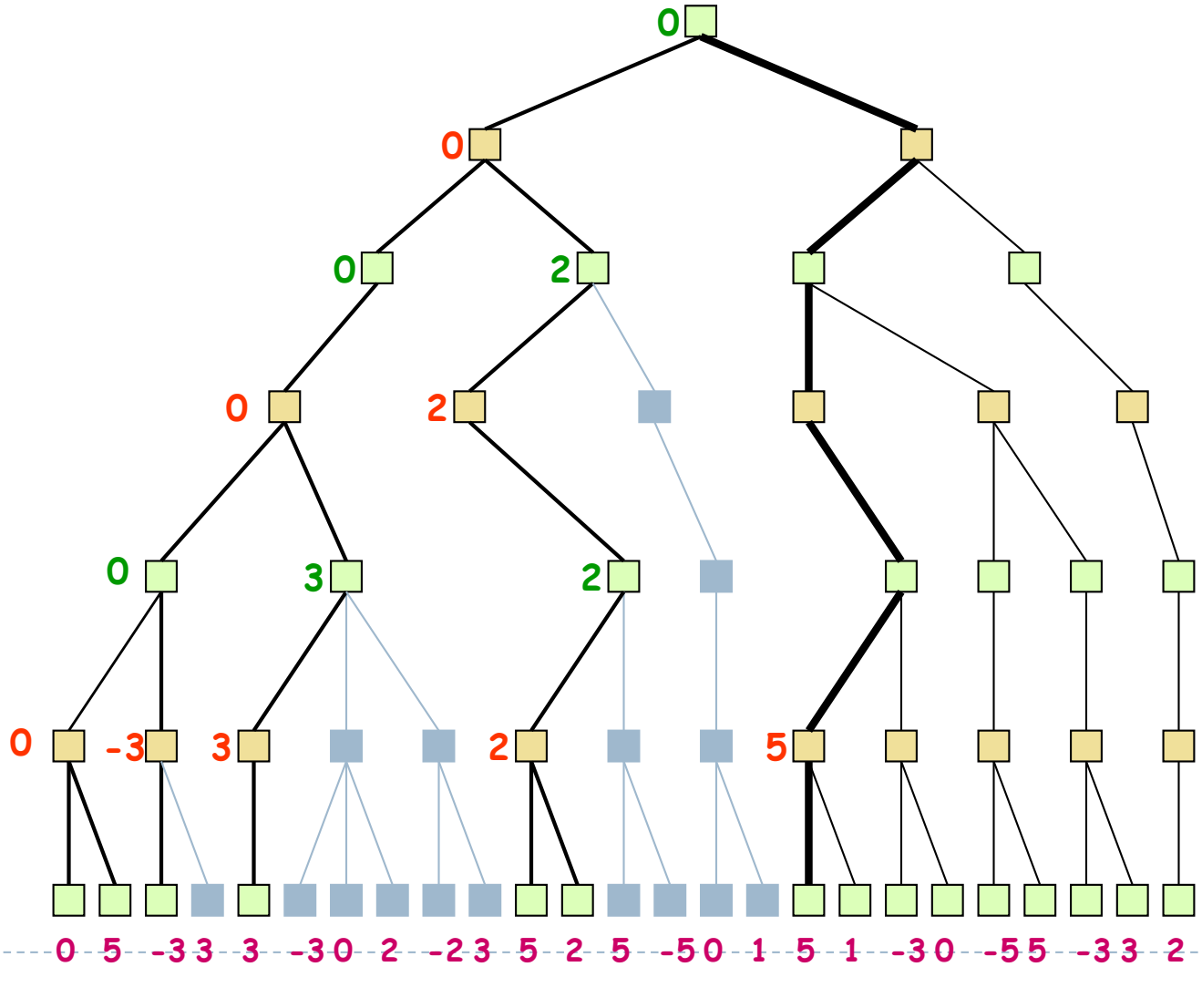

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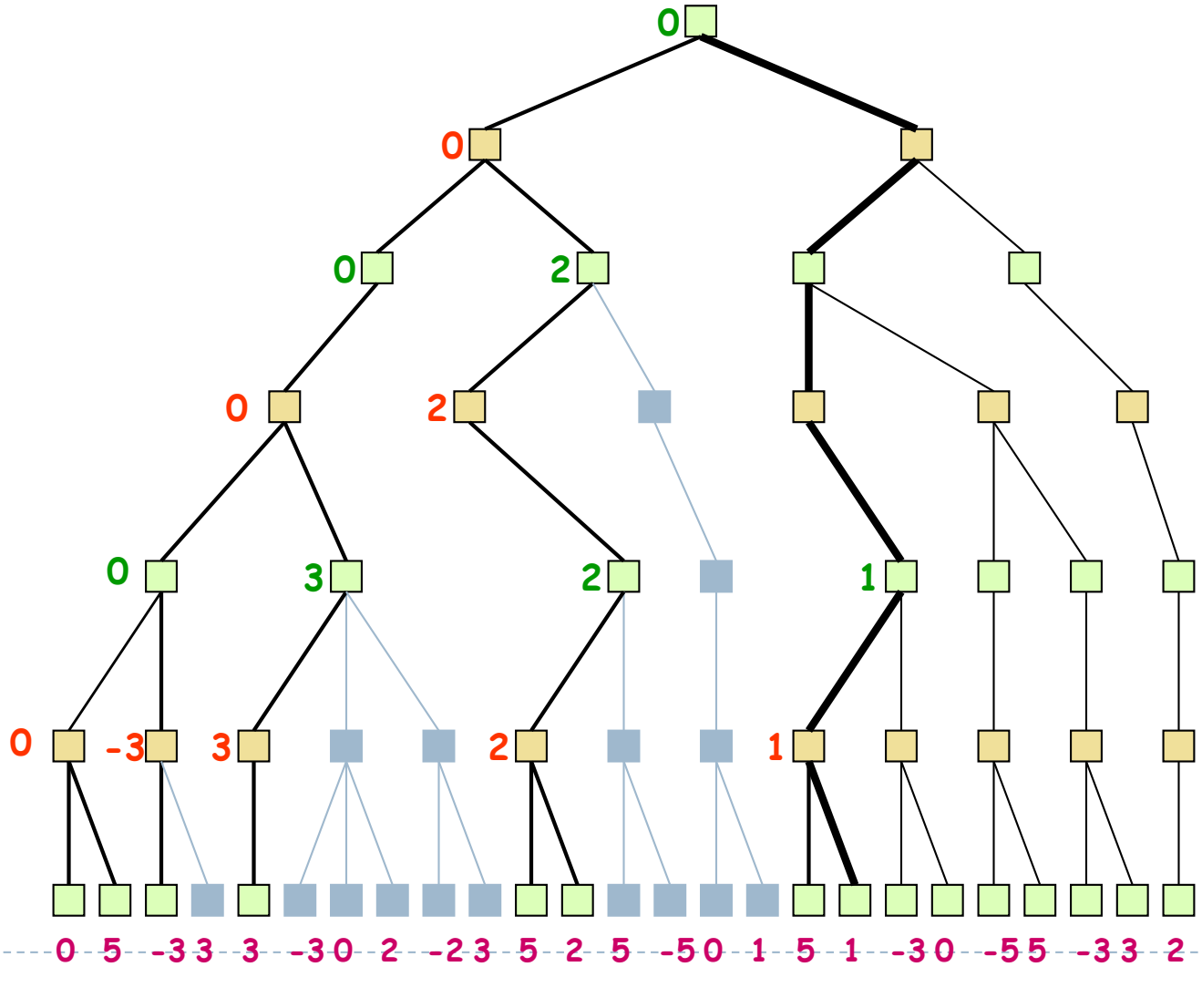

