$\mathbb{R}$eal Analysis

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§1. Sets and stuff

§2. Topology

1. Let (X, d) be a metric space and $x \in X$. Let $\delta > 0$. Define the following sets:

$$B_\delta(x) := \{y \in X \mid d(x, y) < \delta\},$$
$$C_\delta(x) := \{y \in X \mid d(x, y) \leq \delta\}.$$

   Show that $\overline{B_\delta(x)} \subset C_\delta(x)$.
   Can this inclusion be proper?
   HINT:

2. **Topological Nim**
   You and your friend want to play Topological Nim. Here's how it works:
   Let X be your favourite compact metric space and $r > 0$ your favourite (positive) real number.
   Each player removes an open disk of radius r from the space on their turn (only the center of the disk must not have been removed in a prior move), until one player—the winner—removes what remains of the space on his turn.

   Show that no matter what moves are played, the game stops after a finite number of moves. (In other words, there is no infinite sequence of legal moves.)

   Bonus: Fix $n \in \mathbb{N}$ and $r > 0$. Assuming optimal play, who will win the game if

$$X = S^n = \{\mathbf{x} \in \mathbb{R}^{n+1} \mid \|x\| = 1\}$$

   with the standard metric?
   (The answer will depend on r.)

   Credits: https://puzzling.stackexchange.com/questions/99859/

§3. Continuity and derivatives

§4. Integration

§5. Sequence and series of functions