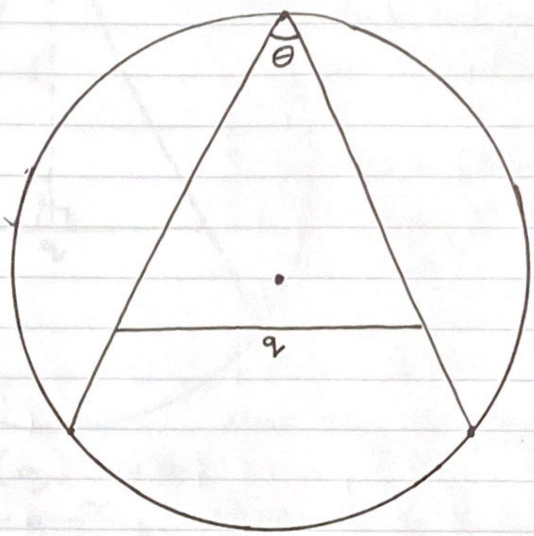
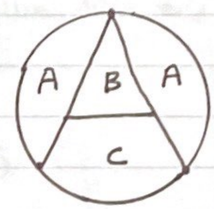
aryanagrawal@berkeley.edu
Aryan Agrawal

Riddler Express, May 22





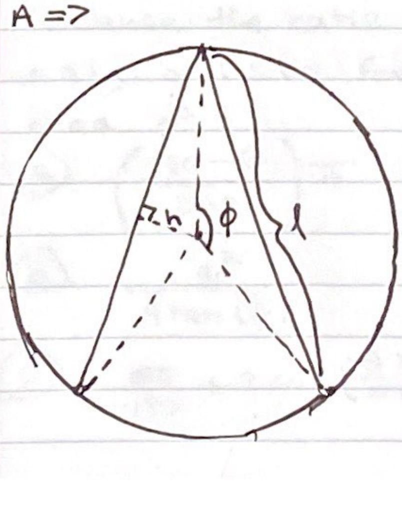
The above diagram is a top view of the cylindrical muffin.

Let $r \Rightarrow$ radius of the circle
$q \Rightarrow$ length of the A's midbar
$\theta \Rightarrow$ angle at which the A "opens"

Note that the midbar, q, does not necessarily have to fall along a diameter of the circle.

First, derive expressions representing the areas of each zone.

$A \Rightarrow$

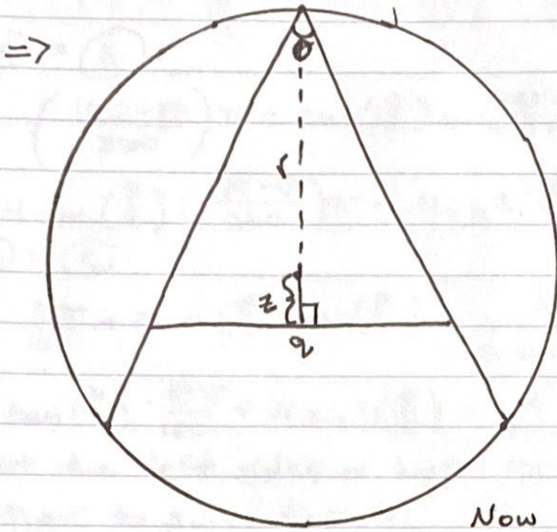


$$\phi = 180 - \theta$$
$$l = 2r\cos\left(\frac{\theta}{2}\right)$$
$$h = r\sin\left(\frac{\theta}{2}\right)$$

Area of A: $\frac{\phi}{360} \cdot \pi r^2 - \frac{1}{2} l h$

$$= \left(\frac{180-\theta}{360}\right) \cdot \pi r^2 - r^2 \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$$