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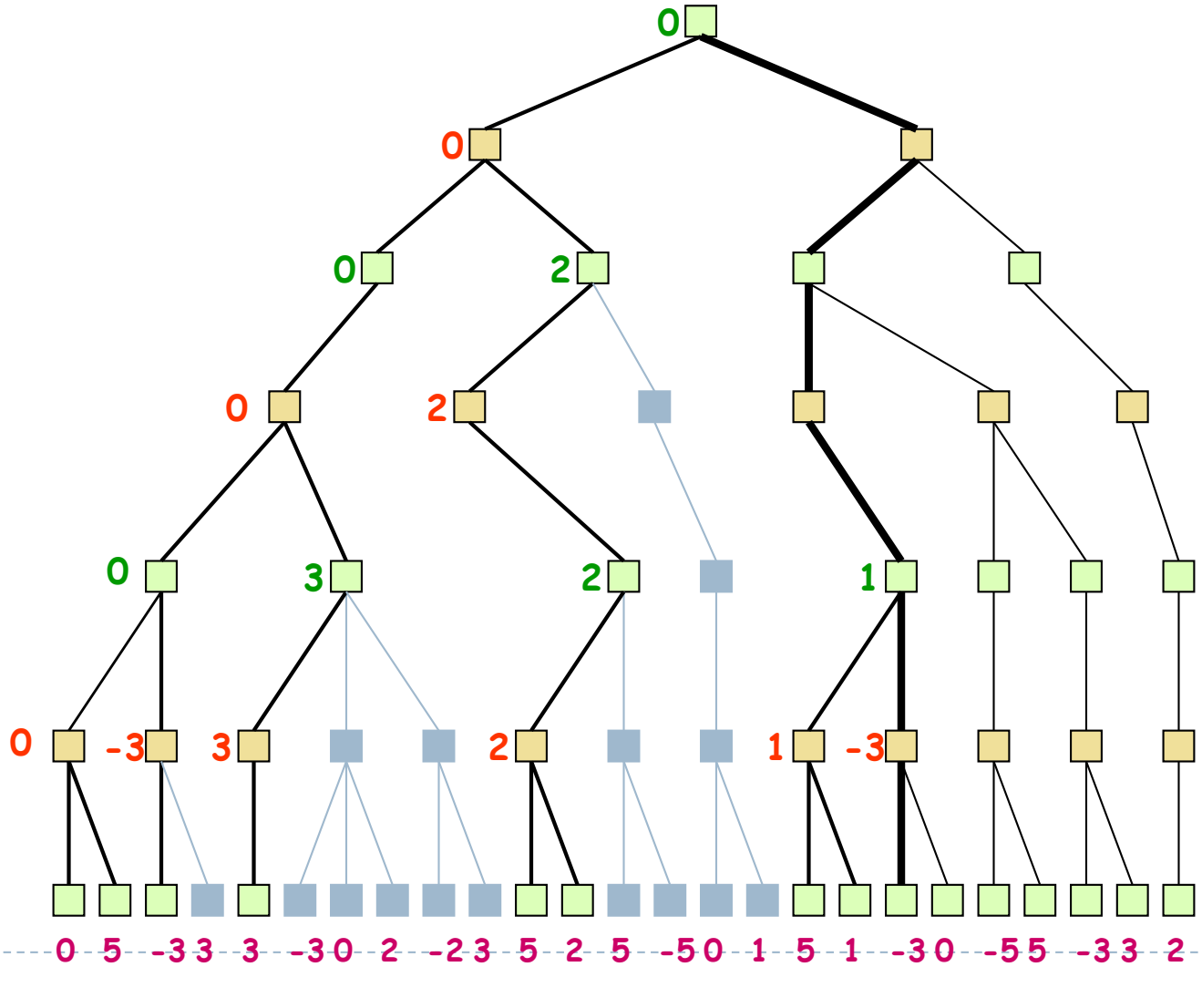

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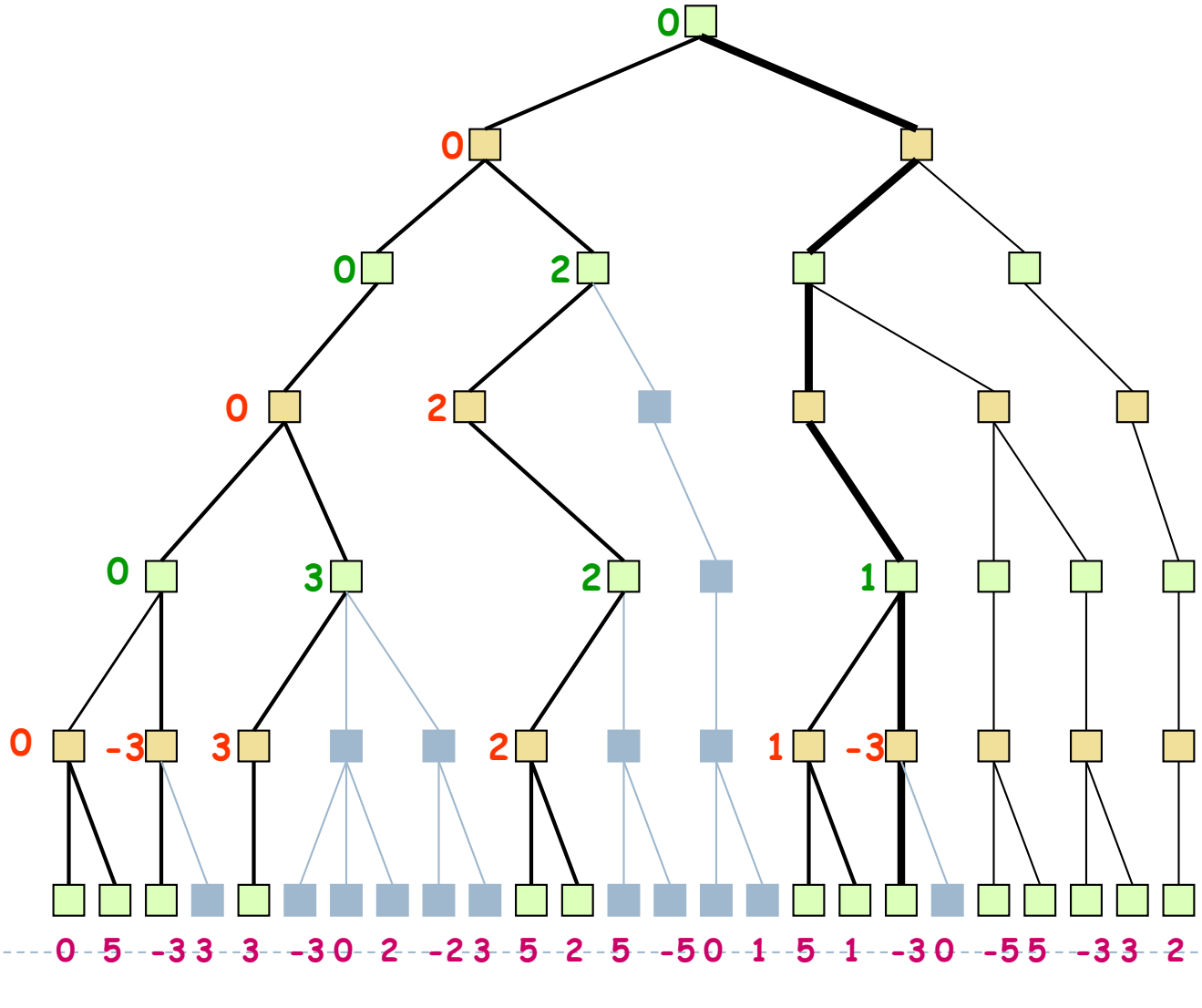

