COMP3702 Tutorial 12

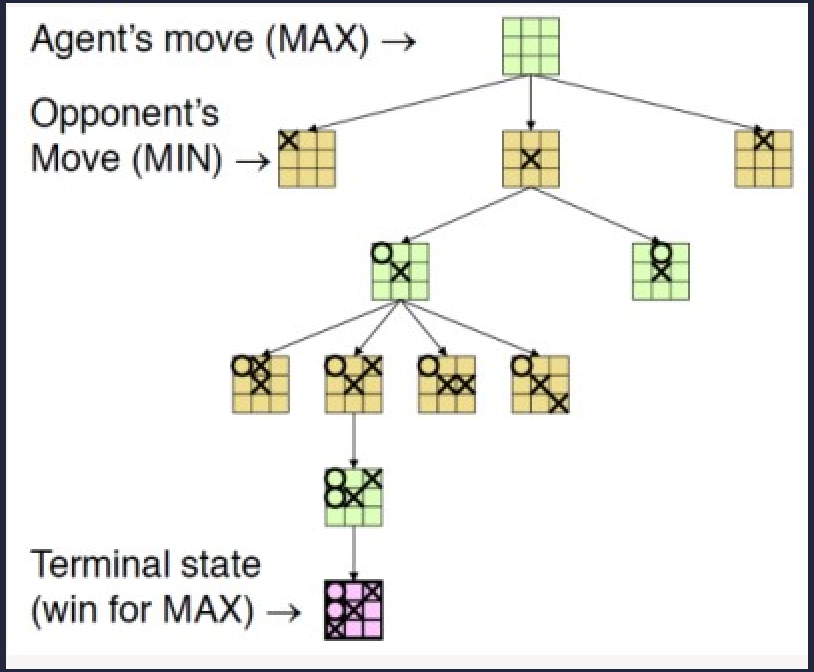
Matt Choy | matthew.choy@uq.edu.au

Mini-Max Trees

- When we have a game that has the following conditions, we can represent it as a minimax tree.
  - Two agents that perform actions sequentially
  - Deterministic actions
  - We can observe the entire state before acting
  - Is a zero-sum game (pure conflict, one agent's gain is another agent's loss)
- These games include Tic-tac-toe, Chess, Go, etc.
- We can represent the game as a minimax tree, in which at each stage the respective agent performs the action they believe maximises their profits.

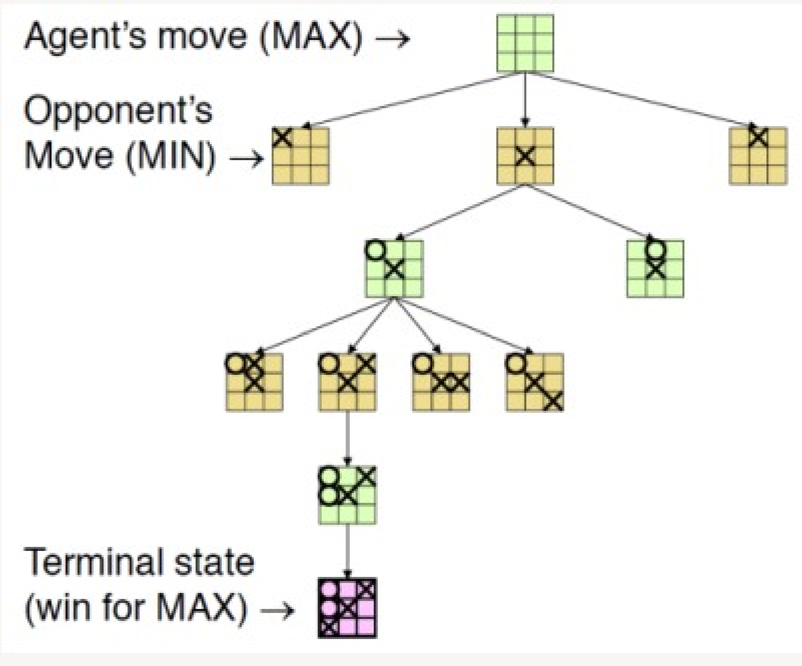
Mini-Max Trees

- These minimax trees are similar to AND-OR trees:
  - At the “OR” level, it is the agent’s move, in which they are trying to maximise their value
  - At the “AND” level, it is the opponent’s move, in which the opponent is trying to maximise their own score (and thus minimise the agent’s score).



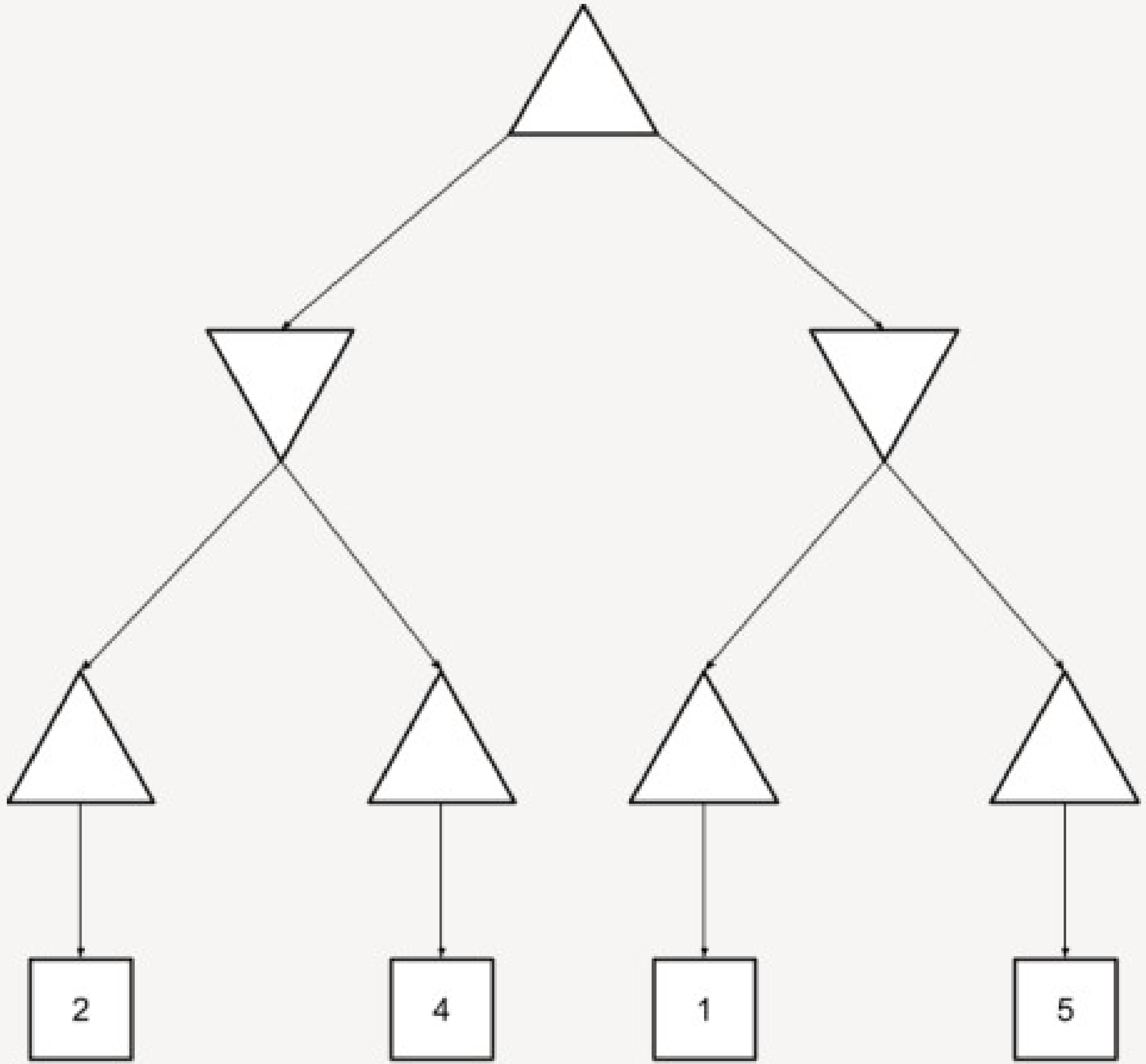
Mini-Max Algorithm

- Each node in the minimax tree is has three values that we keep track of:
  - V: The current best value (maximum of children for MAX nodes, minimum of children for MIN nodes)
  - : Maximum lower bound of all possible solutions (children)
  - : Minimum upper bound of possible solutions
- To find the minimax value of a node :
  - If is a leaf node, return the value of the state.
  - If is a MAX node, we recurse down to find the minimax value of all children. We take the action that gives the **highest** score for child nodes
  - If is a MIN node, we recurse down to find the minimax value of all children. We take the action that gives the **lowest** score for child nodes



