Week 11

Sorting

(Merge Sort, Counting Sort, Radix Sort, Bucket Sort)

Merge Sort

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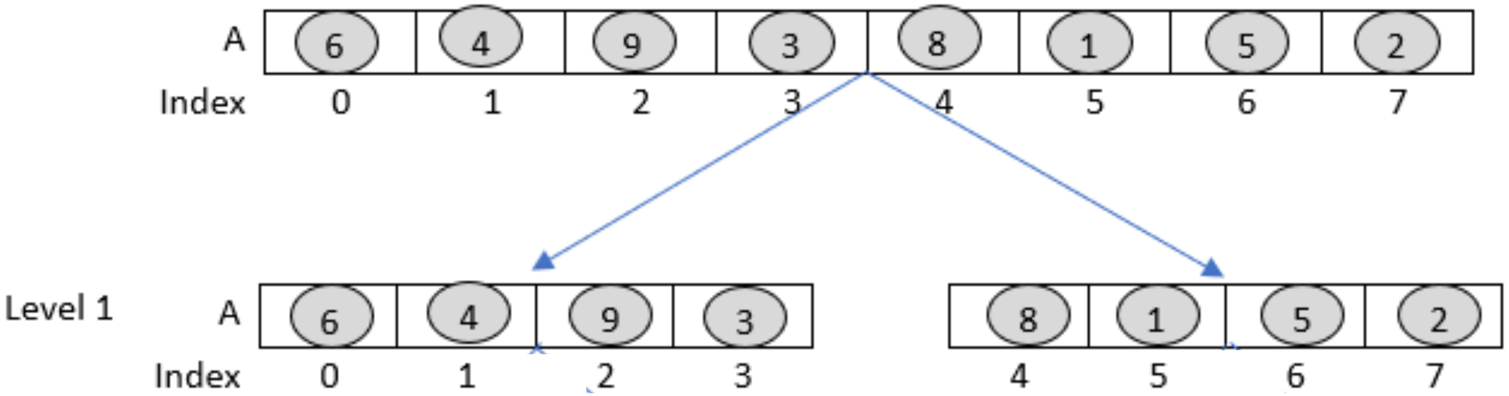
It is an external memory algorithm. That means, algorithm can process the data collection part by part

Merge Sort

A	6	4	9	3	8	1	5	2
Index	0	1	2	3	4	5	6	7

Merge Sort – Divide into smaller problems



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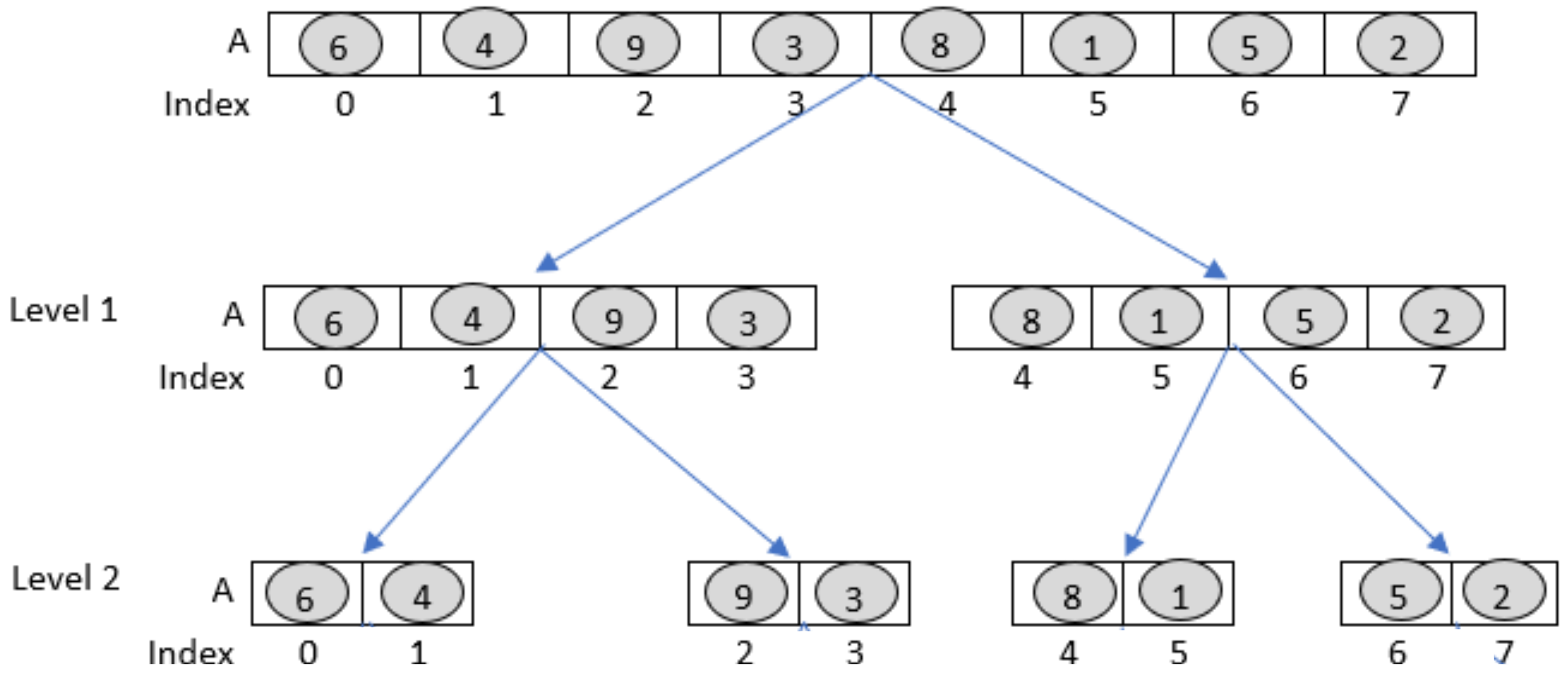