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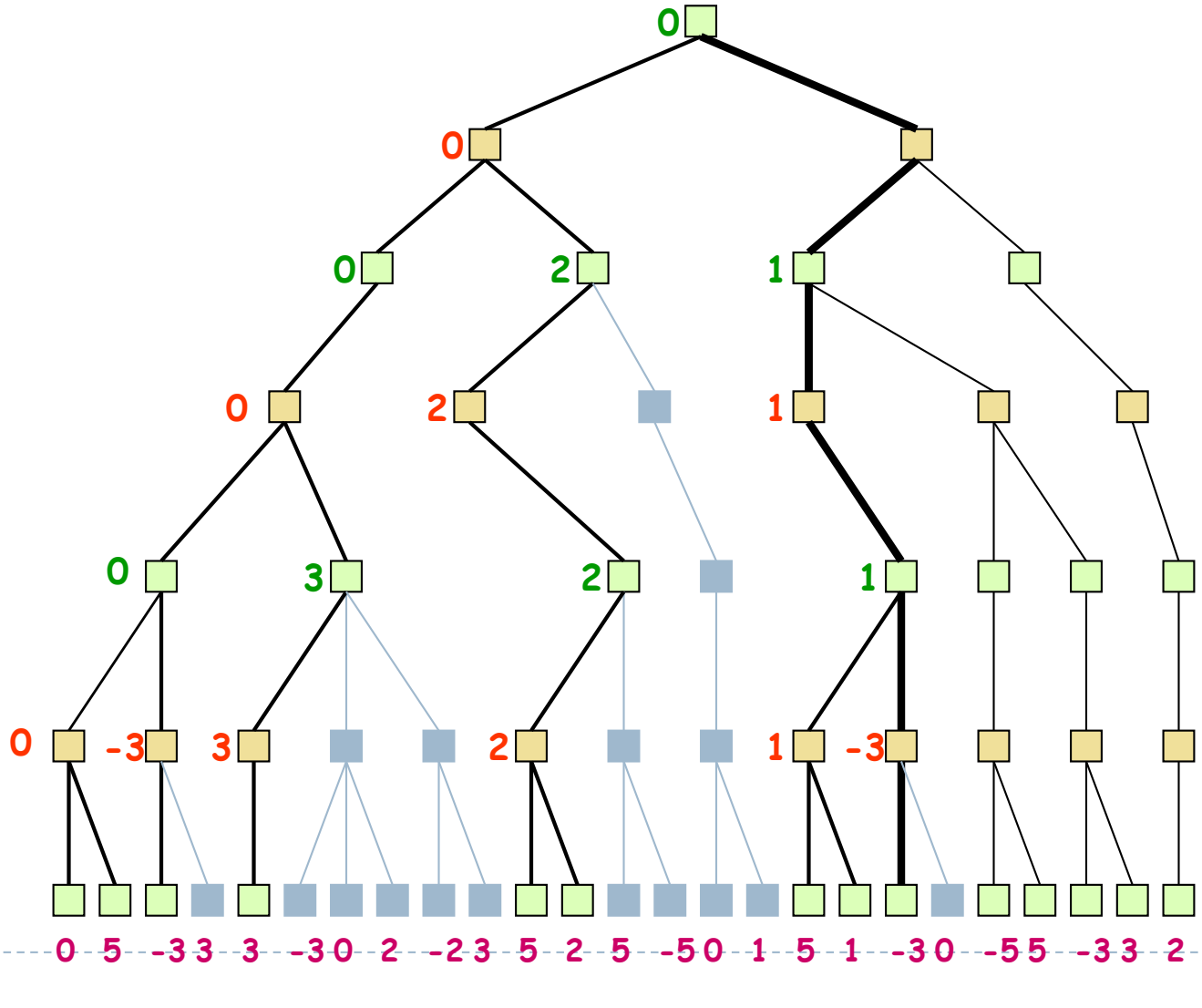