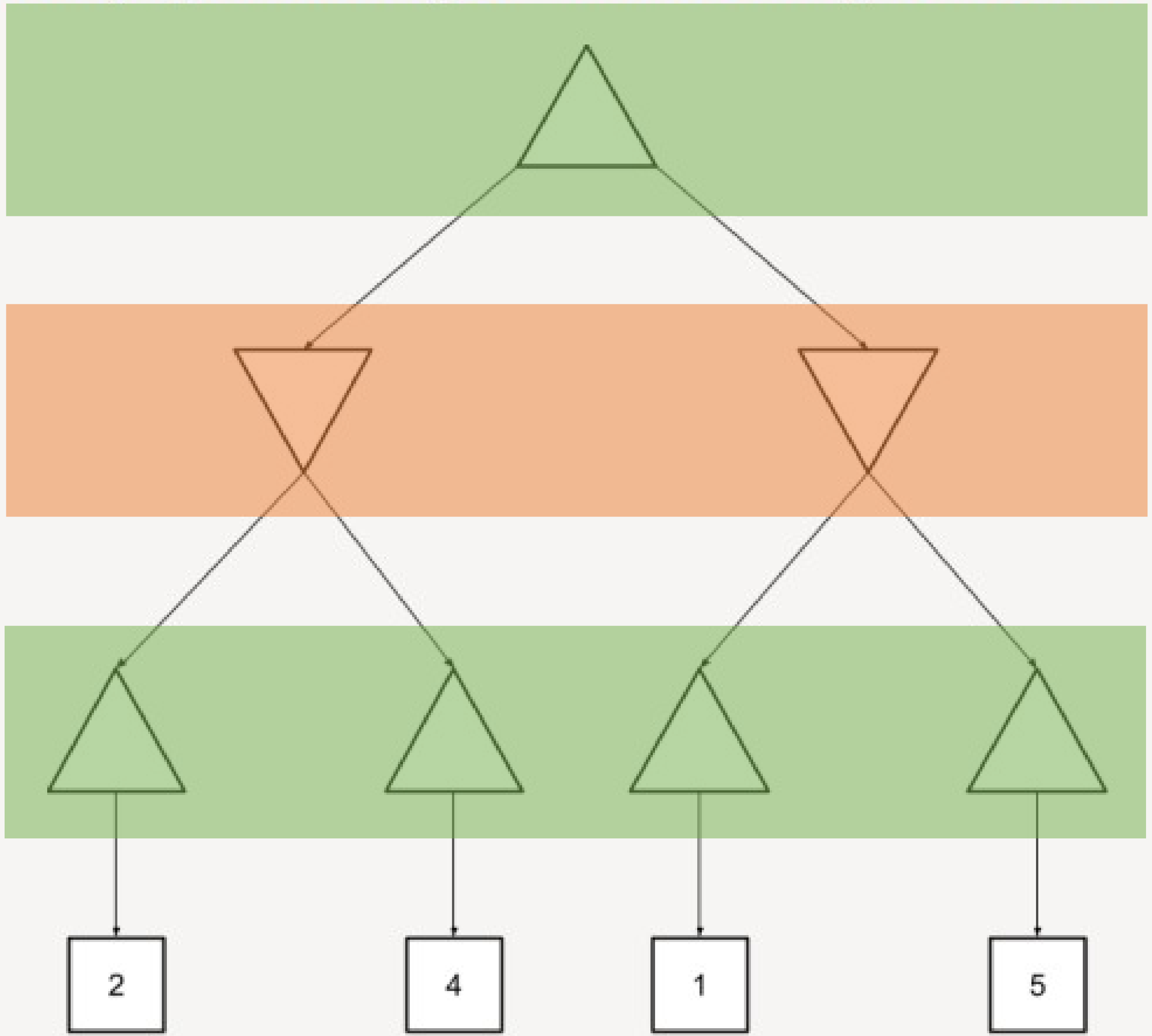
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



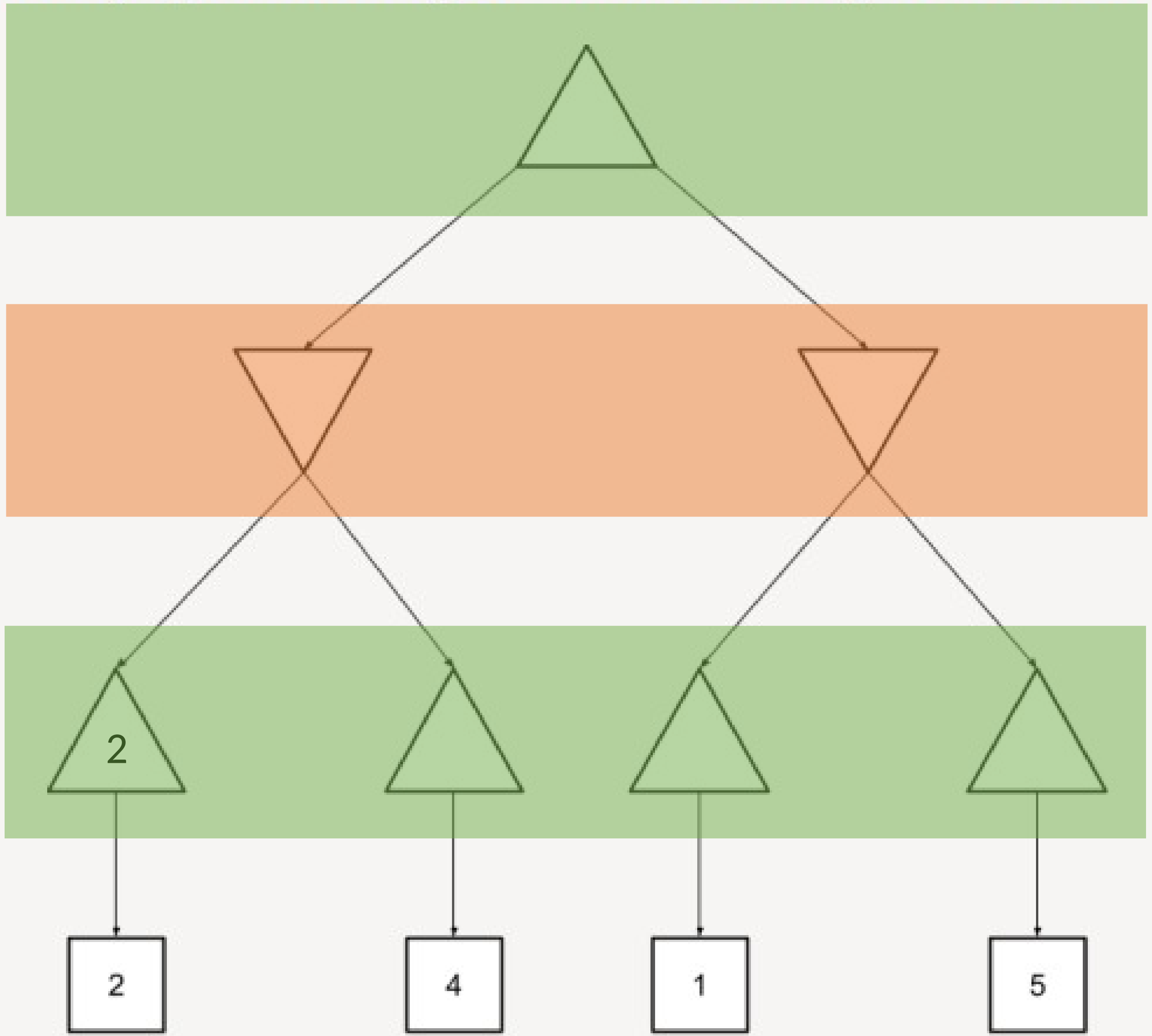
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



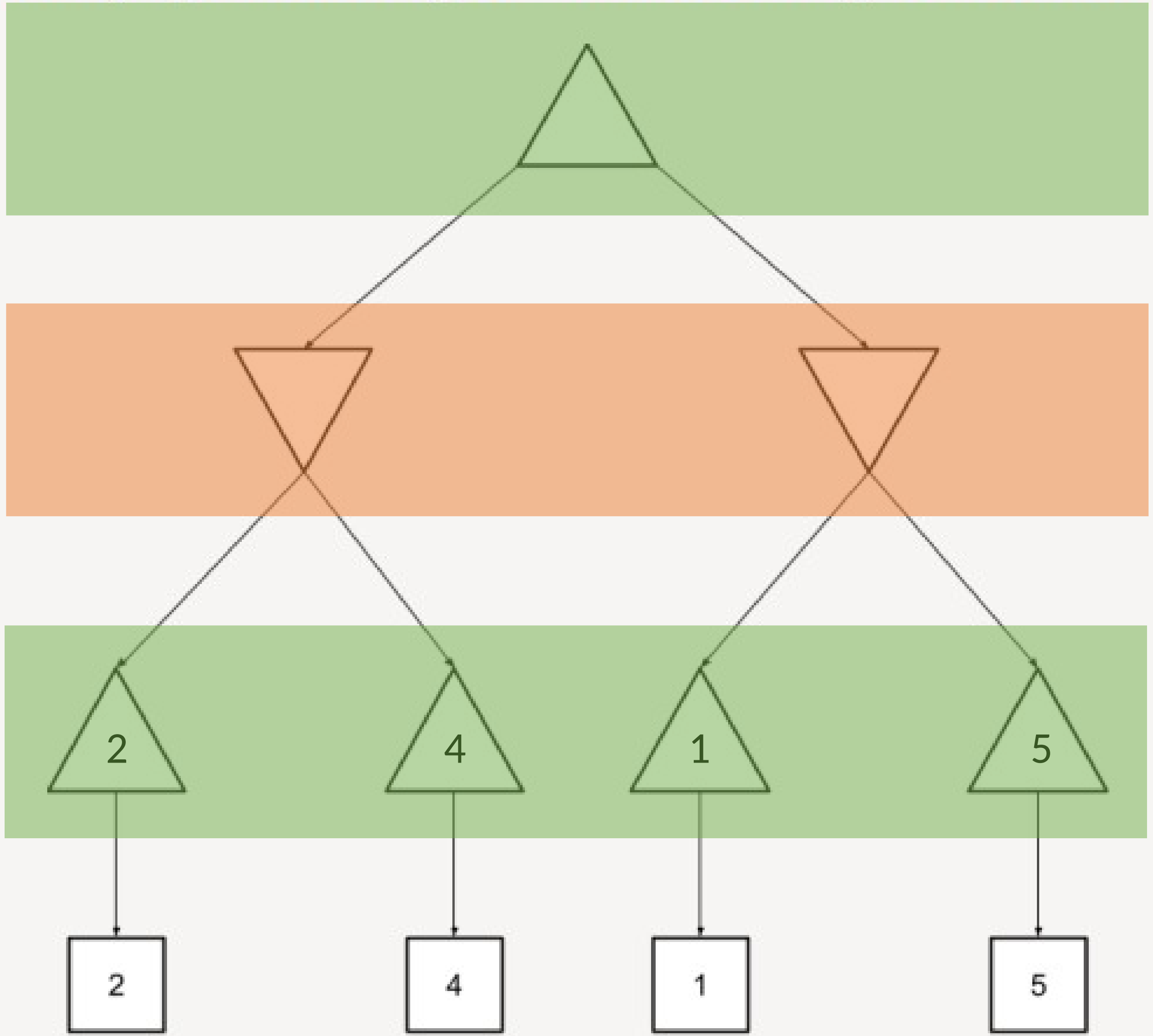
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