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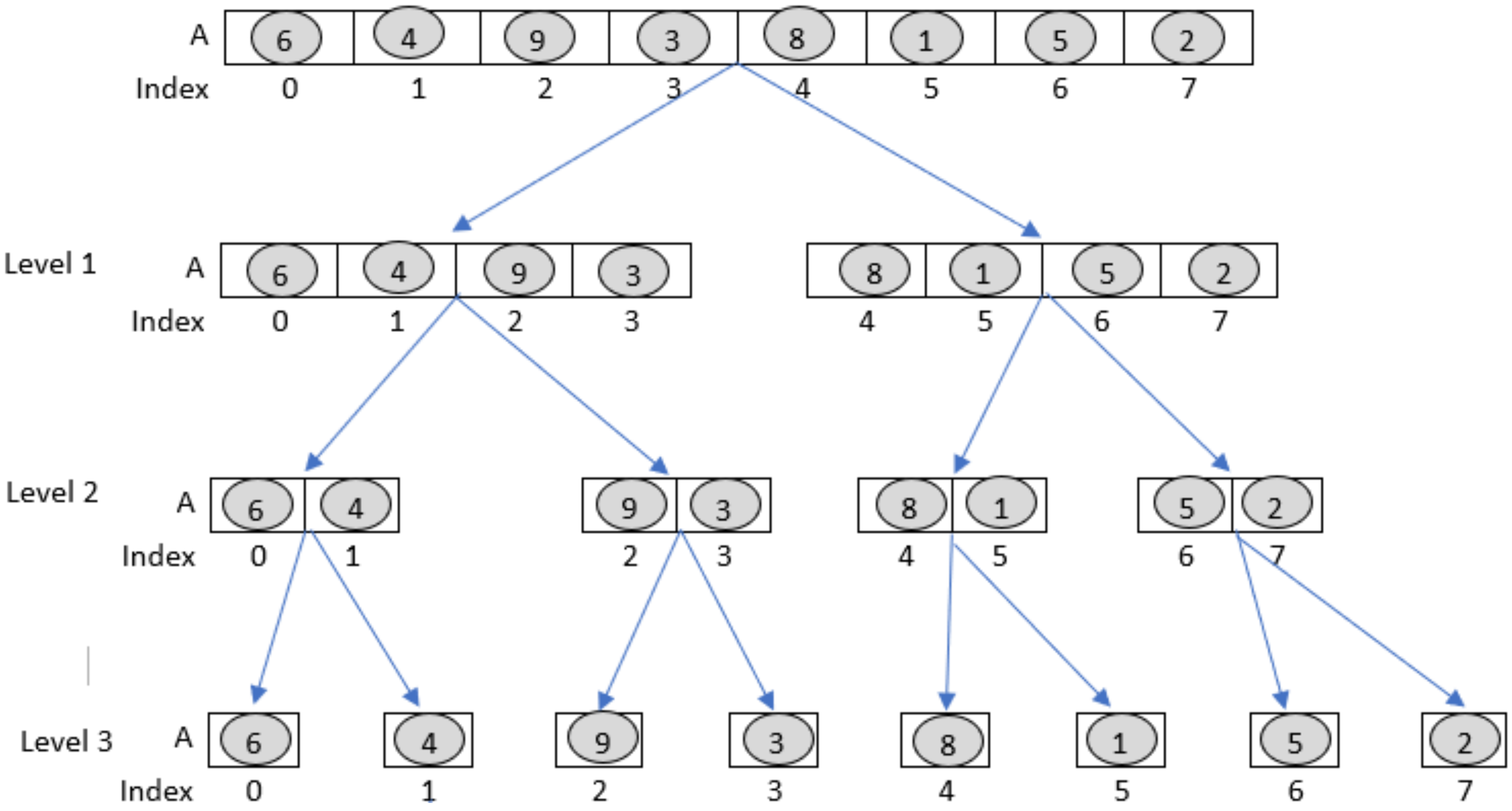

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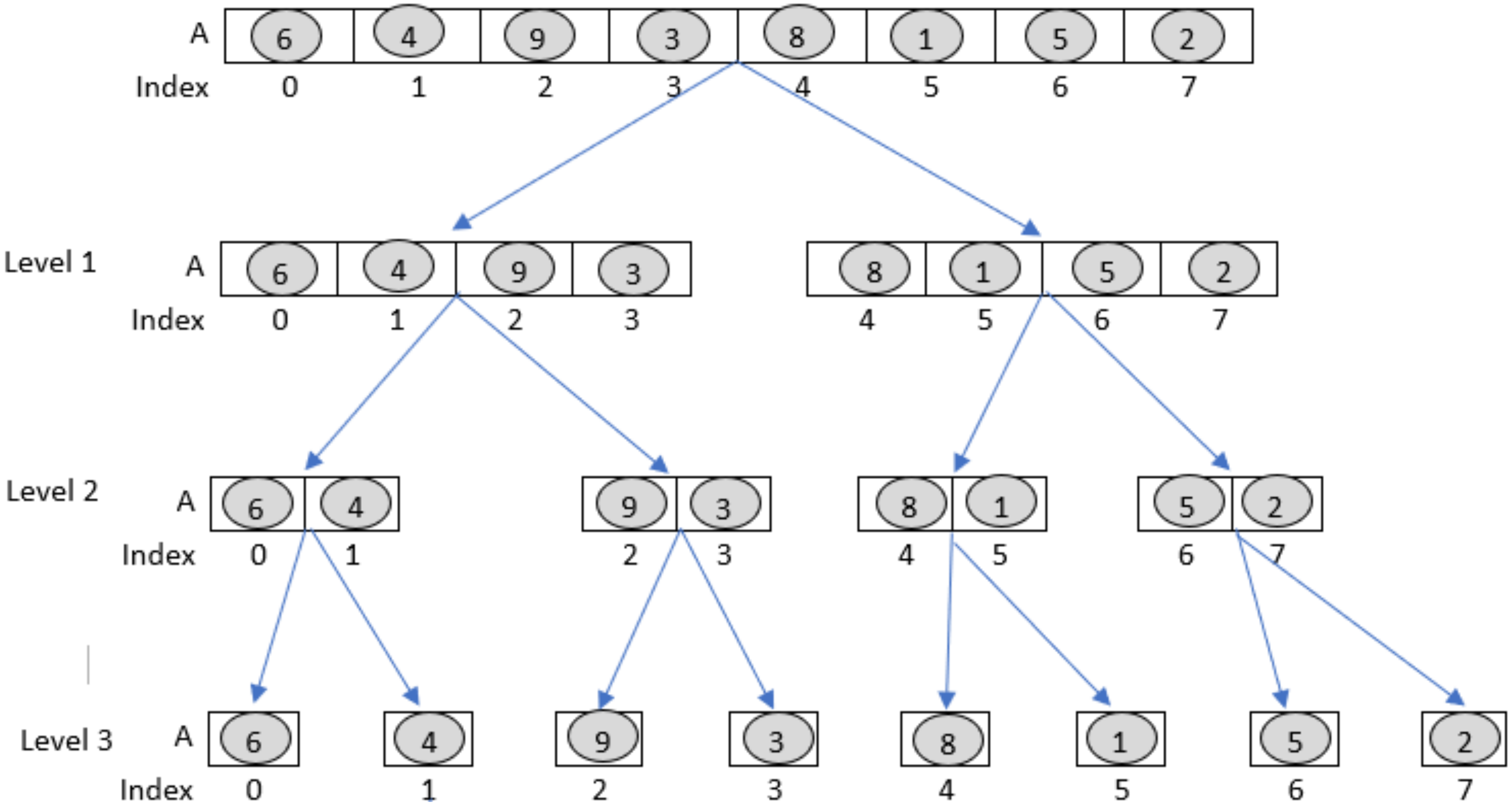