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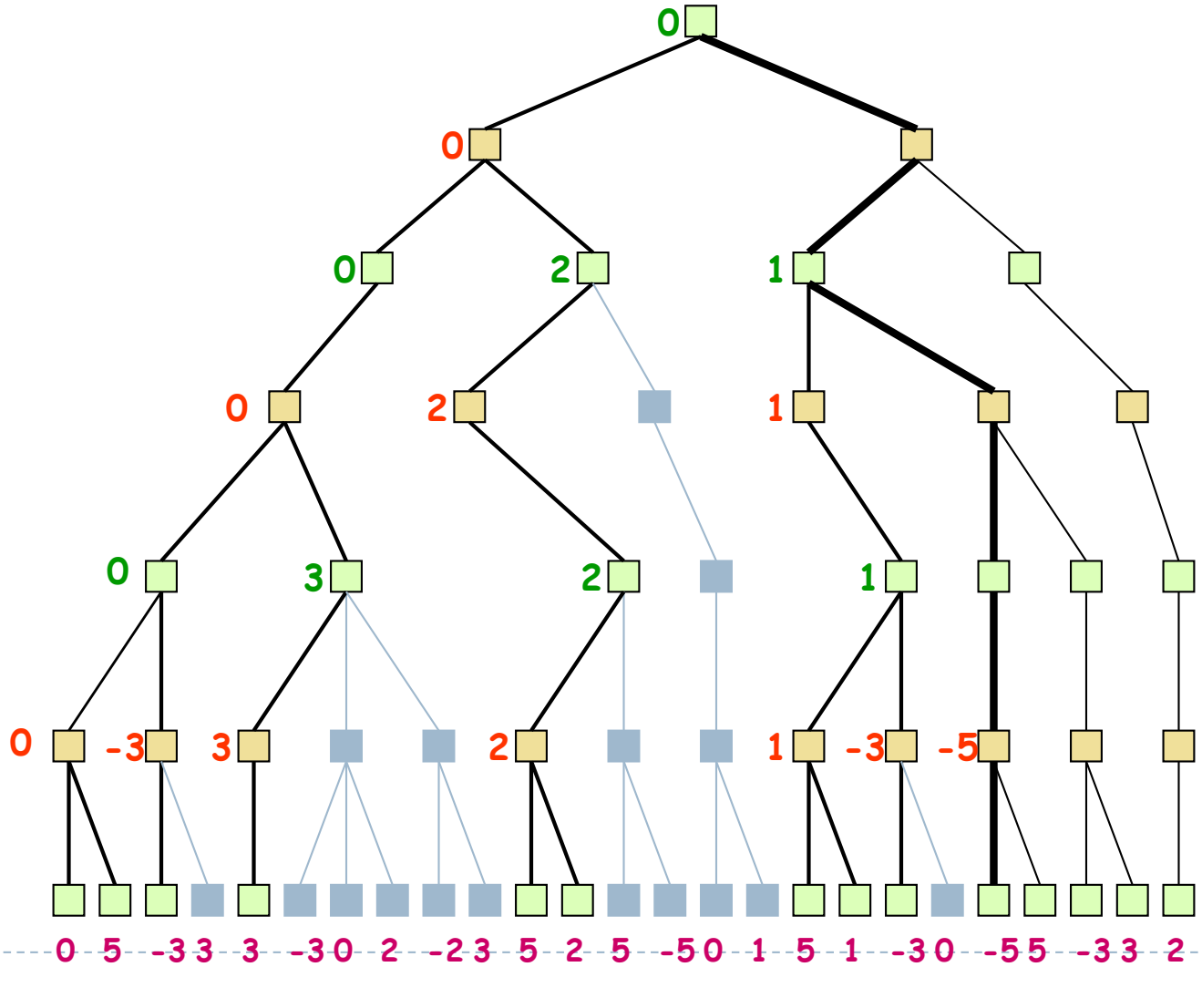

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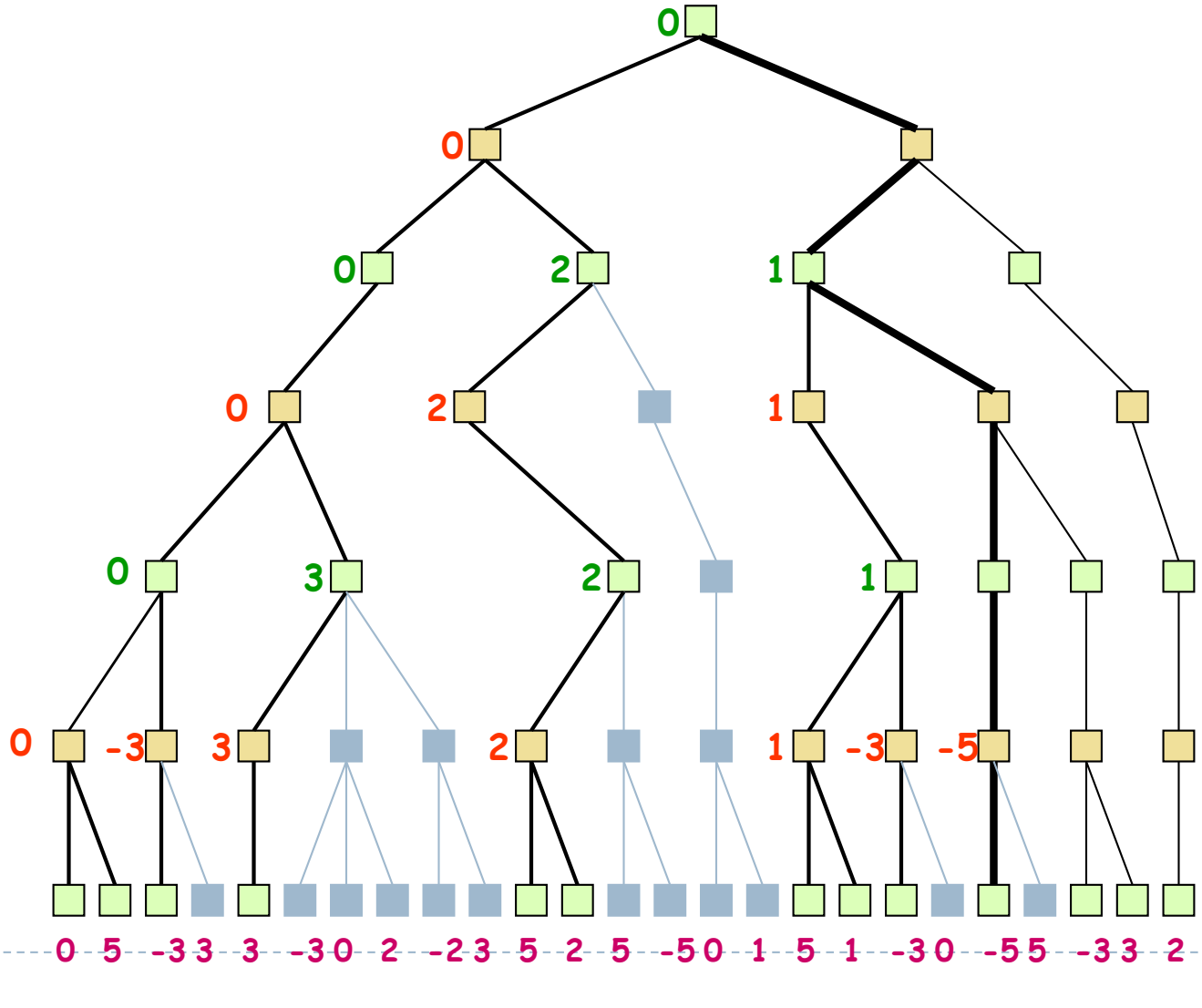