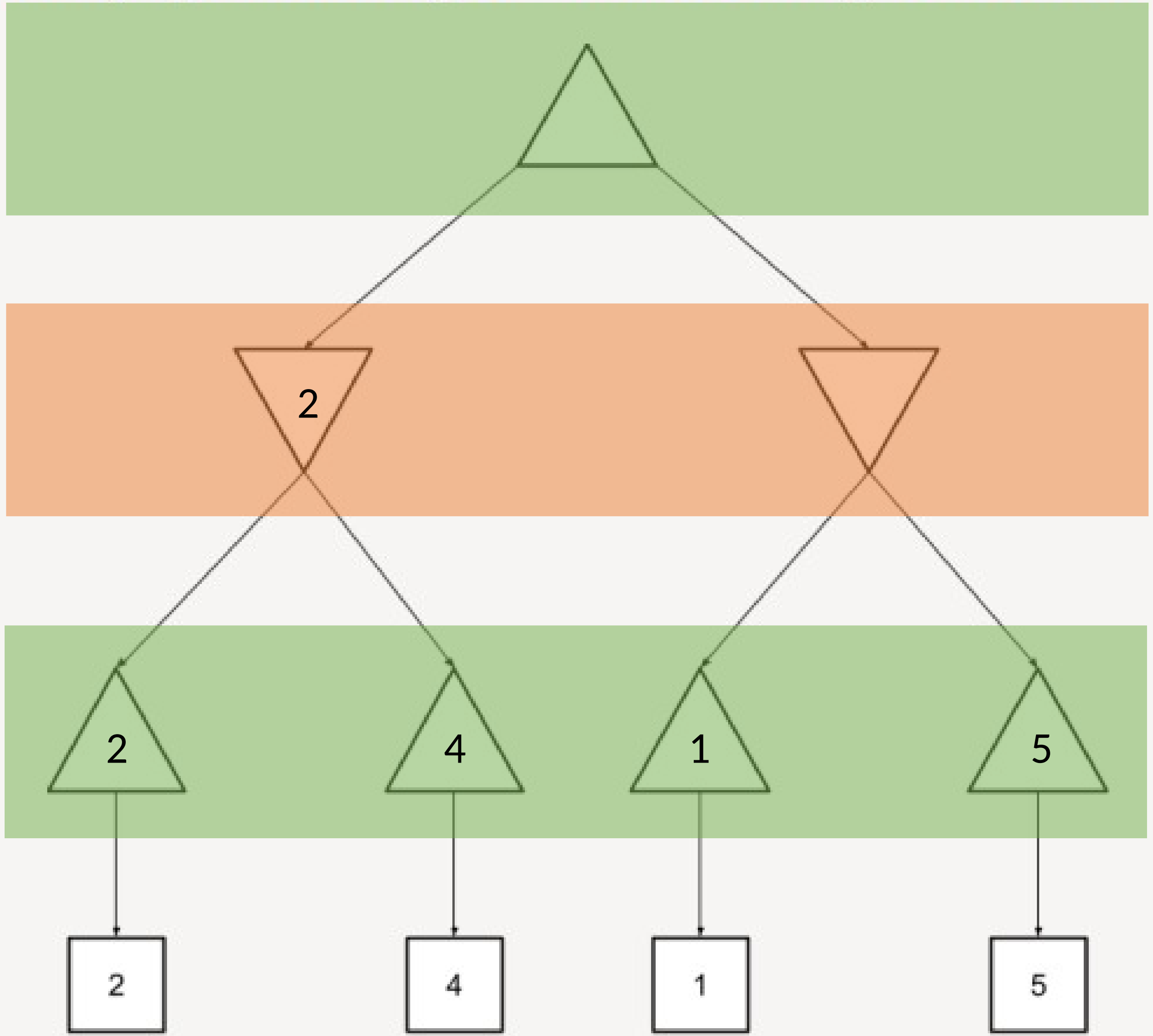
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



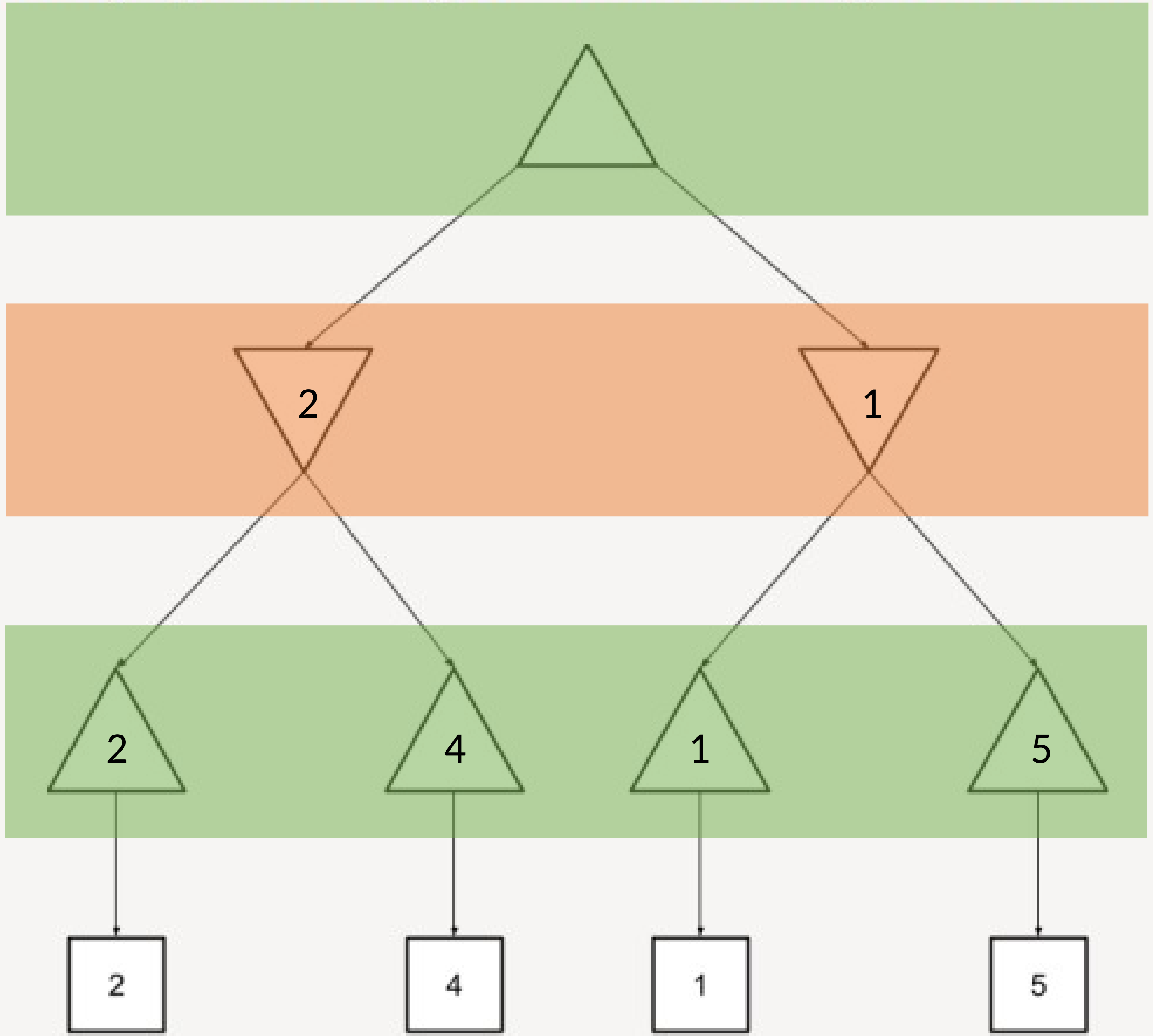
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