..................$\theta(2^0)$

..................$\theta(2^1)$

..................$\theta(2^2)$

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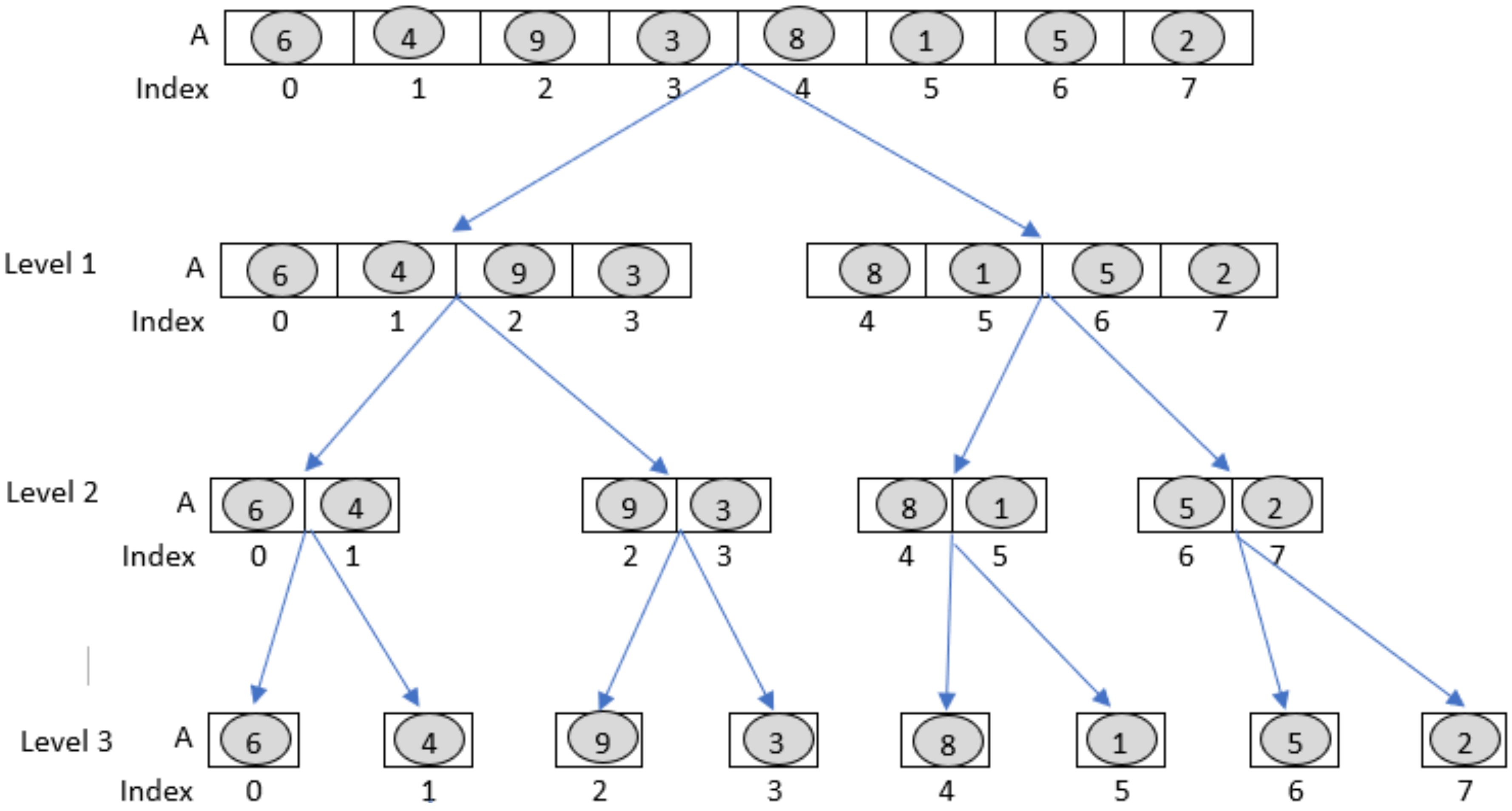

………………$\theta(2^0)$

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Total: $2^0 + 2^1 + 2^2 + \cdots + 2^{l-1} = \theta(n)$

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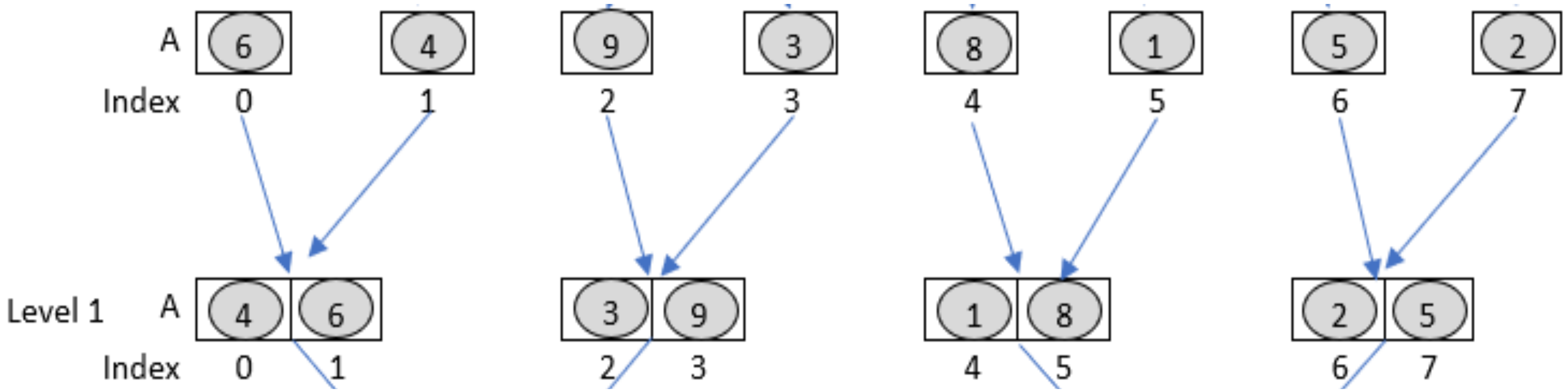

