1 Problem Set

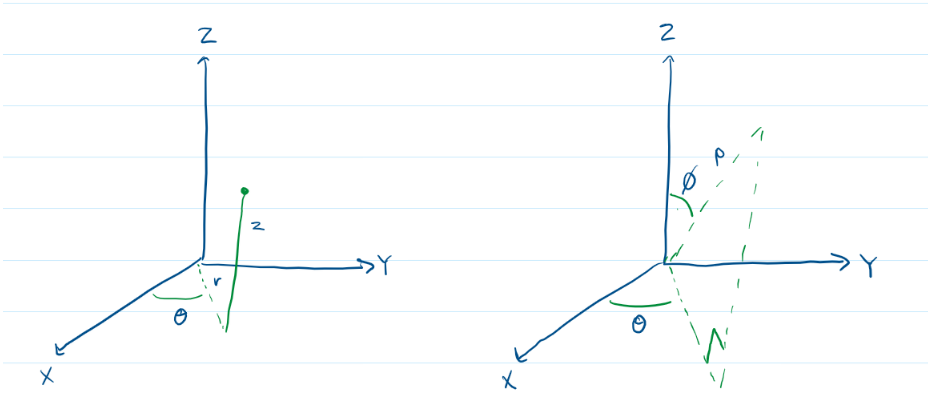
If you are curious about visualising 3D points, lines and planes, I highly recommend GeoGebra's 3D calculator.

Throughout the problem set, we work in $\mathbb{R}^3$. Before we proceed, we recall the new coordinate transformations we've seen:

Cylindrical coordinates $x = r\cos(\theta)$; $y = r\sin(\theta)$; $z = z$, where $\theta \in [0, 2\pi]$, $z, r \in \mathbb{R}$

Spherical coordinates $x = r\sin(\theta)\cos(\varphi)$; $y = r\sin(\theta)\cos(\varphi)$; $z = r\cos(\theta)$, where $\theta \in [0, 2\pi), \varphi \in [0, pi], \rho \in \mathbb{R}_{\geq 0}$

For revision, you can derive the above by annotating these to express x, y, z in the respective coordinates:



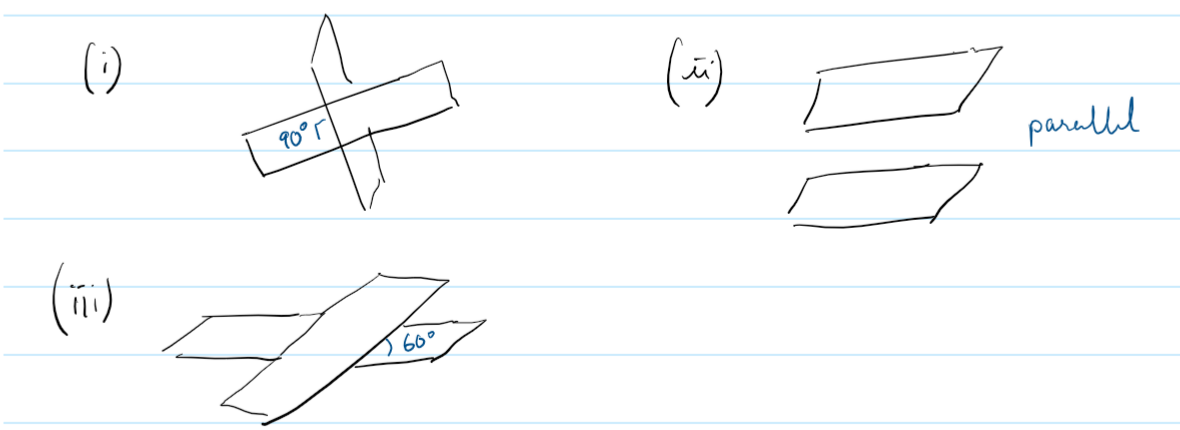
1. Interesting structures in new coordinate systems: Sketch the following solids/graphs. (You should get that they resemble an annulus, box or helix for the first three.):

1. $x \in (3, 4), y \in [0, 5], z = 0$
2. $\rho \in [3, 4], \theta \in [0, 2\pi), \phi \in [0, \pi)$
3. $r = z \in \mathbb{R}^3, \theta = 2\pi r$ (Note here that some parameters depend on the value that others take)
4. **a cool bonus problem**: $\bigcup_{n \in \mathbb{N}} \rho \in [2n, 2n + 1], \theta \in [0, \pi], \phi \in [0, \frac{\pi}{2n}]$

2. Area of triangle Suppose you are given three vectors, $\vec{u}, \vec{v}, \vec{w}$ such that they intersect to form a triangle. Express the area of the triangle in terms of these vectors (and operations between them).

(ctd.)

3. **Finding angles between planes:** **Recall that, to find the angle between two vectors, we use the dot product formula: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$. Now, consider the following ways in which planes intersect: can you infer the angle between their normal vectors? With your observation, can you find a formula for angle between two planes using the dot product?**



4. **What maketh a plane?:** **Similar to last time, do you think that there exists some plane satisfying the properties below? If yes, is there a unique such plane, or a collection of them? Can you characterize them?**

1. A plane perpendicular to a given, non-zero vector $\vec{v}$ and consisting of a point $\vec{u}$
2. A plane containing two non-parallel, non-zero vectors $\vec{a}, \vec{b}$.
3. A plane that contains the line $\vec{v}(t) = (0, 0, 1) + t(1, 1, 1)$.
4. A plane that contains the line $\vec{v}(t) = (0, 0, 1) + t(1, 1, 1)$ and is perpendicular to the plane $x + y + z = 0$.
5. A plane that contains the line $\vec{v}(t) = (0, 0, 1) + t(1, 1, 1)$ and is perpendicular to the plane $x - y + z = 0$.