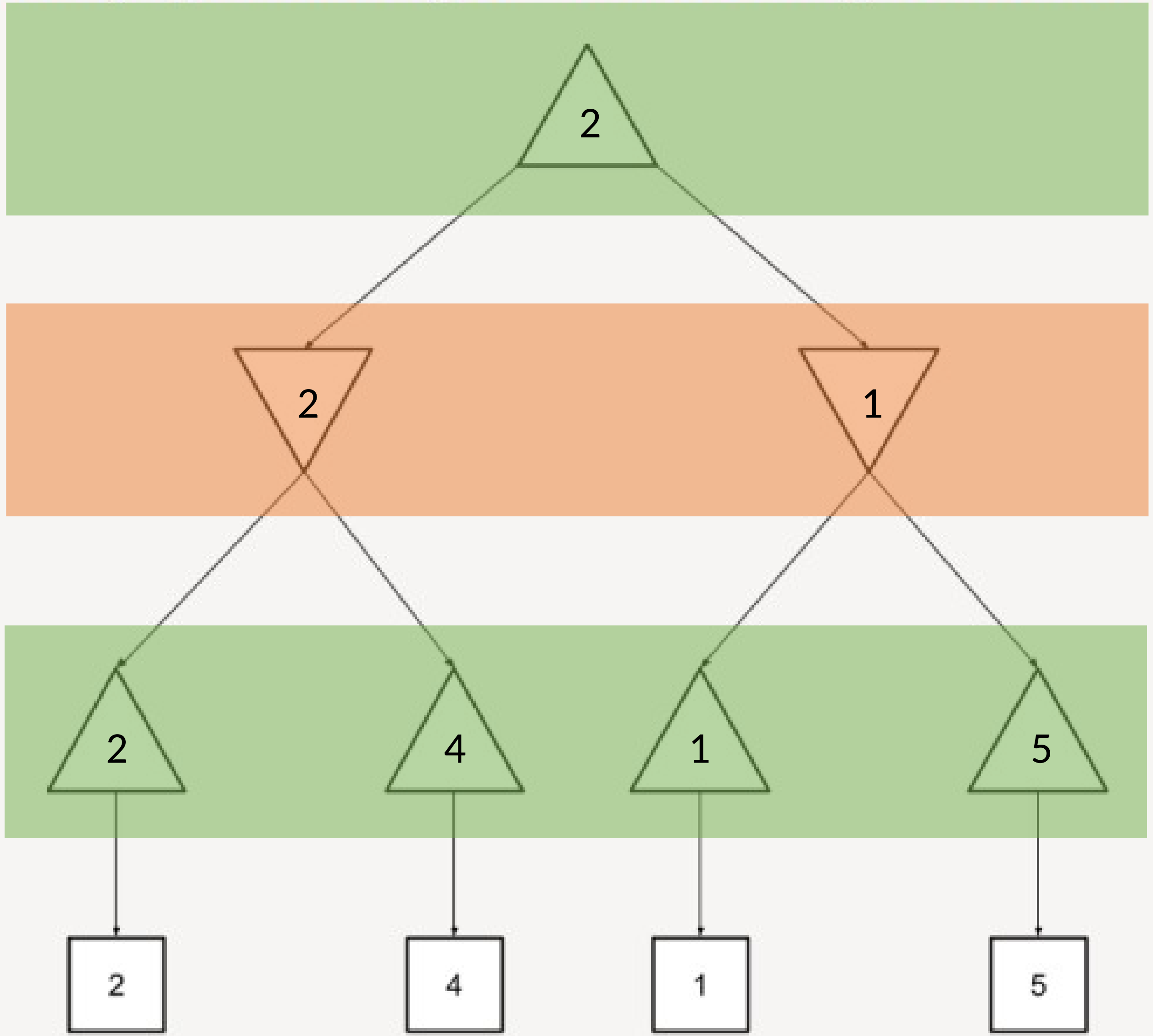
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.



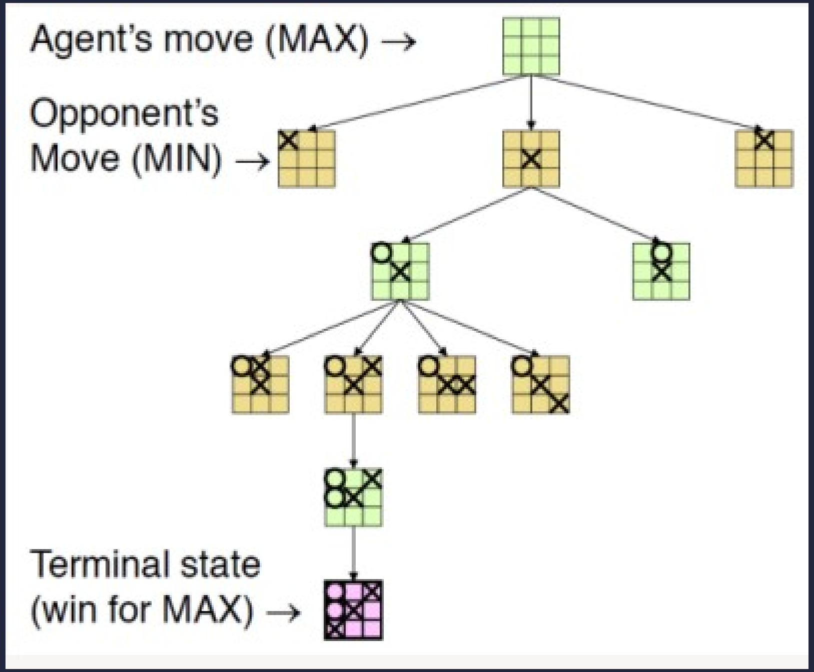
Minimax: simple example

MAX nodes represented by upwards triangles. MIN nodes represented as inverted triangles.