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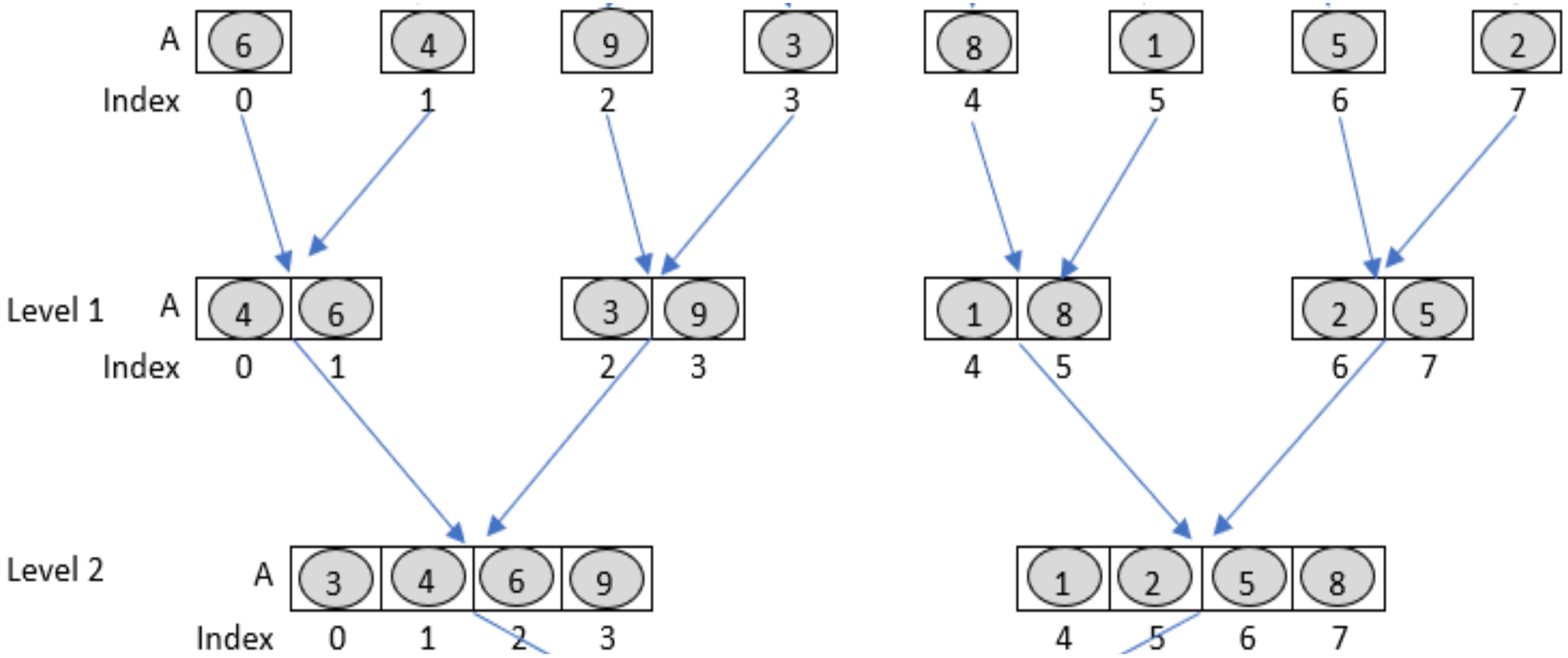

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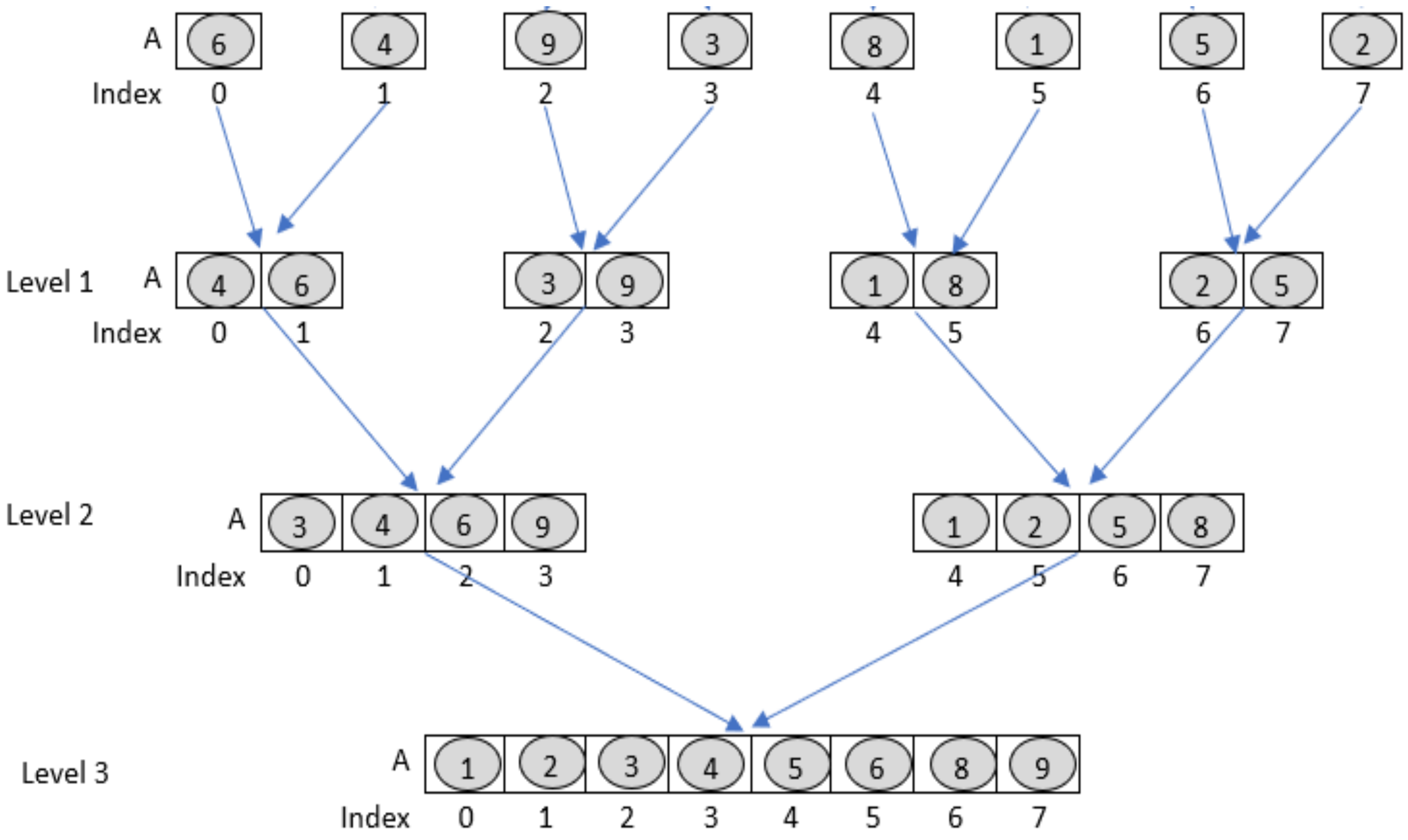