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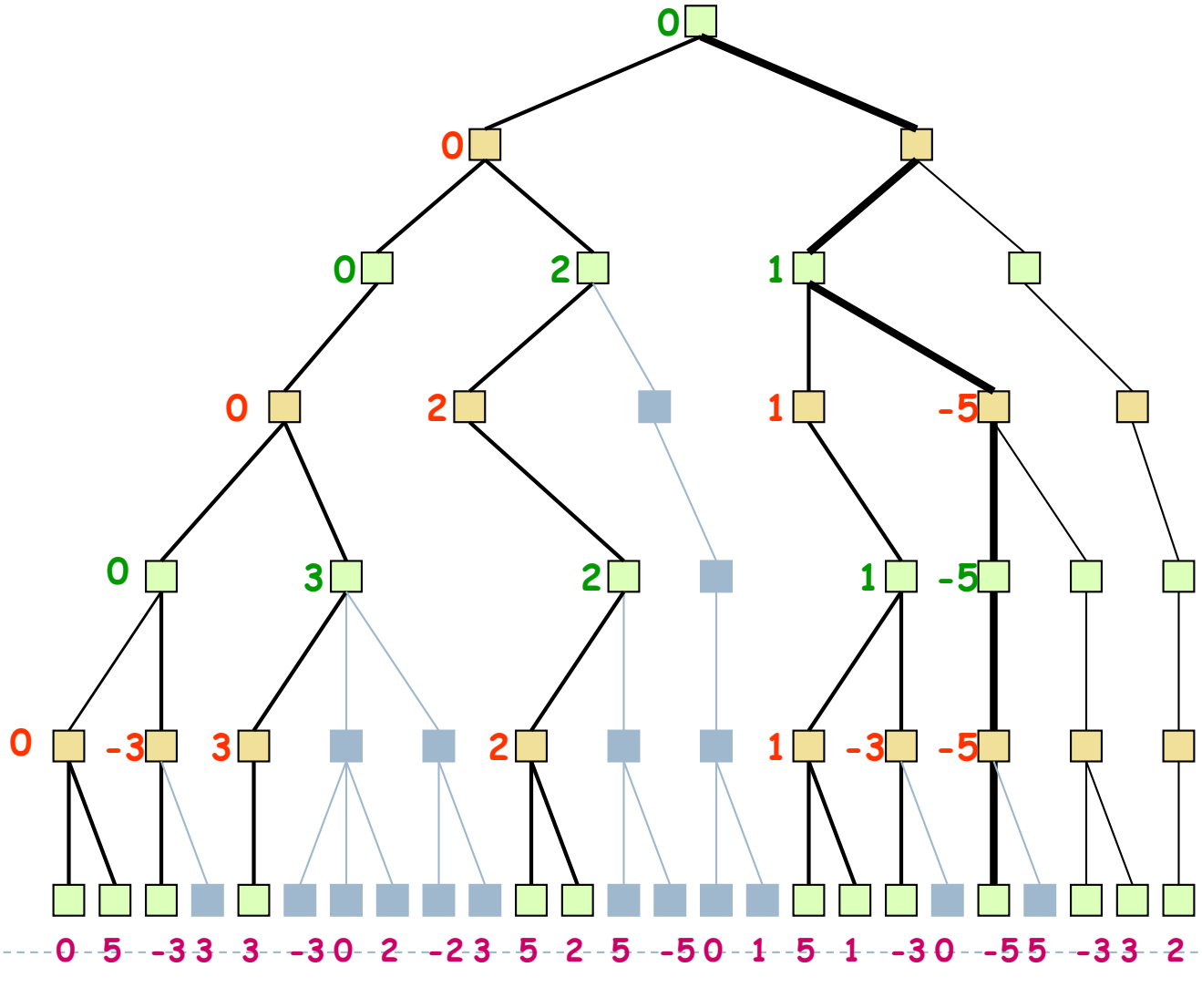

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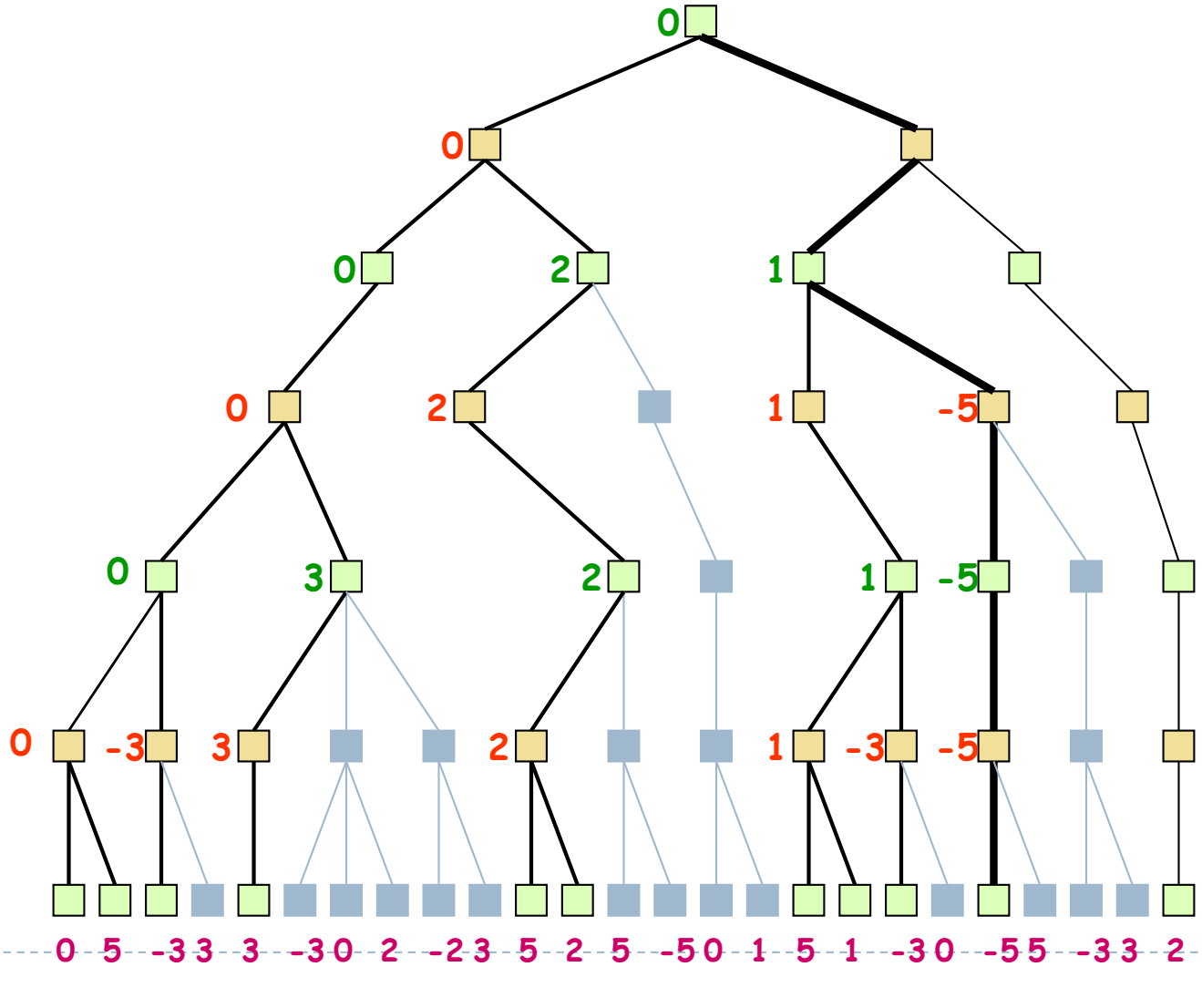