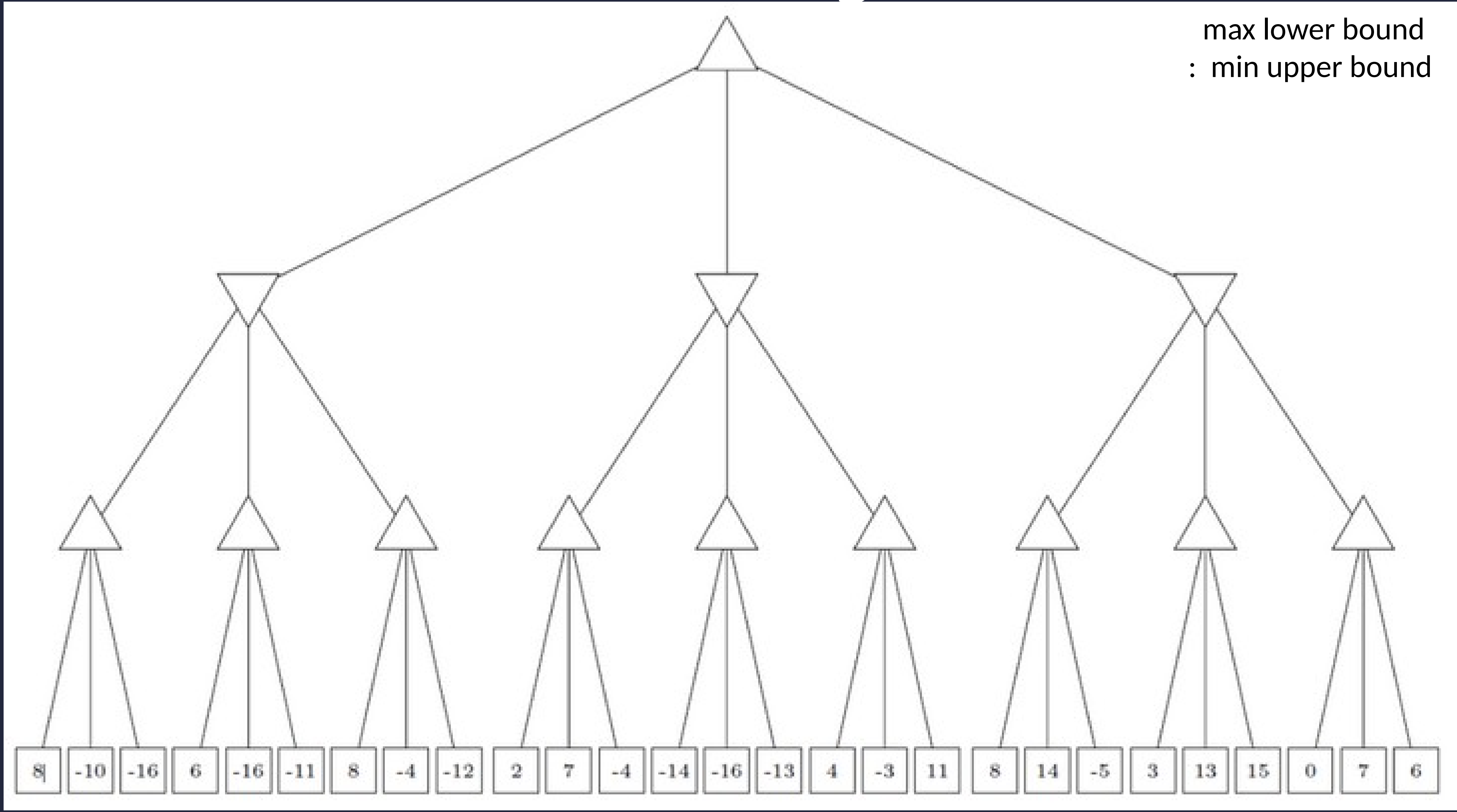


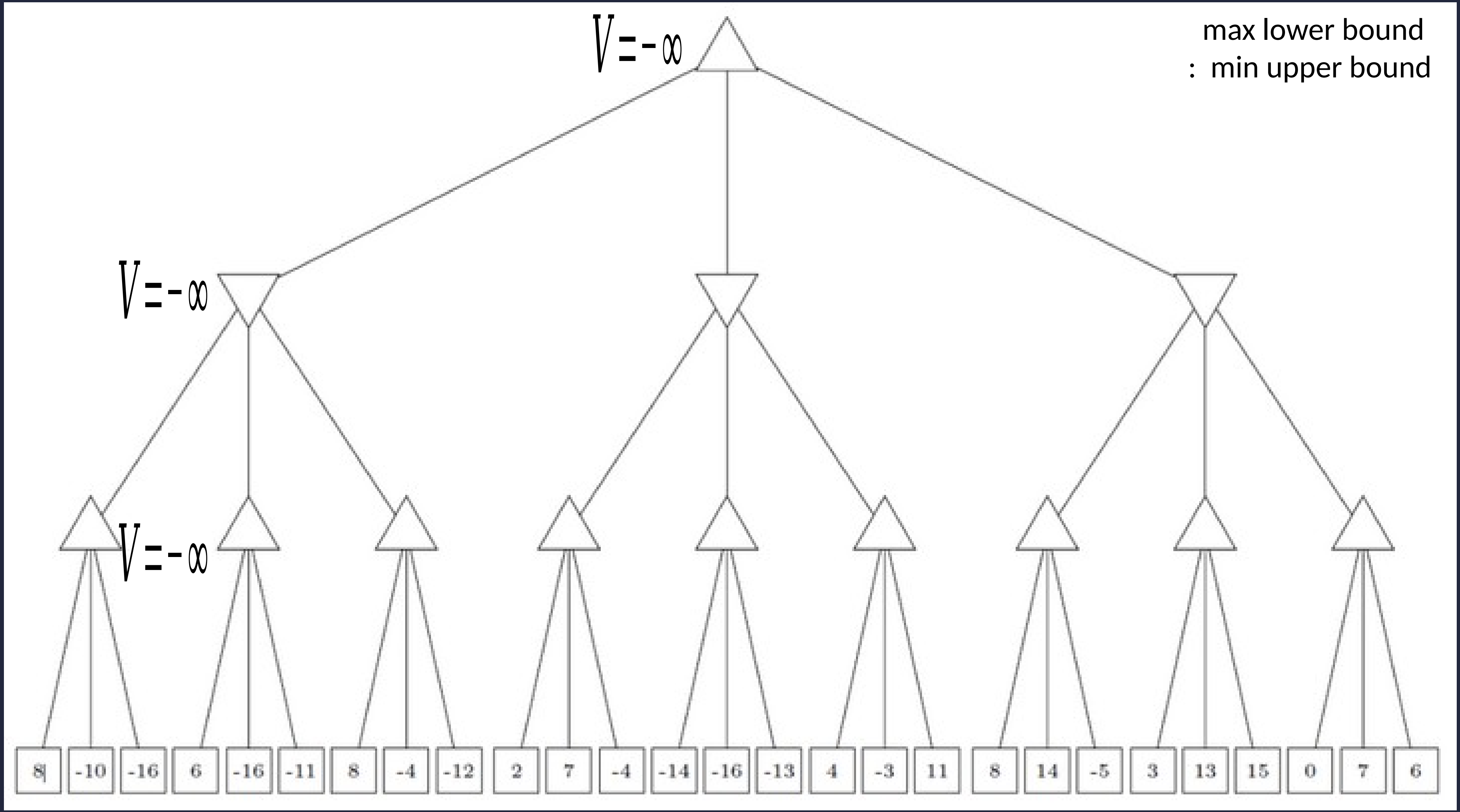
Mini-Max Algorithm

- Each node in the minimax tree is has three values that we keep track of:
  - V: The current best value (maximum of children for MAX nodes, minimum of children for MIN nodes)
  - : Maximum lower bound of all possible solutions (children)
  - : Minimum upper bound of possible solutions
- To find the minimax value of a node :
  - If is a leaf node, return the value of the state.
  - If is a MAX node, we recurse down to find the minimax value of all children. We take the action that gives the **highest** score for child nodes
  - If is a MIN node, we recurse down to find the minimax value of all children. We take the action that gives the **lowest** score for child nodes