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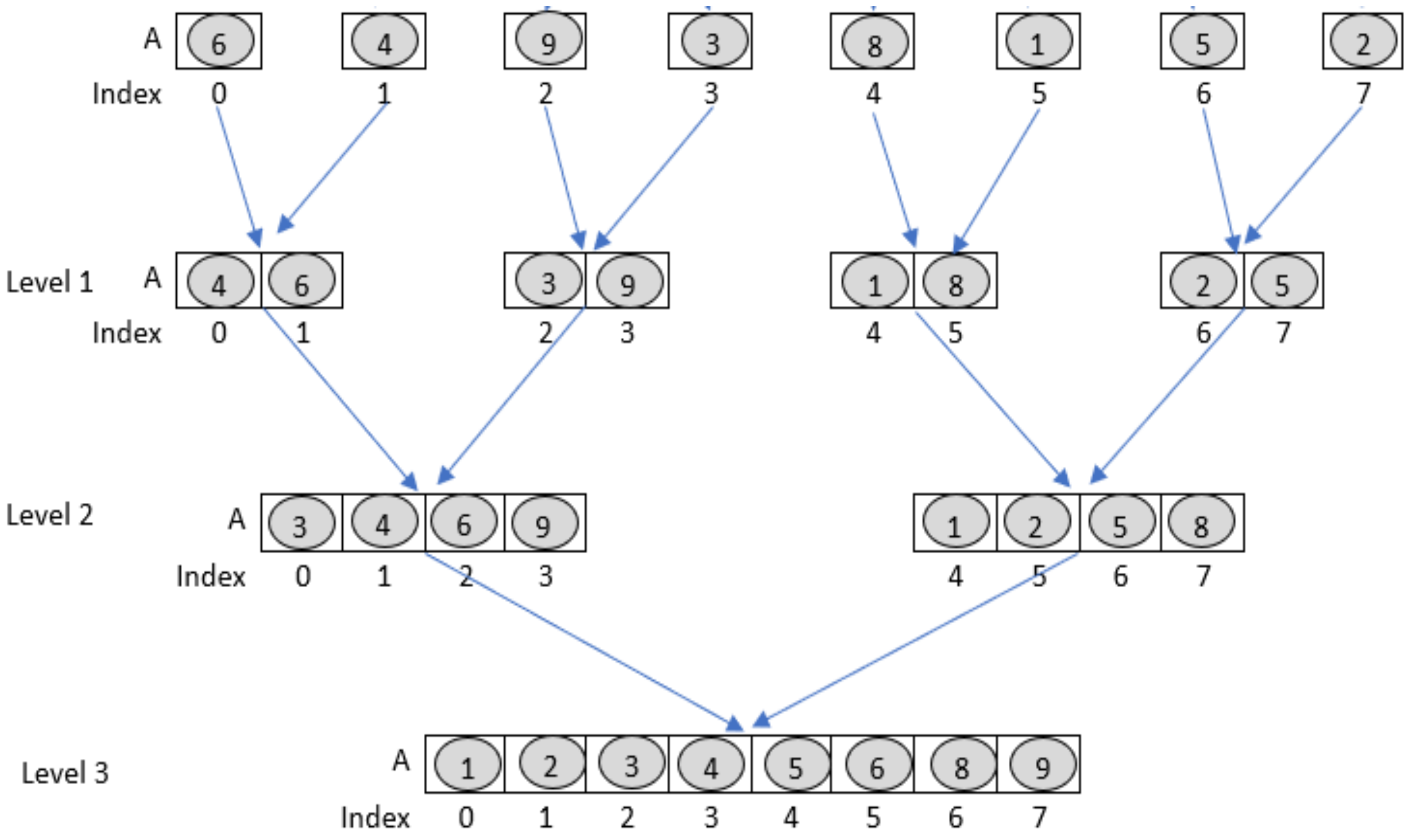

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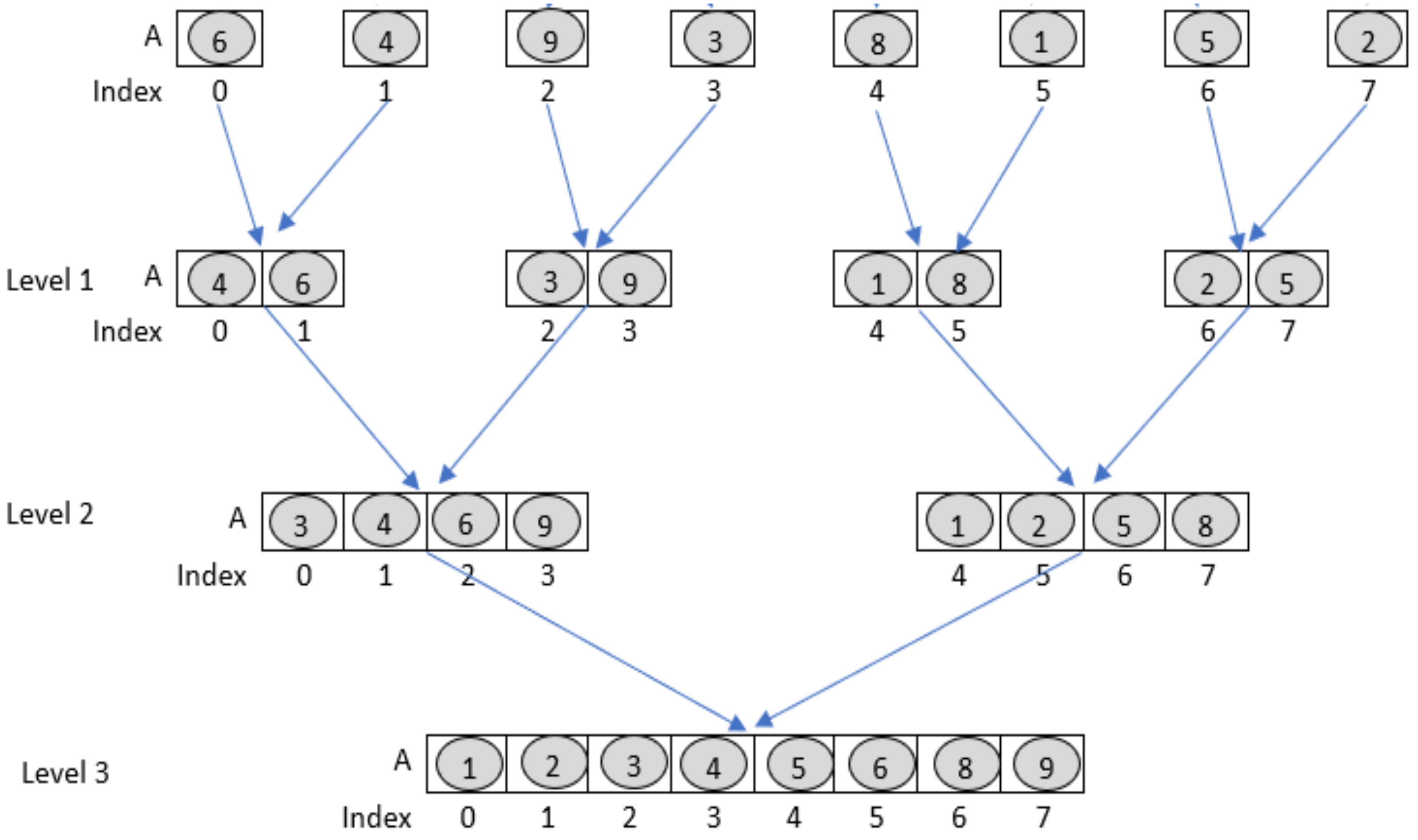