B =>



To account for the midbar's offset from the center, solve for z, in terms of θ and q:

$$\tan\left(\frac{\theta}{2}\right) = \frac{q/2}{(r+z)}$$

$$\frac{2\tan\left(\frac{\theta}{2}\right)}{q} = \frac{1}{r+z}$$

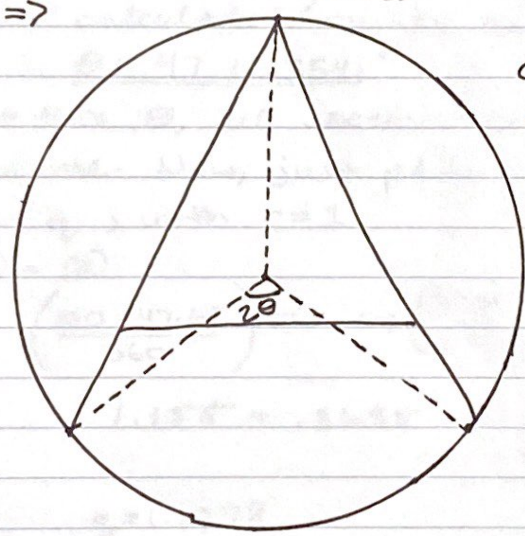
$$z = \frac{q}{2\tan\left(\frac{\theta}{2}\right)} - r$$

Now solve for the area of B:

$$\frac{1}{2}(q)(r+z)$$

$$= \frac{1}{2}q\left(r + \frac{q}{2\tan\left(\frac{\theta}{2}\right)} - r\right) = \frac{1}{2}q\left(\frac{q}{2\tan\left(\frac{\theta}{2}\right)}\right) = \frac{q^2}{4\tan\left(\frac{\theta}{2}\right)}$$

C =>



Calculate the combined area of zones B and C, then subtracted the area of B:

$$\frac{\theta}{180}\pi r^2 + 2r^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) - \frac{q^2}{4\tan\left(\frac{\theta}{2}\right)}$$

Because the ratio between q and r is constant for $r > 0$, let $r = 1$, and solve for θ such that the area of A = area of B = area of C.

A $\left(\frac{180-\theta}{360}\right)\pi - \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$

B $\frac{q^2}{4\tan\left(\frac{\theta}{2}\right)}$

C $\frac{\theta\pi}{180} + 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) - \frac{q^2}{4\tan\left(\frac{\theta}{2}\right)}$

Ⓐ = Ⓑ:

$$\left(\frac{180-\theta}{360}\right)\pi - \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \frac{q^2}{4\tan\left(\frac{\theta}{2}\right)}$$

$$4\tan\left(\frac{\theta}{2}\right)\left(\frac{180-\theta}{360}\right)\pi - 4\sin^2\left(\frac{\theta}{2}\right) = q^2$$

Ⓑ = Ⓒ:

$$\frac{\theta\pi}{180} + 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \frac{q^2}{2\tan\left(\frac{\theta}{2}\right)}$$

$$2\tan\left(\frac{\theta}{2}\right)\cdot\frac{\theta\pi}{180} + 4\sin^2\left(\frac{\theta}{2}\right) = q^2$$

Set the left sides of both resultant expressions equal to each other to solve for θ:

$$4\tan\left(\frac{\theta}{2}\right)\left(\frac{180-\theta}{360}\right)\pi - 4\sin^2\left(\frac{\theta}{2}\right) = 2\tan\left(\frac{\theta}{2}\right)\cdot\frac{\theta\pi}{180} + 4\sin^2\left(\frac{\theta}{2}\right)$$

$$4\tan\left(\frac{\theta}{2}\right)\left(\frac{180-\theta}{360}\right)\pi - 8\sin^2\left(\frac{\theta}{2}\right) - 2\tan\left(\frac{\theta}{2}\right)\cdot\frac{\theta\pi}{180} = 0$$

Use calculator/numpy to solve for θ:

$$\underline{\theta = 47.653541°}$$

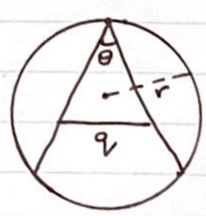
At this θ, all sections of the cylinder will be of equal volume. Now, just plug in θ into a prior equation to solve for q, with $r=1$.

Ⓐ = Ⓑ

$$\left(\frac{180-47.65}{360}\right)\pi - \cos\left(\frac{47.65}{2}\right)\sin\left(\frac{47.65}{2}\right) = \frac{q^2}{4\tan\left(\frac{47.65}{2}\right)}$$

$$1.155 - .3695 = \frac{q^2}{1.766}$$

$q = 1.1778$
$r = 1$
$\theta = 47.653541°$



$\boxed{q : r = 1 : .84900}$

The ratio of the length of the midbar to the radius must be 1 : .84900 for $r > 0$, where all sections of the cylinder are of equal volume.