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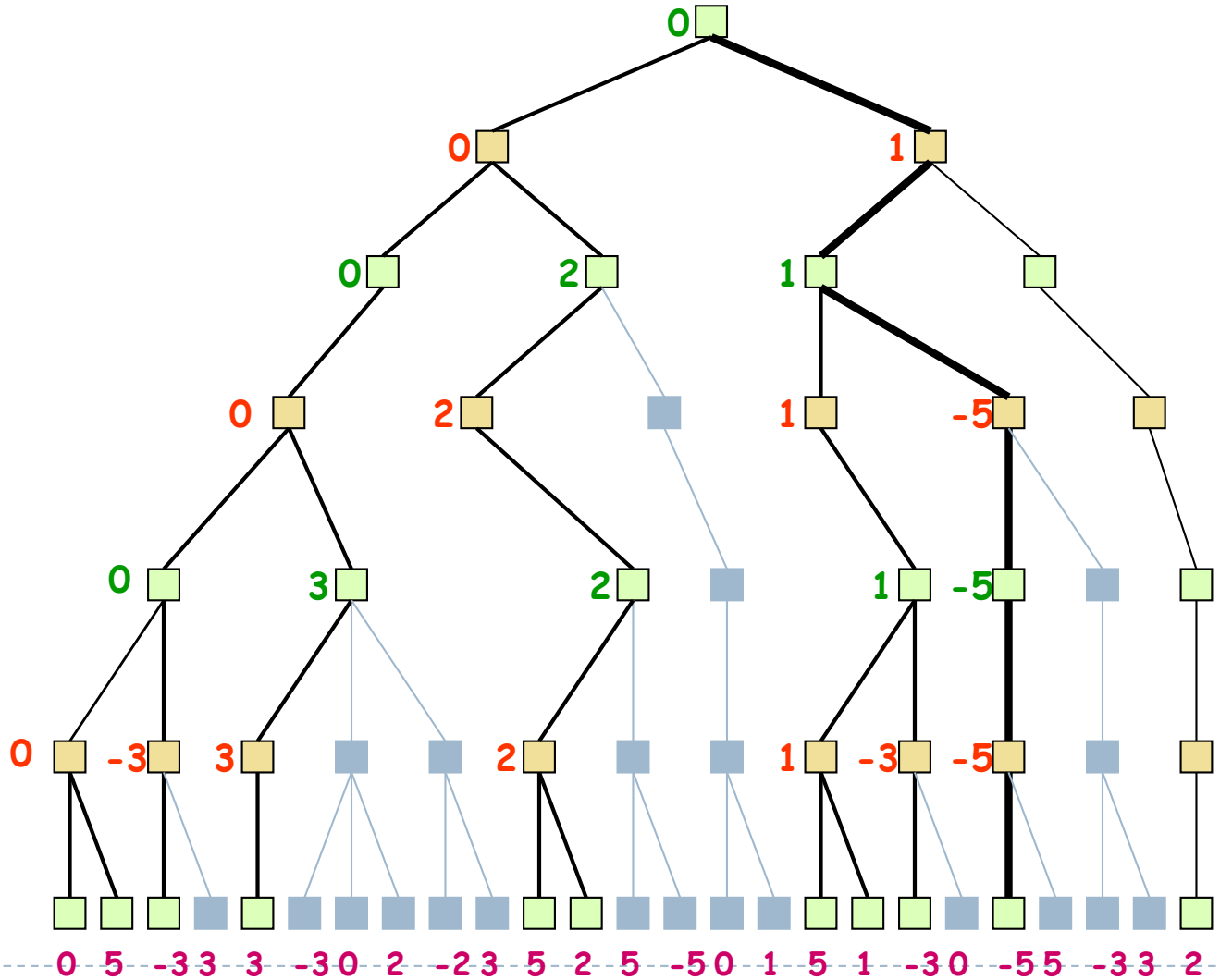