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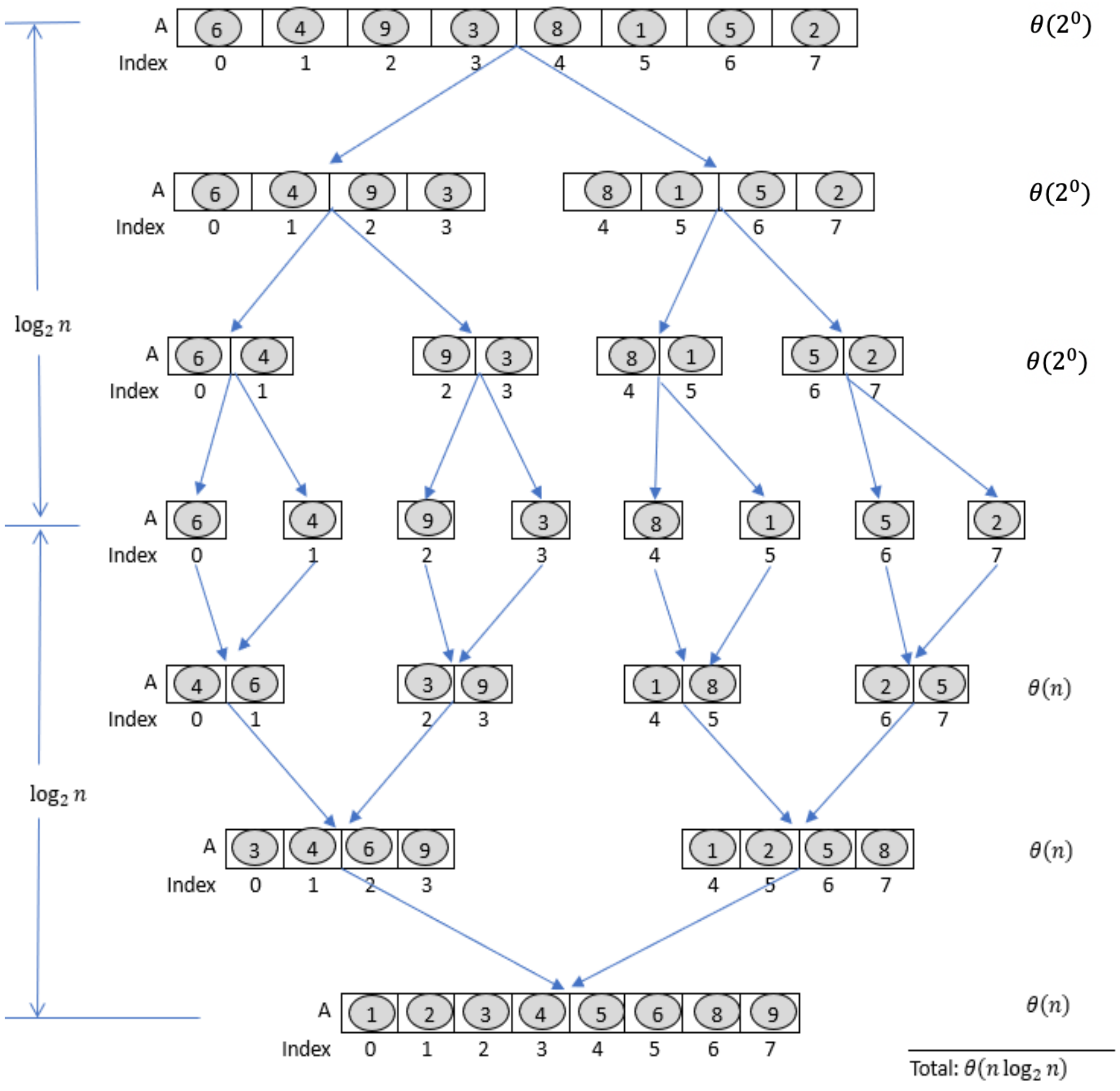
Total: $\theta(n \log_2 n)$

Merge Sort



A
Index
6 4 9 3 8 1 5 2
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$\theta(2^0)$
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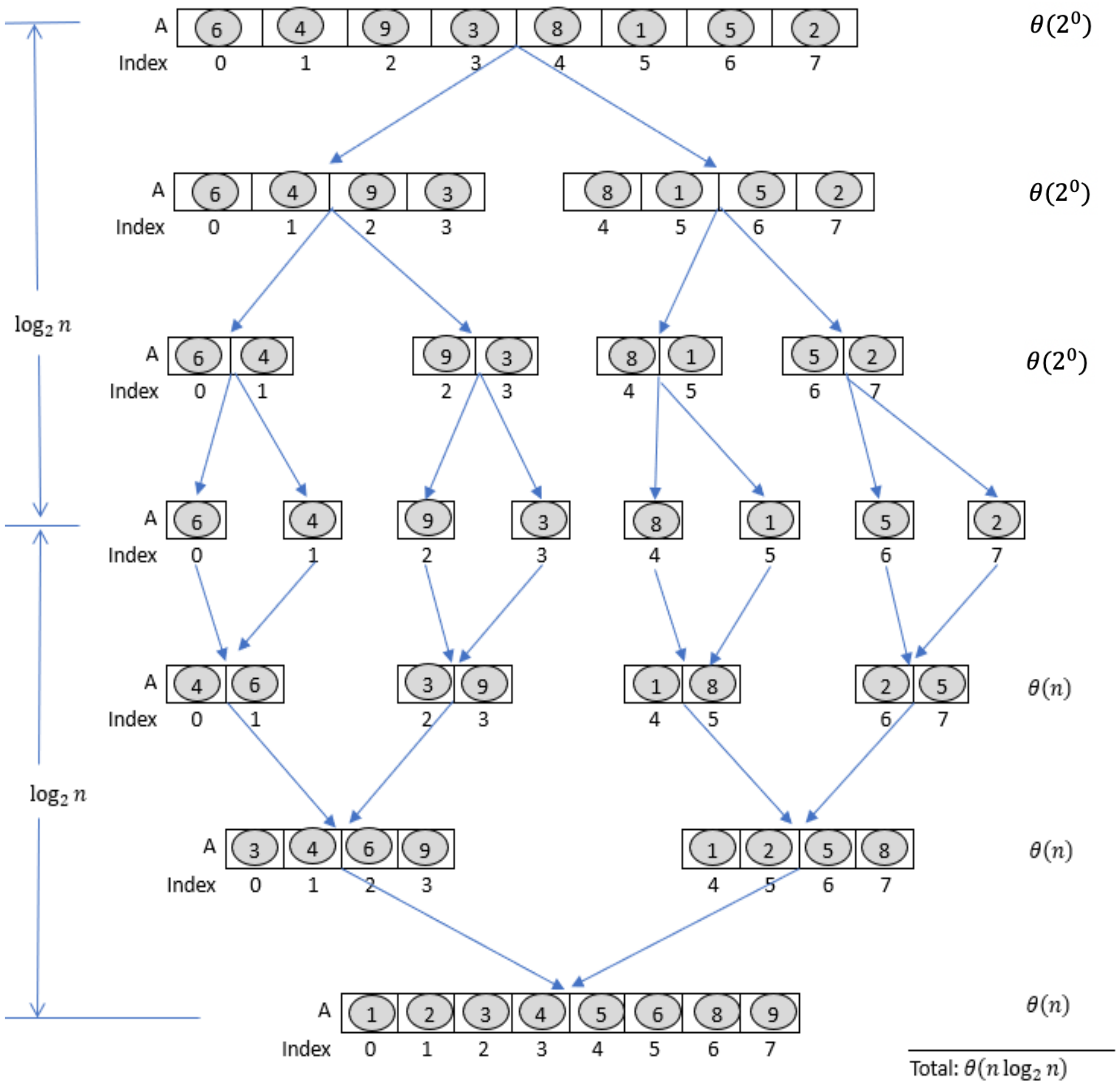