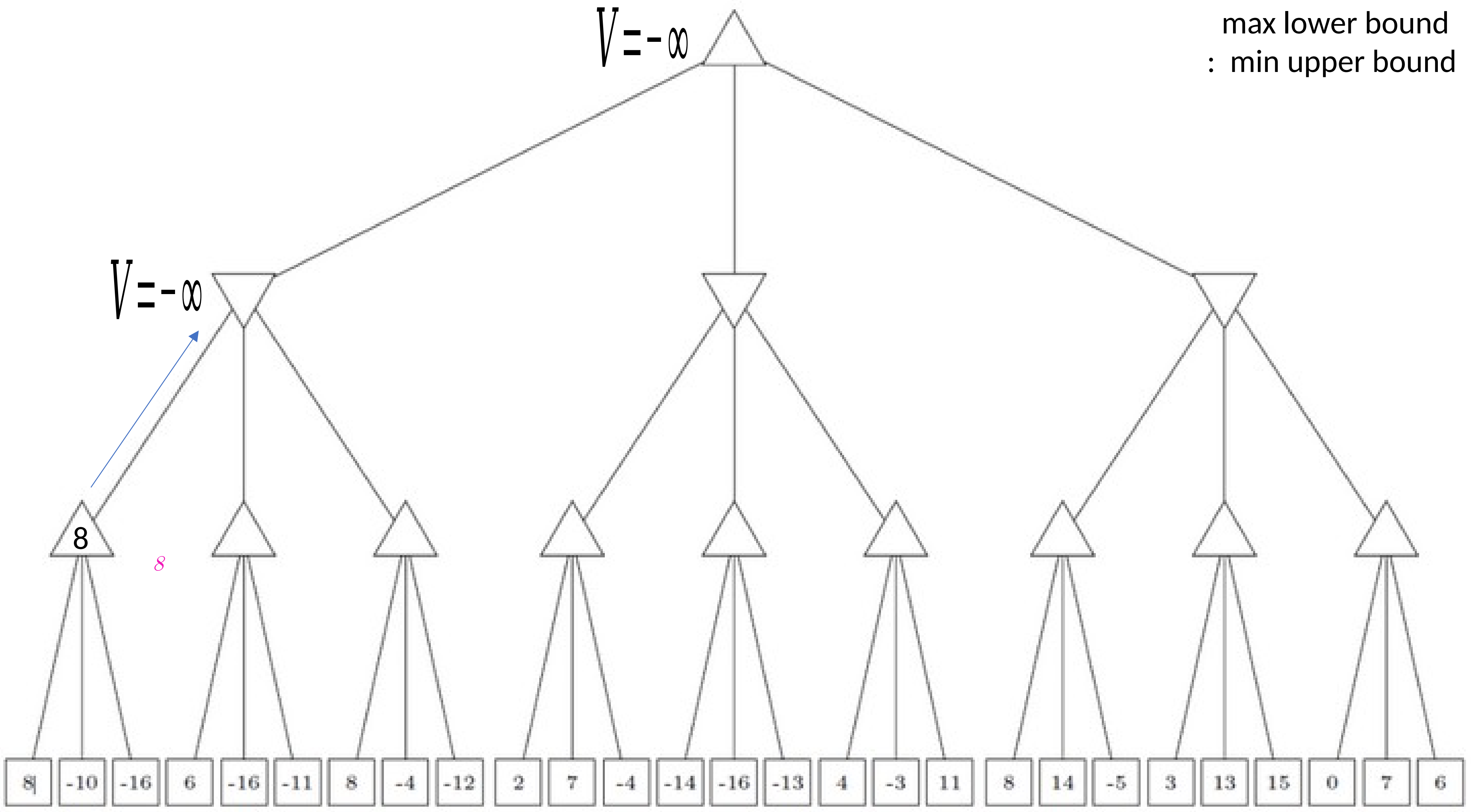


Exercise 12.1 – Solving Minimax Tree

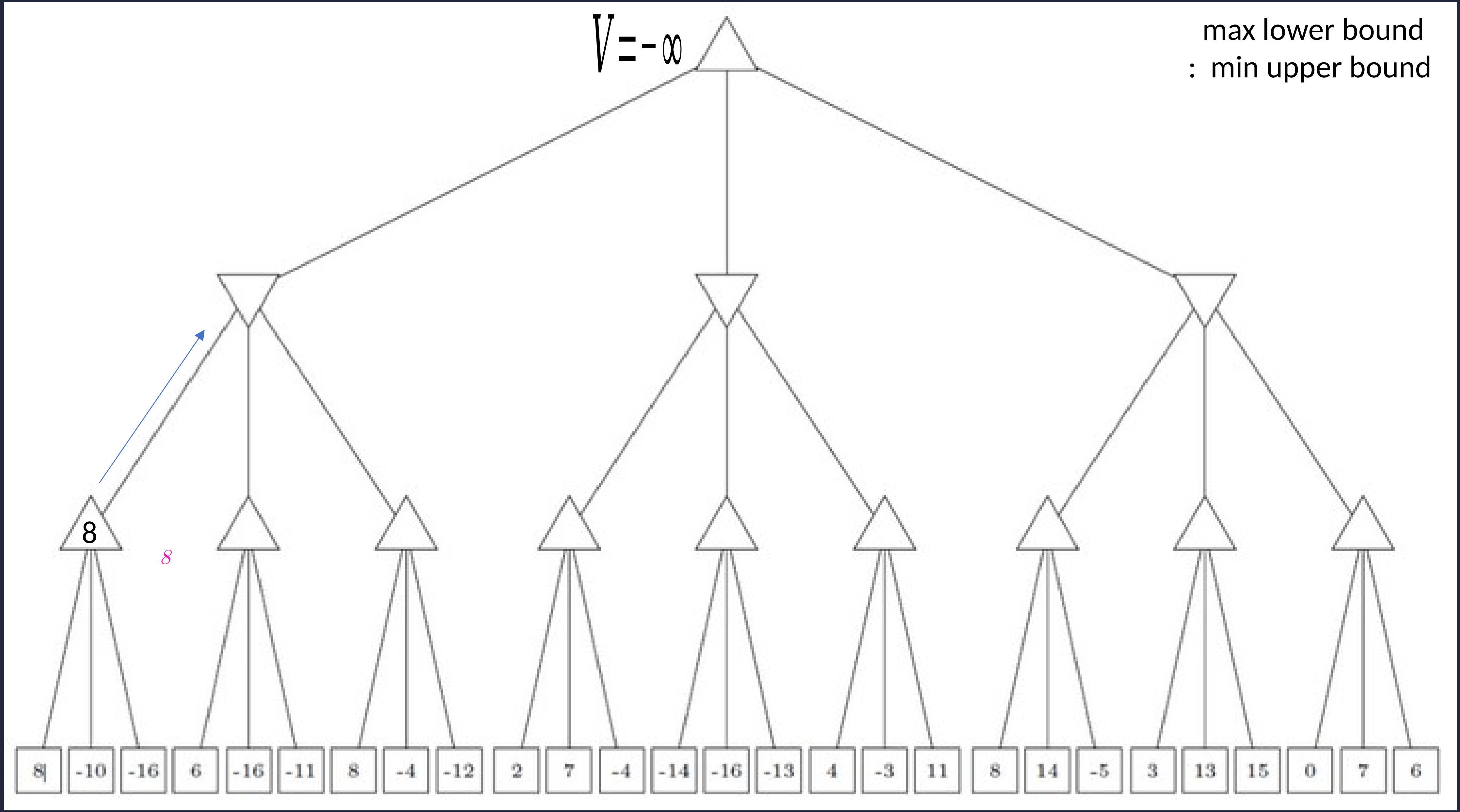




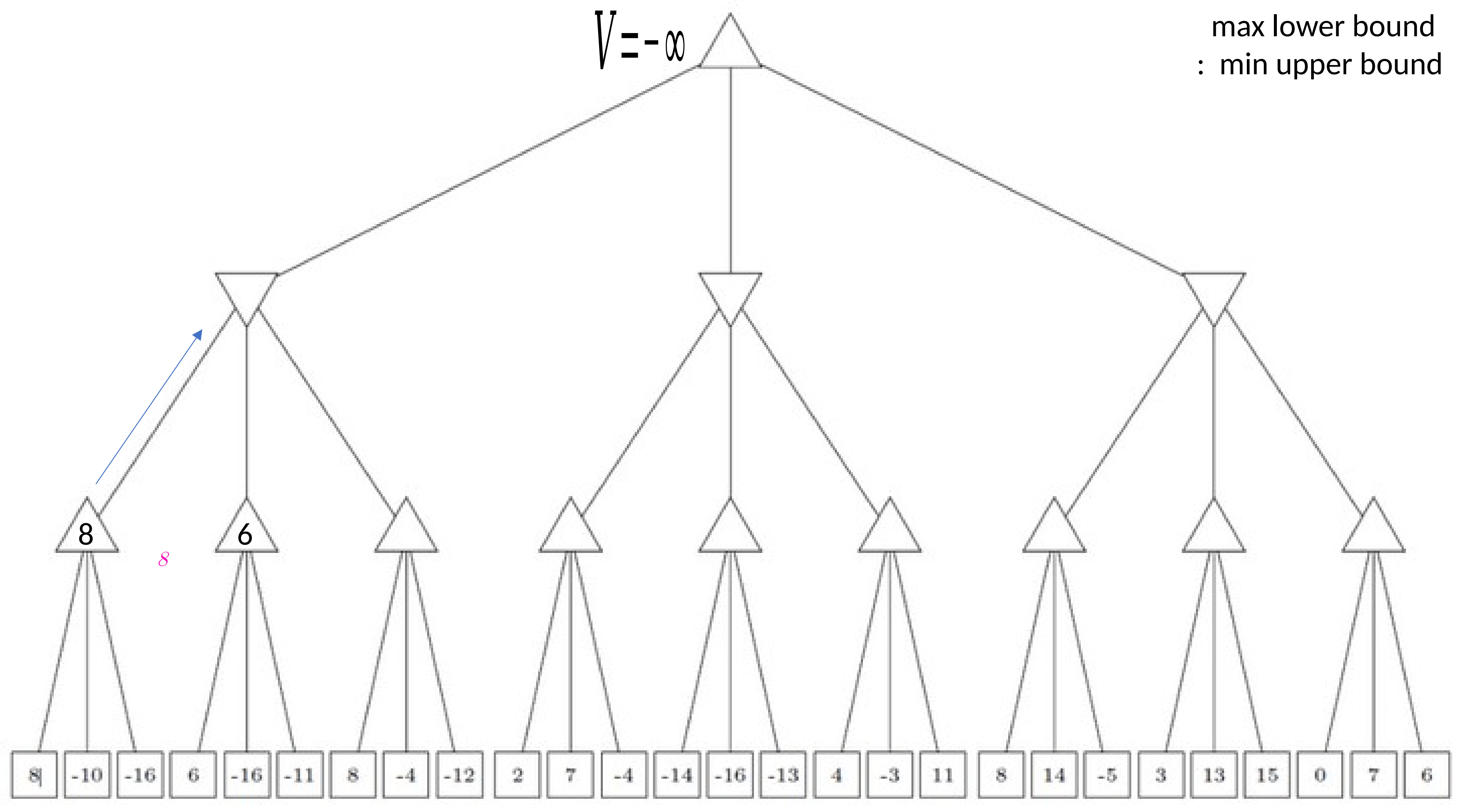
max lower bound
: min upper bound
V=-∞
V=-∞
V=-∞
8
-10
-16
6
-16
-11
8
-4
-12
2
7
-4
-14
-16
-13
4
-3
11
8
14
-5
3
13
15
0
7
6



