Section 2

Thursday, September 8, 2022 11:46 AM

From last week, recall that....

$\boxed{\text{BASIC COUNTING RULE}}$

If we have r distinct position, with $\{|a_i|\}_{i=1}^{r}$ choices of distinct objects for each,

pos : 1 2 3 ... r

choices : $|a_1|$ $|a_2|$ $|a_3|$ ... $|a_r|$

we have $|a_1| \times |a_2| \ldots \times |a_r|$ total sequence in Ω. ⇒ (Pf) in PDF Document from My Section Notes (Week 1 + 2)

$\boxed{\text{Special case}}$

Say, you have n distinct objects, and you draw r without replacement.

pos : 1 2 3 ... r

choices : $n \times n-1 \times n-2 \cdots \times n-(r+1)$

$$|\Omega| = n(n-1)\ldots(n-(r+1)) = (n)_r = \frac{n!}{(n-r)!}$$

There are then $(n)_r$ total sequences in Ω.

BUT WHAT IF SOME OBJECTS AREN'T DISTINCT?

(Ex) Count # of ANAGRAMS of the word BANDANA.

A particular anagram is a particular sequence of letters - if there are 3 A's, it is indifferent to which A lands in which position.

If we count as in $\boxed{\text{Special case}}$, we count any particular anagram too many times.

$\Omega_s = \{ \{B A_1 N_1 D A_2 N_2 A_3\}, \{B A_2 N_1 D A_1 N_2 A_3\}, \ldots \}$ ⟹ $3!\,2!$ total counts

For any 'word', $2!$ diff. permutation counted for varying position of the 2 N's.

So, to account for overcounting, we simply 'quotient' Ω_s. That is, we assert some notion of equivalence of elements in Ω_s.

For $\omega_1, \omega_2 \in \Omega$, we want to say, $\omega_1 = \omega_2$ WHEN they are same up to permutation of identical letters.

In other words, we adjust our counting such that,

$3!\,2!$ 'equivalent' sequences — counted as 1 anagram

$$\Omega_s = \left\{ \{B A_1 N_1 D A_2 N_2 A_3\}, \{B A_2 N_1 D A_1 N_2 A_3\}, \ldots \right\} \qquad \Omega_{true} = \left\{ \{BANDANA\}, \ldots \right\}$$

So, this teaches us to count Ω_{true}.

Namely, $\boxed{\#\text{ of ANAGRAMS} = |\Omega_{true}| = \frac{7!}{3!\,2!}}$

all possible sequences ($7!$)

ways to permute the identical letters A and N ($3!\,2!$)

In general, when we have some objects being identical, it is nice to consider the # of distinct groups of objects. We have the following result

# of sequences when objects are considered distinct

$$\frac{7!}{1!\,2!\,1!\,3!}$$

Say, we have r distinct groups, w/ multiplicity $m_1, \ldots m_r$, s.t. $\sum_{i=1}^{r} m_i = n$. (total # of obj $= n$)

i.e.

$\rightarrow \binom{n}{\underbrace{m_1, m_2 \ldots m_r}_{\sum m_i = n}}$ is a multinomial coefficient.

objects are considered distinct

Say, we have r distinct groups, w/ multiplicity $m_1 \dots m_r$,
s.t. $\sum_{i=1}^{r} m_i = n$. (total # of obj = n)

Then, $\exists \dfrac{n!}{m_1! m_2! \cdots m_r!}$ distinct (i.e. permutation-invariant) sequence of length n

$\dfrac{7!}{1!2!1!3!}$

this accounts for permutation of identical objects

$\sum m_i = n$

What is this notion of grouping?

(Q) Above example

positions $\Rightarrow$ 1 2 3 4 5 6 7

_ _ _ _ _ _ _

① Groups = letters $\Rightarrow \{B, N, D, A\} \Rightarrow \dfrac{7!}{2!3!1!1!}$

Multiplicity = # of each letter $\Rightarrow \{1, 2, 1, 3\}$

$\Omega = \{$ BANDANA, AABANDN, ... $\}$

(Q) Say, there are 24 people, and there are 11, 8 5 tickets to Ravens, Orioles and Lakers game. How many diff. outings can be planned?

② Groups = teams $\Rightarrow \{$ Raven, Orioles, Lakers $\} \Rightarrow \dfrac{24!}{11!8!5!}$

Multiplicity = # of players in each team $\{11, 8, 5\}$

people $\Rightarrow$ 1 2 3 ... 24

_ _ _ ... _

$\Omega = \{$ RRR ..., OOO ..., ... $\}$

QUESTIONS

(Try to visualise the sequences, groups and multiplicities of things!)

(Q) Say, there are 5 science books, 3 english books, and 22 math books. We arrange them on a rack.

(Q) If each book is distinct, how many different arrangements are possible?

$30!$ basic counting rule

(Q) What if books of the same subject are considered the same?

$\binom{30}{5, 3, 22}$ straight-forward multinom.

(Q) What if all 5 science books are together? (Note, consider books of same subject to be identical).

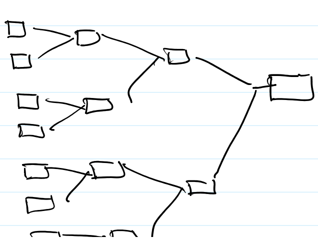
$\binom{26}{1, 3, 22}$ (treat science books as 1 unit)

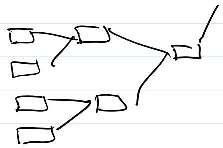
(Q) If no English books are together? (All books of same subject are identical)

$\underbrace{\binom{28}{3}}_{\text{pick out spots for E-books}} \underbrace{\binom{27}{22, 5}}_{\text{arrange non-E books}}$

(Q) Outcomes in Competition

Suppose there are 8 boxers entering a competition. They are split into matches of 2.
The format of the competition is like a tournament:





① How many possible <u>matches</u> for round 1?

$$\underbrace{\frac{1}{4!}}\ \underbrace{\binom{8}{2,2,2,2}}$$

Since the 4 matches aren't distinct in a relevant real life sense, we count them up to permutation of labels.

choosing 4 (distinct) groups of 2 players each.

② How many possible <u>outcomes</u> for round 1?

$$\underbrace{2^4}_{} \cdot \underbrace{\frac{1}{4!}\binom{8}{2,2,2,2}}_{\#\text{ of match}} = \frac{8!}{4!}$$

choose 1 out of 2 to be winner in each of 4 matches

<u>BONUS</u> ⇒ How many total outcomes of the competition?

$8!$ (8 choices for 1st, 7 for 2nd, & so on...) → can you derive this from part (b)?

Q Say, there are 15 people, out of which 3 are children. They are to travel in cars that carry 7, 5 and 3 people.

Find $\mathbb{P}$[all 3 children travel together].

$$\frac{\binom{12}{7,5} + \binom{12}{4,5,3} + \binom{12}{7,2,3}}{\binom{15}{7,5,3}}$$

→ put children into 3, 7 or 5-people car. We add because they're disjoint events.

⟶ total