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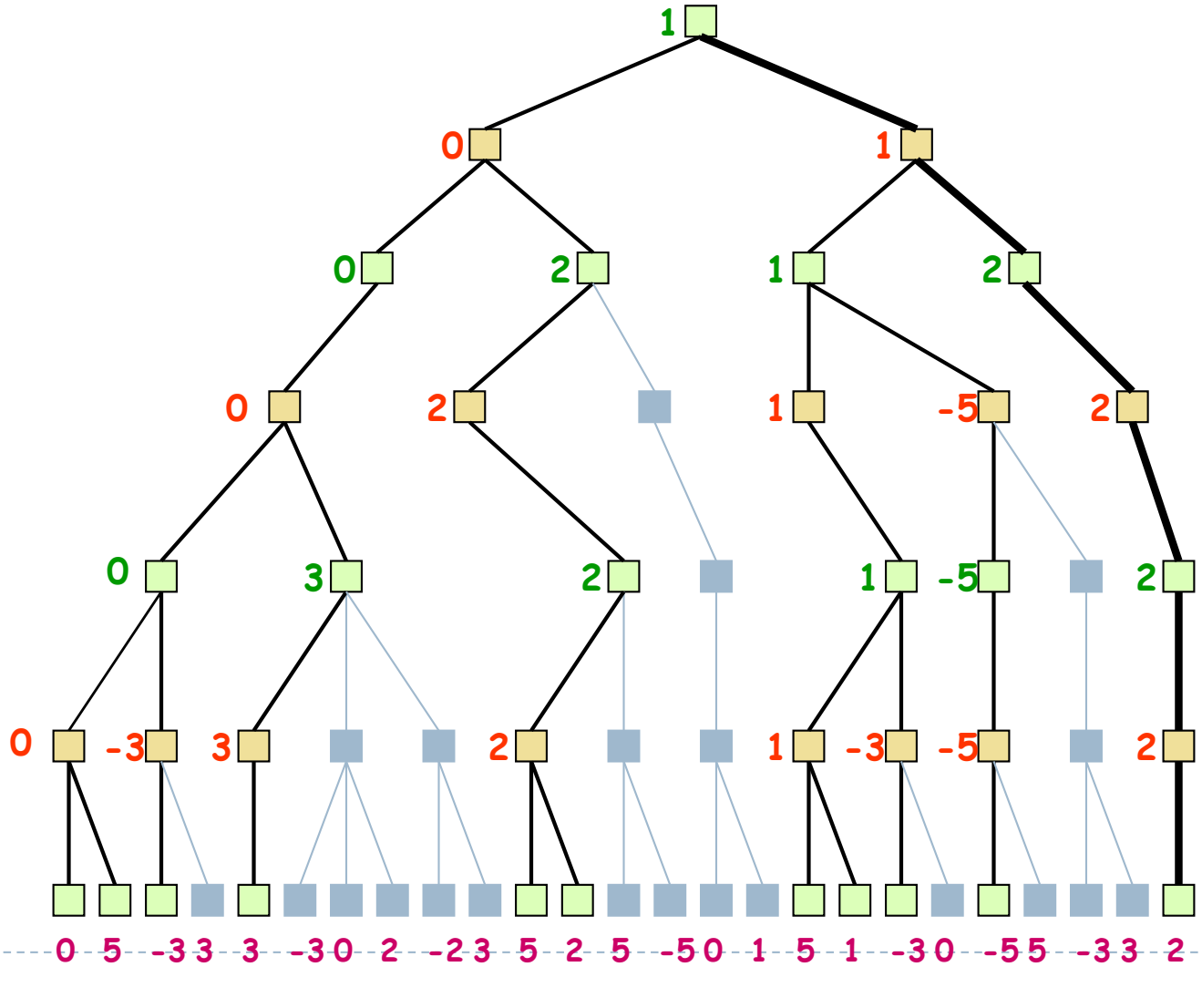
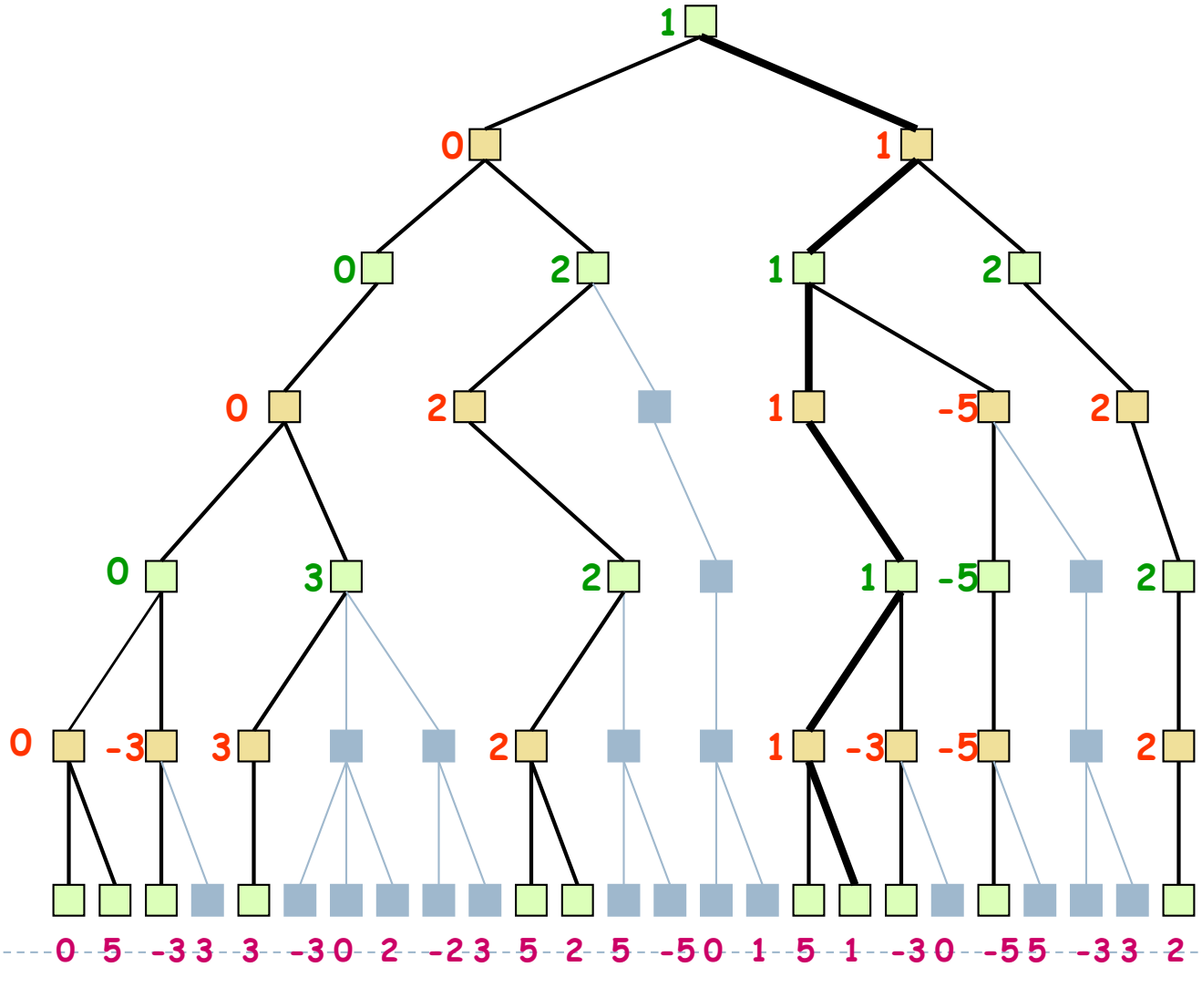
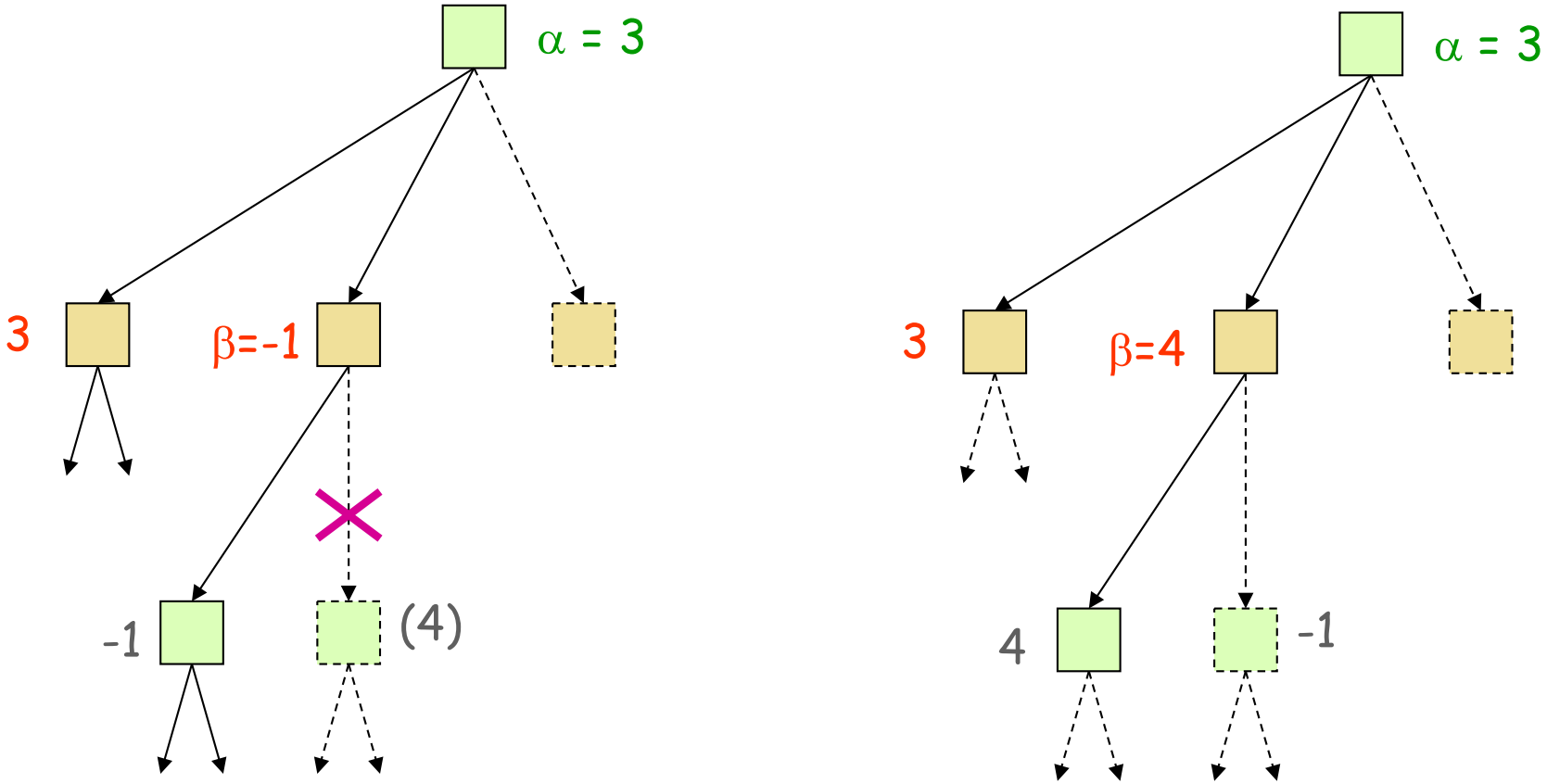