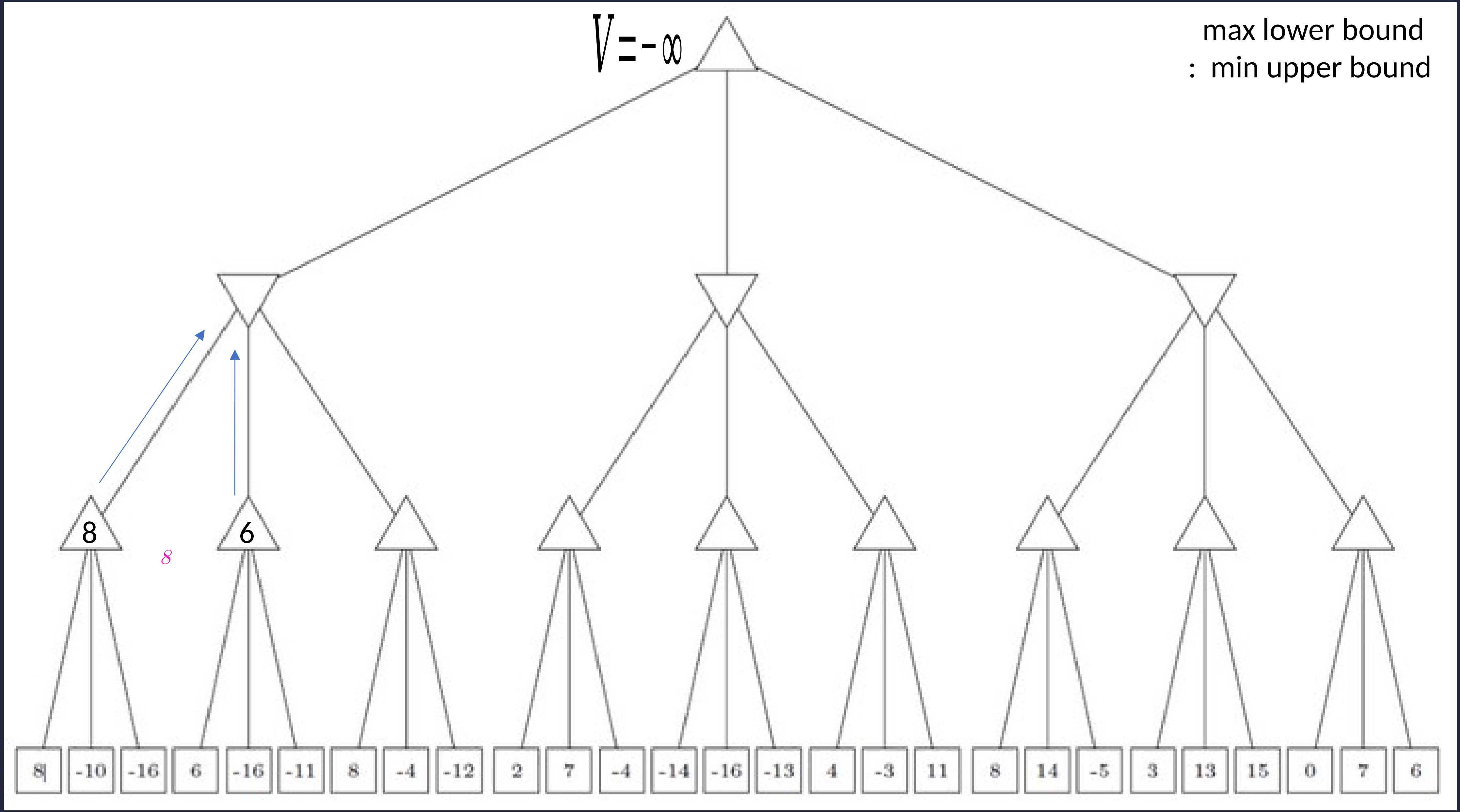
max lower bound
: min upper bound



We then back up the value to the parent node.
Parent min node Update and

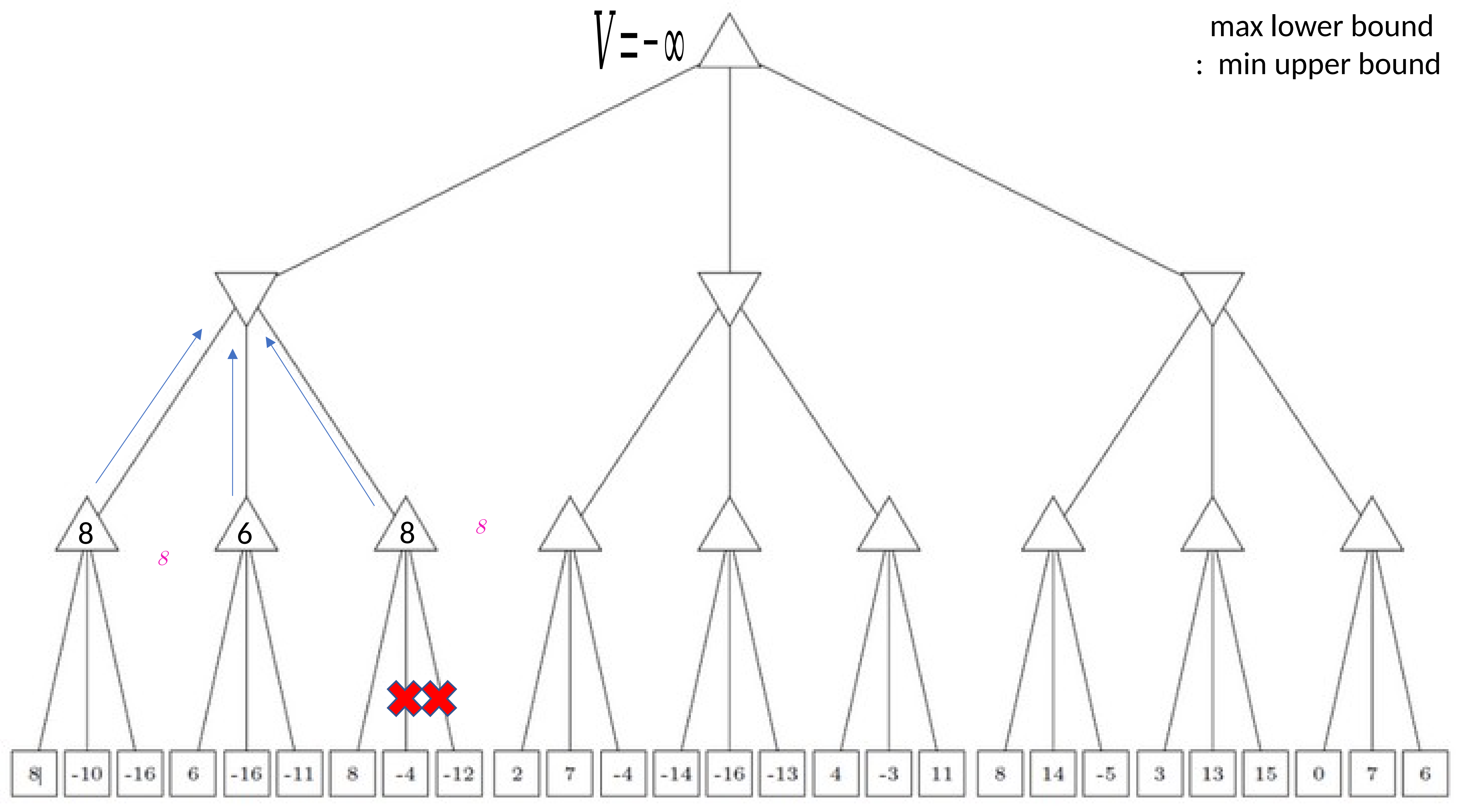


V=-∞
max lower bound
: min upper bound
8
6
8
8
-10
-16
6
-16
-11
8
-4
-12
2
7
-4
-14
-16
-13
4
-3
11
8
14
-5
3
13
15
0
7
6