Time Complexity:

$$T(n) = 2T\left(\frac{n}{2}\right) + n, T(1) = 1$$

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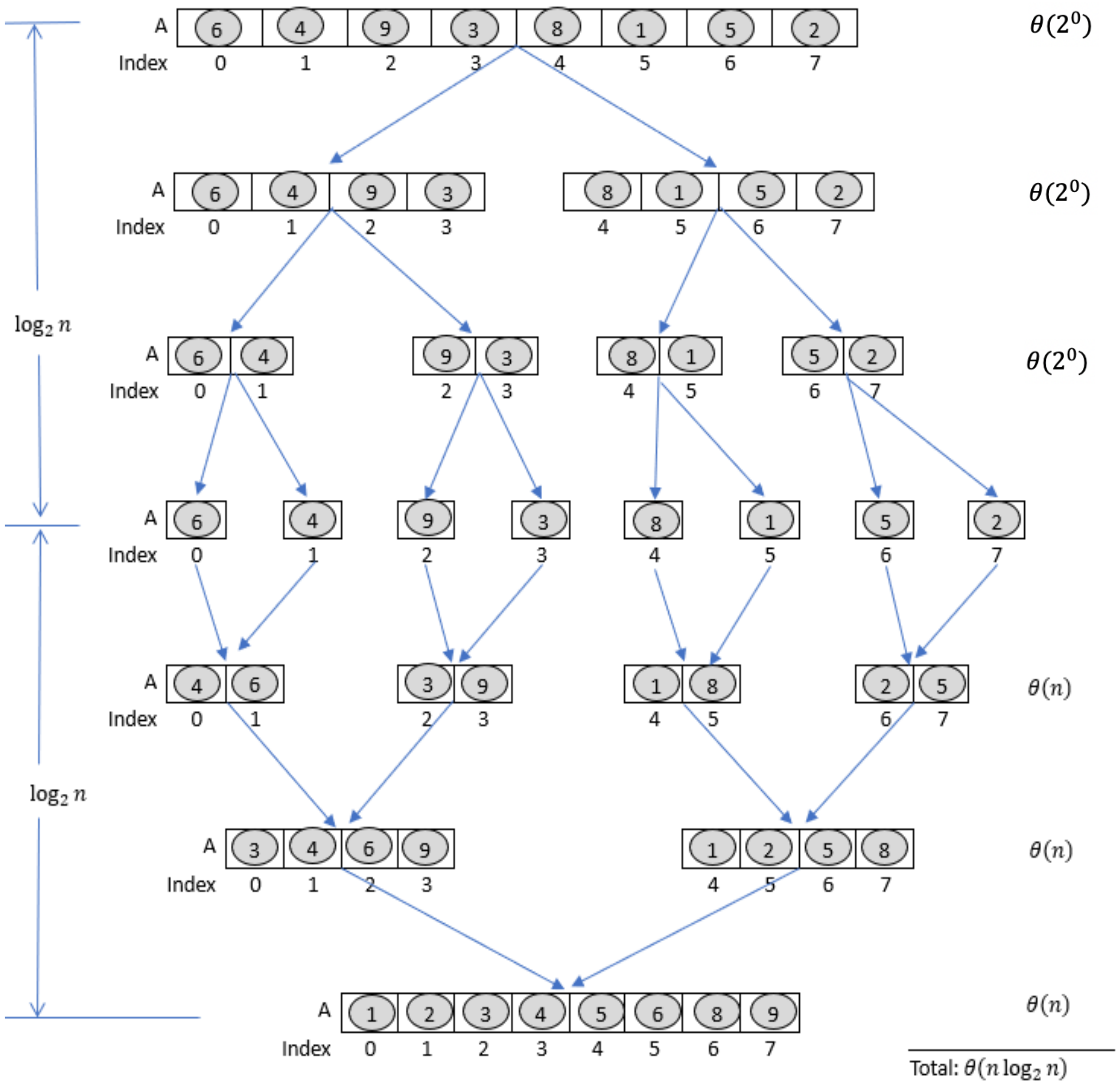
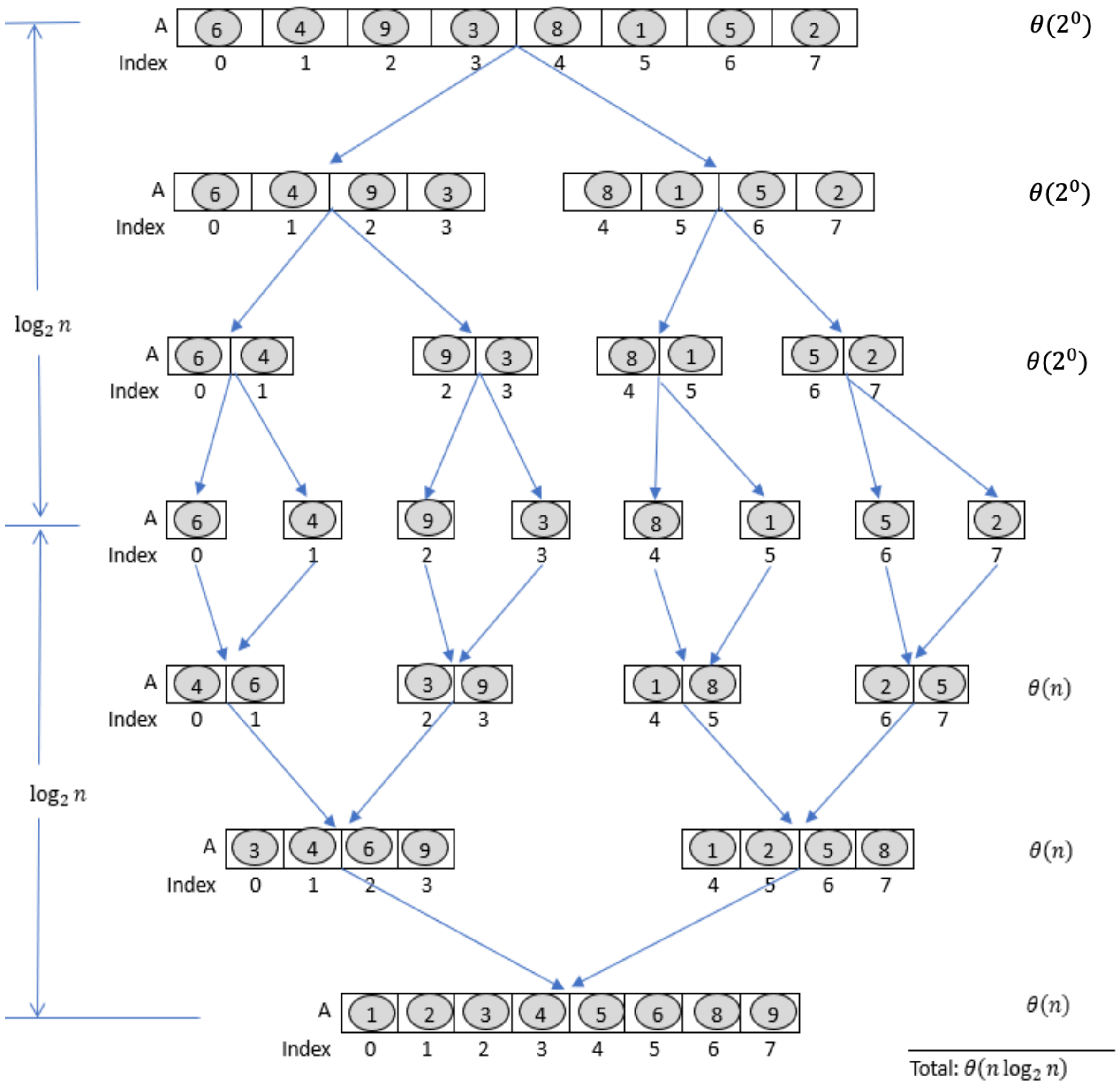