1
0 1
0 2 1 2
0 2 1 -5 2
0 3 2 1 -5 2
0 -3 3 2 1 -3 -5 2
0 5 -3 3 3 -3 0 2 -2 3 5 2 5 -5 0 1 5 1 -3 0 -5 5 -3 3 2

Example


1
0 1
0 2 1 2
0 2 1 -5 2
0 3 2 1 -5 2
0 -3 3 2 1 -3 -5 2
0 5 -3 3 3 -3 0 2 -2 3 5 2 5 -5 0 1 5 1 -3 0 -5 5 -3 3 2

How much do we gain?

Consider these two cases:



Node ordering matters for efficiency!

How much do we gain?

- Assume a game tree of uniform branching factor b
- h: depth of the tree.
- Minimax examines $O(b^h)$ nodes, so does alpha-beta in the worst-case

How much do we gain?

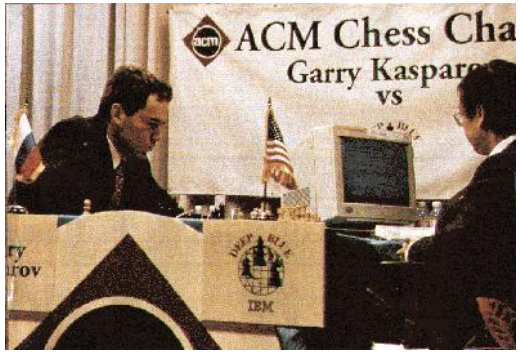
- The gain for alpha-beta is maximum when:
  - The children of a MAX node are ordered in decreasing backed up values
  - The children of a MIN node are ordered in increasing backed up values
- Then alpha-beta examines $O(b^{h/2})$ nodes [Knuth and Moore, 1975]
- But this requires an oracle (if we knew how to order nodes perfectly, we would not need to search the game tree)
- If nodes are ordered at random, then the average number of nodes examined by alpha-beta is $\sim O(b^{3h/4})$

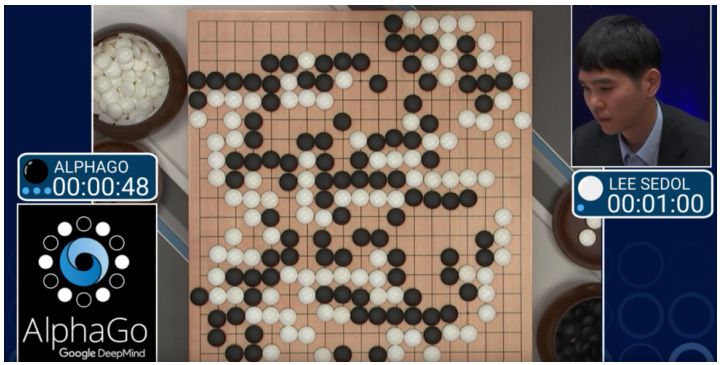
Heuristic Ordering of Nodes

- Order the children of a node according to the values backed-up at the previous iteration

Computer programs have beaten some of the best human players

- 1994: Chinook beats Mr. Tinsley in Checkers.
  - Mr. Tinsley is world champion of checkers for over 40 years.
- 1997: Deep blue beats Mr. Kasparov.
  - Mr. Kasparov is world champion in chess during1985-2000.
- 2016: AlphaGo beats Mr. Lee Sedol
  - Mr. Lee Sedol is the world #2 Go Player, considered as one of the strongest player in the history of professional Go.
- 2017: OpenAI Bot beats Dendi
  - Dendi is a top professional Dota 2 gamer





How they (the comp. game makers) did it?

- Strong strategies to decide which path to evaluate
  - Pruning strategy if we know the values (alpha-beta)
  - Huge databases to learn the opponent's pattern, so as to improve the evaluation function and able to be more focus on which path to expand.
- The methods are general, but their implementation is dramatically improved by many specifically tuned-up enhancements (e.g., the evaluation functions).
- Recently: Let computers play again each other, but don't forget to add noise!!!