Week 1 (10-03-2021)

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- Matrices $\rightarrow$ Multiply them.

(1) $\rightarrow$ $\begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$

$$= a_1 b_1 + \cdots + a_n b_n$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m \times n} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} \\ \\ \end{bmatrix}_{m \times 1}$$

$$A = \begin{bmatrix} A_1 \\ \vdots \\ A_m \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$A_1, \ldots, A_m \in \mathbb{R}^{1 \times n}$$

$$Ab = \begin{bmatrix} A_1 b \\ \vdots \\ A_m b \end{bmatrix}$$

$A \in \mathbb{R}^{m \times n}$ $\qquad B \in \mathbb{R}^{n \times p}$

$B = \begin{bmatrix} b_1 & \cdots & b_p \end{bmatrix}$; $b_1, \ldots, b_p \in \mathbb{R}^{n \times 1}$

$$AB = \begin{bmatrix} Ab_1 & \cdots & Ab_p \end{bmatrix} \in \mathbb{R}^{m \times p}$$

$\in \mathbb{R}^{m \times 1}$

$A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$

We say that B is an inverse of A if

$$AB = I = BA.$$

Fact. (Will see later) $AB = I \Rightarrow BA = I$

(this was not clear, a priori)

→ Functions $f, g : X \longrightarrow X$. ($X \neq \emptyset$ is some set.)

If $(f \circ g)(x) = x \quad \forall x \in X$,

is it necessary that $(g \circ f)(x) = x \quad \forall x \in X$?

No. Find example.

$Ax = b$. — (*) $\quad A \in \mathbb{R}^{m\times n}$, $x \in \mathbb{R}^{n\times 1}$, $b \in \mathbb{R}^{m\times 1}$

If A is upper triangular, it is easy by back-substitution. (Whether consistent or not is also clear.)

Idea: Do operations on both A and b to get something as above.

→ If $Ax_0 = b$, i.e., x_0 is a particular sol^n, and $S = \{x \in \mathbb{R}^{n\times 1} : Ax = 0\}$.

Then, all solutions of (*) are precisely of the form $x_0 + s$ for some $s \in S$.

Idea: Row echelon form (REF)

(i) All zero rows at bottom. (Possibly none.)

(No zero row can be above a nonzero row.)

first $\neq 0$ element from left

(2) Pivots should be strictly from left to right as you go from top to bottom.