# 1 Problem Set

Throughout the problem set, we work in $\mathbb{R}^3$.

**1. For each of the following properties, mention if there exists an object satisfying it or not. If such an object exists, write if it is (i) unique, and find an expression for it (ii) non-unique, and characterise it as much as you are able to.**

1. Line through point $c_0$ with slope $\vec{v}$.
2. Line segment through points $(2, 1, 2), (3, 2, 3)$ and $(4, 0, 4)$.
3. Line segment through points $(1, 2, 1), (5, 4, 7)$ and $(-1, 1, -2)$.
4. Line orthogonal to that found in part (1) and passing through $c_0$.
5. Line orthogonal to that found in part (1).

**2. True or false? If false, then correct it. Write the result that you are using from class.**

1. For $\vec{x}, \vec{y} \in \mathbb{R}^3, |\vec{x} \cdot \vec{y}| = ||\vec{x}||||\vec{y}||$
2. For $\vec{a_1}, \vec{a_2}, \vec{a_3}, \vec{a_4}$,
$$|\vec{a_1} - \vec{a_4}| < |\vec{a_1} - \vec{a_3}| + |\vec{a_3} - \vec{a_2}| + |\vec{a_2} - \vec{a_3}| + |\vec{a_3} - \vec{a_4}|$$

**3. Suppose you are given a point $P$ and a line $l(t) = \vec{a_0} + t\vec{v}, t \in \mathbb{R}$, where $\vec{v} \neq \vec{0}$. Find an expression for the point on the line that is the shortest distance from $P$ (for a hint, consider the illustration we'll draw on the board.**

**4. Suppose we have two lines intersecting at the origin, $\mathbb{R}^3 : l_1(t) = t\vec{v}$ and $l_2(s) = s\vec{u}$, where $s, t \in \mathbb{R}$, such that that $\vec{u}, \vec{v}$ are not parallel. Find the line that (i) passes through their point of intersection, (ii) is orthogonal to both lines.**