Week 1

11 May 2022 13:30

→ Hi. I'm Aryaman. Almost an alumnus.

→ bit.ly/ma-108 (Recordings of recap and tut - MS Team)

PDFs of whatever I write

MA-108

ODE: A relation involving $x, y, y', \ldots, y^{(n)}$.

$$F(x, y, \ldots, y^{(n)}) = 0.$$

interested in

eg. ① $y' + y = 0$

② $(y')^2 + xe^y + \sin(y'') = 0$

NOT

NOT example: $y(y(x)) = y'(x)$.

$$y^{(n)} = G(x, y, \ldots, y^{(n-1)}) \quad \text{---} (*)$$

EXPLICIT: (explicit)

A solution of $(*)$ is a function ϕ which is defined on some (open) interval I s.t.

$$\phi^{(n)}(x) = G(x, \phi(x), \ldots, \phi^{(n-1)}(x)) \quad \text{for all } x \in I.$$

IVP:

$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

Here, we will try to find an interval I containing x_0 s.t. $\exists\, \phi : I \to \mathbb{R}$ satisfying the above.

Linear ODE: Is an ODE of the form:

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_0(x)y = b(x).$$

The above is said to have order n if $a_n(x) \neq 0$.

EXAMPLE: $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ $(n \in \mathbb{Z})$

NON-EXAMPLES: ① $yy' + x = 43$

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Legendre's equation (MA 207)

NON-EXAMPLES: ① $yy' + x = 4^3$

② $e^y = y'$

EXPLICIT SOLUTION:

IVP: $y' = e^{x^2}$; $y(0) = 0$.

$$\phi(x) := \int_0^x e^{t^2}\,dt$$ is an EXPLICIT SOLUTION.

IMPLICIT: "Don't have $y = \phi(x)$.
Rather, a relation $F(x,y) = 0$
s.t. on some interval I, we can
get a ϕ s.t. $F(x, \phi(x)) = 0$
$\forall x \in I$."

EXAMPLE: ① $yy' + x = 0$.

$y^2 + x^2 = C$

Particular

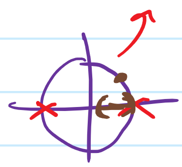
$y^2 + x^2 = 4 \longrightarrow$ Implicit

But this

$y = \sqrt{4-x^2}$ and $y = -\sqrt{4-x^2}$

as two solutions.

Use IFT.



② $\left[F(x,y) = y\sin x + x^2 e^y - y \right]$

Let's say $y\sin x + x^2 e^y - y = 0$

From the graph, it is "clear" that we can get a function of x on $(-1/2, 1/2)$.

Formally: IMPLICIT FUNCTION THEOREM

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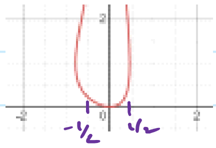


Note: ① $F(0,0) = 0.$

② $\dfrac{\partial F}{\partial y}(0,0) = \left(\sin(x) + x^2 e^y - 1\right)\Big|_{(x,y)=(0,0)}$

$= -1 \neq 0.$

The above conditions tell you that you can get y as a function x **locally**.

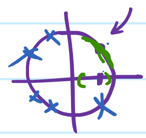
$$F(x, y) = c$$

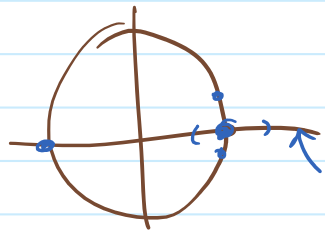
Given (x_0, y_0) s.t. $F(x_0, y_0) = c.$

$x^2 + y^2 = 4$

$\downarrow \partial/\partial y$

$2y$





Q: Can you can get ϕ on some interval I around x_0 s.t. $F(x, \phi(x)) = c$ $\forall\, x \in I$.

A: (IFT) Yes, if $\dfrac{\partial F}{\partial y}(x_0, y_0) \neq 0.$