Thursday, September 29, 2022 1:07 PM

(1) Martha and Stuart are baking cakes. M
Suppose that the probability that Martha burns her cake is 0.2. $P(M)$, $P(M^c)$
Suppose that the probability that Stuart burns his cake is 0.4. $P(S)$, $P(S^c)$ S
You know that exactly one cake was burned.
What is the probability that Stuart burnt it?

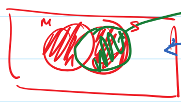
① Designate events

What we know
① M and S are independent

② What are we interested in?

$P[S \mid \underbrace{\text{either } M \text{ or } S, \text{ but not both}}]$ ⇒

$[(S \cap M^c) \cup (M \cap S^c)]$ = symmetric difference of M & S



$\boxed{S \cap M^c}$

$P[\ \cdots\ ] = P$

① set theory
② venn diagram

③ $P[S \mid (S \cap M^c) \cup (M \cap S^c)] = P[S \cap [(S \cap M^c) \cup (M \cap S^c)]]$

disjoint

$P[S \cap M^c] + P[M \cap S^c]$

$$= \frac{P[S \cap M^c]}{P[S \cap M^c] + P[M \cap S^c]} = \frac{P[S]P[M^c]}{P[S]P[M^c] + P[M]P[S^c]}$$

(2) Suppose you're generating random numbers, choosing from 0 and 1.
You generate 2 numbers initially.
If you get two zero's, then you get to generate 2 more numbers.
If you get one zero, then you get to generate 1 more number.
Else, you don't get more.
What is the probability you get exactly 2 one's?

$P[2 \text{ ones}] = P[2 \text{ ones} \cap 2 \text{ zeros}] + P[2 \text{ ones} \cap 1 \text{ zero}] + P[2 \text{ ones} \cap 0 \text{ zeros}]$

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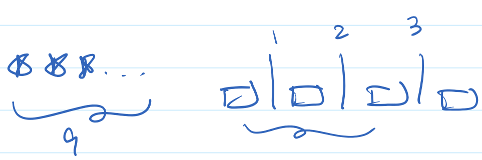
$= \frac{1}{4} \cdot \frac{1}{4} + \left(\frac{1}{2}\right) \frac{1}{2} \cdot \frac{1}{2} + \left(\frac{1}{2}\right) \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}$

(3) Say, John has 9 identical gold rings to put on his 4 fingers, excluding his thumb.
How many ways can he do so?

What we want
① # of $|\Omega|$

What we know
① 9 identical rings, 4 distinct fingers
② You can have ≥ 0 rings on a finger

box & bars



$$\binom{9+3}{3} = \frac{12!}{9!\,3!} = \binom{12}{3}$$

(4) Say, you have a jar with 9 jellybeans – 4 red and 5 blue. You draw them with replacement – let Y be the trial on which you choose first red jellybean. Given $Y=y$, you then roll a fair die y times. Let X be the number of 6's you get.
What is $\mathbb{P}[X=0]$?

[J] B B B B ... ~~R~~ ... B R

$\cancel{5} = y$

$\times \frac{5}{9}$

B R

$\frac{5}{9} \times \frac{4}{9}$

Roll your dice 8 times!
↳ $\mathbb{P}[X=0] = \left(\frac{5}{6}\right)^{8 = y}$

(1) $\mathbb{P}[X=0 \mid Y=8] = \left(\frac{5}{6}\right)^8$

(2) $\mathbb{P}[X=0] = \mathbb{P}[X=0 \mid Y=1]\,\mathbb{P}[Y=1] + \mathbb{P}[X=0 \mid Y=2]\,\mathbb{P}[Y=2] + \dots$

$= \left(\frac{5}{6} \cdot \frac{4}{9}\right) + \left(\left(\frac{5}{6}\right)^2 \times \frac{5}{9} \times \frac{4}{9}\right) + \dots = \frac{a_1}{1-r_n} = \frac{\frac{5}{6}\cdot\frac{4}{9}}{1-\frac{5}{6}\cdot\frac{5}{9}} =$

a_1 = first term

$\times \frac{5}{6} \cdot \frac{5}{9}$

(5) The number $17! = \underbrace{2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17}_{\text{prime factors}}$.

(i) How many different divisors exist of $17!$?

(ii) How many are odd?

(i) Note, a divisor of $17!$ is a product of some of its factors.
How many ways can we choose factors to multiply?

choose $0 \le i \le 15$ number of 2
$0 \le i \le 6$ number of 3

$\underbrace{2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17}$

$16 \times 6 \times 4 \times 3 \times 2 \times 2 \times 2$ choices

(ii) Play the same game as above, but with just the odd factors.

$6 \times 4 \times 3 \times 2 \times 2 \times 2$

(In particular, $P(\text{getting odd factor of } 17!) = \frac{1}{16}$)