Tutorial 1

Aryaman Maithani

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Disclaimer

These are **not** complete solutions and should not be regarded as such. The purpose of this is to basically get you started and you must fill in the gaps. To be more explicit, if what you care about is marks, then just the solutions written here won't suffice.

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1. (a) Use the definition and note that $(AB)^t = B^t A^t$. Make sure you prove both the directions of the *iff* statement.

   (b) $S = \frac{1}{2}(A + A^t)$ and $T = \frac{1}{2}(A - A^t)$.
   Argue that the above matrices do have the properties required. This shows the existence part.

   For the uniqueness part, assume that S' and T' are some other matrices with the same properties. Conclude that $S = S'$ by computing $A + A^t$ and likewise for T'.

2. (a) $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

   You still have to argue why these do have the properties required.

   (b) Let $m, n \in \mathbb{N}$ be such that $A^m = O = B^n$.
   Compute $(A + B)^{m+n}$ and $(AB)^n$ to conclude.

3. Hint: $(I - BA)(I + B(I - AB)^{-1}A) = I$.

4. The system of linear equations can be written in the form $Ax = b$.
   Consider the augmented matrix $[A \mid b]$ and convert to reduced echelon form.
   You should arrive at pivots in the second, third, and fourth columns.
   Thus, the free variables are the first and fifth one.
   Back-substitute starting from the last equation to get x_4, x_3, and x_2 in terms of x_1 and x_5. (In fact, they'll only be in terms of x_5 but consider that as $+0x_1$.)

   Thus, the complete set of solutions would be $(x_1, x_2, x_3, x_4, x_5)$ after substituting the values of x_2, x_3, x_4 in terms of x_1 and x_5.
   If my calculations are correct, you should get it finally to be of the form:

   $$(0, 8, 0, -1, 0) + x_1(1, 0, 0, 0, 0) + x_5\left(0, -\frac{33}{10}, -\frac{3}{10}, -\frac{1}{2}, 1\right),$$

   where $(x_1, x_5) \in \mathbb{R}^2$.
   (That is, a two-parameter family of solutions.)

   You may also write it as the following in set notation:

   $$\left\{ (0, 8, 0, -1, 0) + x_1(1, 0, 0, 0, 0) + x_5\left(0, -\frac{33}{10}, -\frac{3}{10}, -\frac{1}{2}, 1\right) \middle| (x_1, x_5) \in \mathbb{R}^2 \right\}.$$

5. Same concept as previous.