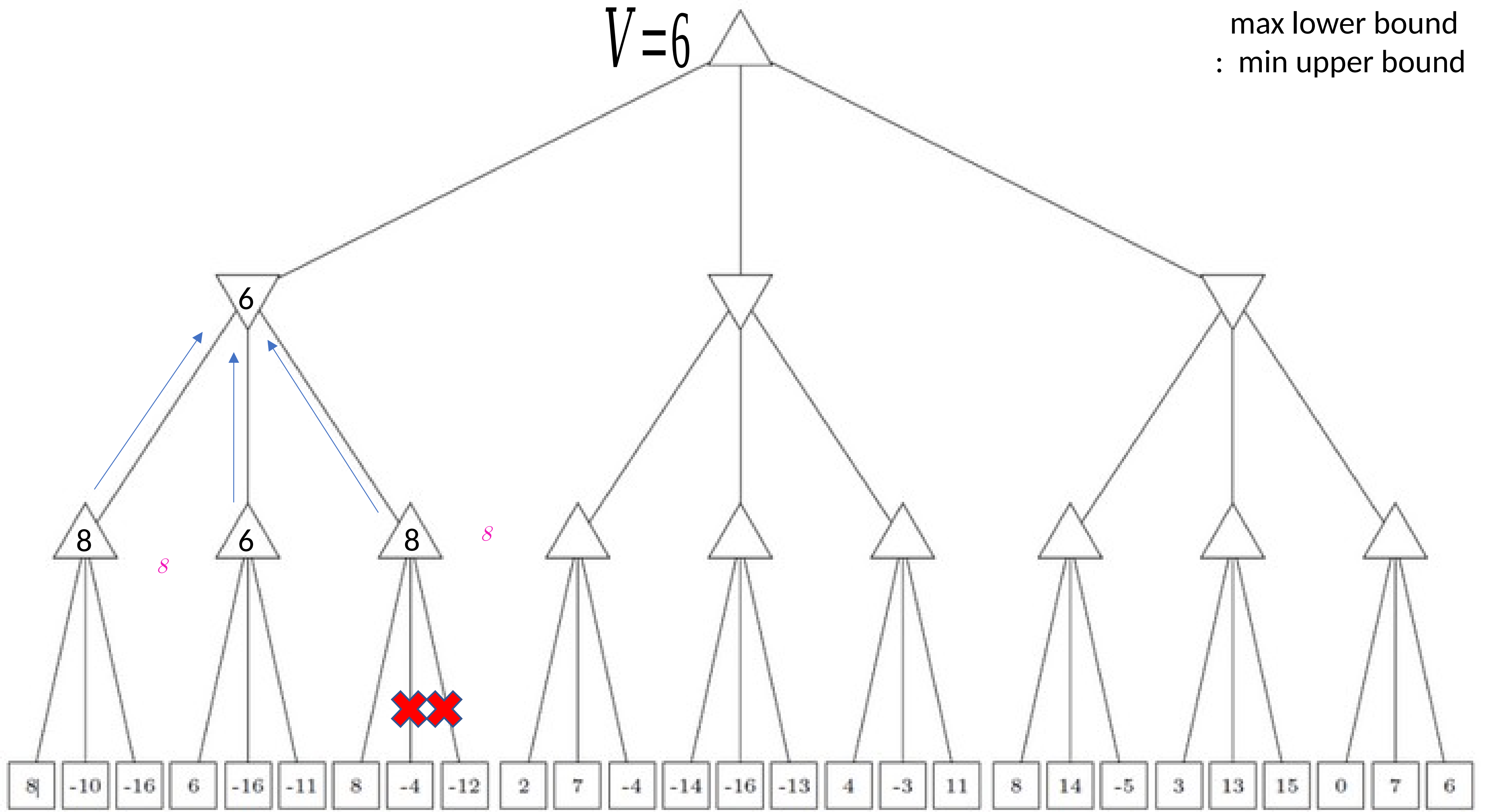


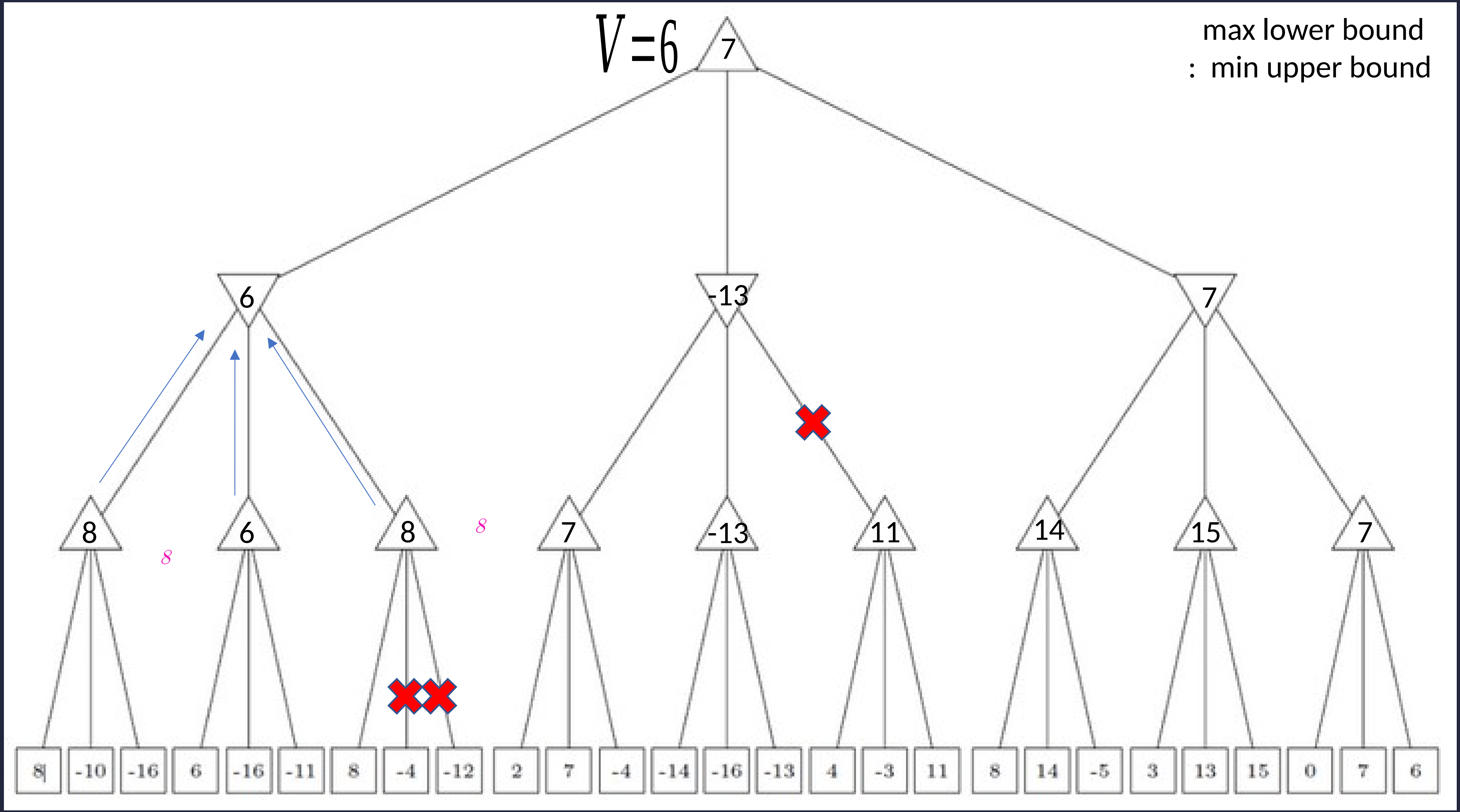
V=-∞
max lower bound
: min upper bound
8
6
8
8
-10
-16
6
-16
-11
8
-4
-12
2
7
-4
-14
-16
-13
4
-3
11
8
14
-5
3
13
15
0
7
6

max lower bound
: min upper bound



max lower bound
: min upper bound




V=6
max lower bound
: min upper bound
7
6
-13
7
8
6
8
7
-13
11
14
15
7
8
8
8| -10 -16 6 -16 -11 8 -4 -12 2 7 -4 -14 -16 -13 4 -3 11 8 14 -5 3 13 15 0 7 6

12.2 – Matching Pennies

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

12.2 – Matching Pennies

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

12.2 – Matching Pen

- If both pennies match, +1 for Even
- If they don't match, +1 for Odd

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

12.2 – Matching Pen

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd heads	*Odd* tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

- If both pennies match, +1 for Even
- If they don't match, +1 for Odd

- Let probability of opponent playing heads.
- For *Even*, the expected payoff when playing heads is given by

: <u>Even plays heads</u>. Probability of Odds playing heads is

From this, the expected payoff to Even is as both pennies match.

: <u>Even plays heads</u>. Probability of Odds playing tails is .

From this, the expected payoff to Even is as pennies mismatch.

12.2 – Matching Pen

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

- If both pennies match, +1 for Even
- If they don't match, +1 for Odd

- Let probability of opponent playing heads.
- For *Even*, the expected payoff when playing heads is given by

- Likewise, the expected payoff for Even when playing tails is given by

12.2 – Matching Pen

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

- If both pennies match, +1 for Even
- If they don’t match, +1 for Odd

- Let probability of opponent playing heads.
- For *Even*, the expected payoff when playing heads is given by

- Likewise, the expected payoff for Even when playing tails is given by

- Since these payoffs must match we can simplify and solve for

12.2 – Matching Pen

Exercise 12.2. *Matching pennies* is a simple game played by two players, *Even* and *Odd*. Each player has a penny and must secretly turn the penny to heads or tails, before the players simultaneously reveal their choices. If the pennies match (both heads or both tails), then *Even* keeps both pennies. If the pennies do not match (one heads and one tails) *Odd* keeps both pennies. Payoffs for *matching pennies* are given in the matrix below.

		Odd	
		heads	tails
Even	heads	1, -1	-1, 1
	tails	-1, 1	1, -1

Question: Find the Nash equilibrium of *matching pennies*. Show your working.

- Since these payoffs must match we can simplify and solve for

- Likewise, the same logic applies to Odd’s payoffs, and thus the Nash Equilibrium is at

12.3 – Morra

Exercise 12.3. Morra is an old game of chance, widely played in Roman times and possibly with origins in ancient Egypt. Understandably, there are many versions of Morra.

In a particular two-player variant, each player simultaneously shows either one or two fingers and announces a number between 2 and 4. If a player's number is equal to the sum of the number of fingers shown, then her opponent must pay her that many dollars. The payoff is the net transfer, so that both players earn zero if both or neither guess the correct number of fingers shown. In this version of Morra, each player has 6 strategies: she may show one finger and guess 2, or guess 3, or 4; or she may show two fingers and guess one of the three numbers.

a) There are two weakly dominated strategies in Morra. What are they? Eliminate them from the action space, then write down the reduced payoff matrix.

b) Imagine that player A can read player B's mind and anticipate how he plays before she chooses her action. Knowing that player A has this ability, what pure strategy should player B use?

c) Player B consults a game theory textbook and decides to use randomisation to improve his performance in Morra. Ideally, if he can find a best mixed strategy to play, what would be his expected payoff?

d) One possible mixed strategy is to play show one finger and call "three" with probability 0.6, and to show two fingers and call "three" with probability 0.4 (and play the other strategies with probability 0). Is this a Nash equilibrium strategy? Assume that Player B is risk neutral with respect to the game payoffs.

12.3a – Morra

- There are two weakly dominated strategies in Morra.
- It never pays to put out one finger and guess that the total number of fingers is 4
  - This is as the other player can at most put out 2 fingers (and thus, the total number of fingers is 3)
- Likewise, it never pays to put out two fingers, and guess that the sum is 2 because the other player must play at least one finger.

	one 2	one 3	two 3	two 4
one 2	0	2	-3	0
one 3	-2	0	0	3
two 3	3	0	0	-4
two 4	0	-3	4	0

12.3b – Morra

- We now suppose that player A can read player B's mind and anticipate how they play before choosing their action. Knowing that player A has this ability, what pure strategy should player B use?

- Player B should use *minimax reasoning* – minimise the greatest gain to their opponent, which is equivalent to maximising the minimum the opponent is guaranteed to receive. Answer is (one 3)

	one 2	one 3	two 3	two 4
one 2	0	2	-3	0
one 3	-2	0	0	3
two 3	3	0	0	-4
two 4	0	-3	4	0

12.3b – Morra

- We now suppose that player A can read player B's mind and anticipate how they play before choosing their action. Knowing that player A has this ability, what pure strategy should player B use?

- Player B should use *minimax reasoning* – minimise the greatest gain to their opponent, which is equivalent to maximising the minimum the opponent is guaranteed to receive. Answer is (one 3)

	one 2	one 3	two 3	two 4
one 2	0	2	-3	0
one 3	-2	0	0	3
two 3	3	0	0	-4
two 4	0	-3	4	0

12.3c – Morra

- We now suppose that Player B chooses actions randomly. If they can find the best mixed strategy to play, what would be their expected payoff?

- The answer to this question is entirely dependent on what the opponent does.
- Consider the worst case, in which Player A adapts to whatever lottery Player B chooses, and can itself randomise (i.e., pick a mixed strategy). If Player A can still read Player B's mind to see their mixed strategy, Player B should choose to make player A indifferent between pure strategies.

1. Because the game is symmetric, there is a symmetric mixed-strategy Nash equilibrium
2. Because the game is zero-sum, the symmetric mixed strategy Nash equilibrium must have payoffs to both players of 0.

12.3d – Morra

- One possible mixed strategy is to show one finger and call three with probability 0.6, and to show two fingers and call “three” with probability 0.4 (and play the other strategies with probability 0).
- Is this a Nash equilibrium strategy? Assume that Player B is risk neutral with respect to game payoffs.

- In a symmetric Nash equilibrium of a zero-sum game, the payoff is zero.
- We also know that both players are indifferent to pure strategies.
- If they are indifferent, they equal 0 in total, and they should also individually be 0.
- If one is greater than the other, then Player A would not mix, but rather exploit this property.

- To check, we see if player A’s expected rewards for each pure action are 0.
  - If they are, player A will have been induced to randomise by Player B’s mixed strategy (consistent with Nash Equilibrium)

Denote $\sigma_B = P_B(one3) : 0.6, P_B(two3) : 0.4$, and then:

$$R_A(one\ 2, \sigma_B) = 0.6(2) + 0.4(-3) = 1.2 - 1.2 = 0.0$$

$$R_A(one\ 3, \sigma_B) = 0.6(0) + 0.4(0) = 0$$

$$R_A(two\ 3, \sigma_B) = 0.6(0) + 0.4(0) = 0$$

$$R_A(two\ 4, \sigma_B) = 0.6(-3) + 0.4(4) = -1.8 + 1.6 = -0.2$$

Therefore, is not consistent with a Nash Equilibrium.

Assignment 3 Help