Short Quiz 3: Solution

Aryaman Maithani

21st August, 2019

Question. State whether the following statement is true or false. Justify your answer.
Is it possible to find a function $f : \mathbb{R} \to \mathbb{R}$ such that

(a) f is continuous on $[-7, 0]$

(b) f is differentiable on $(-5, 0)$

(c) $f'(x) \leq 2$

(d) $f(-5) = -3$ and $f(0) = 8$.

[2 marks for correct alternative (T/F); 3 marks for correct justification]

Answer. F **[2]**
Justification:
Assume that such a function f exists.
By hypothesis, f is continuous on $[-7, 0]$; thus, it is continuous on $[-5, 0]$. **[0.5]**
Moreover, f is differentiable on $(-5, 0)$. **[0.5]**
Thus, by MVT, we know that there exists $c \in (-5, 0)$ such that

$$f'(c) = \frac{f(0) - f(-5)}{0 - (-5)} = \frac{11}{5} > 2$$

[2]

However, by hypothesis $f'(c) \leq 2$. Thus, we have arrived at a contradiction. ■

Points to be noted -

1. Those who have not mentioned that f is continuous on $[-5, 0]$ have lost half a mark as per the above scheme. It is important to mention that as we cannot appeal to the Mean Value Theorem otherwise.
2. For those who written that $11/5 = 2.1$ or $11/5 = 2.5$, a quarter mark has been deducted. Note that it was not necessary to calculate $11/5$ in its decimal expansion. However, if you write something that is incorrect and use that to justify your claim, it is not be correct.
3. Half a mark has been deducted for simply writing $f'(c) = \dfrac{f(0) - f(-5)}{0 - (-5)}$ without mentioning what c is, that is, not writing "for some c in $(-5, 0)$" or "$\exists c \in (-5, 0)$."