Time Complexity:

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$\Rightarrow T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2\frac{n}{2} + n$$

Merge Sort



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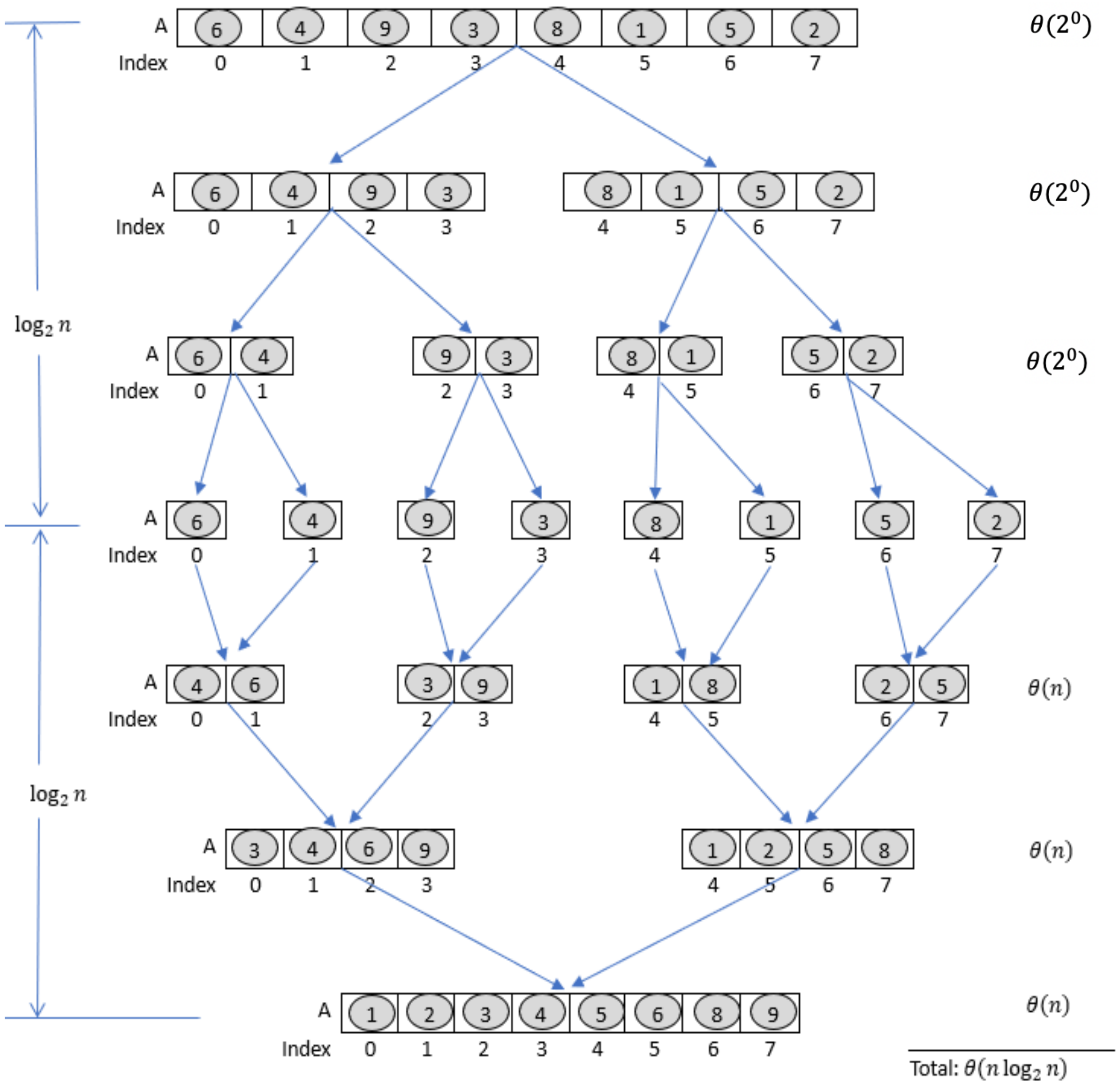

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.....

$$\Rightarrow T(n) = 2^l\, T\left(\frac{n}{2^l}\right) + n + n + \cdots + n + n$$

$$\Rightarrow T(n) = n + n + n + \cdots + n = \theta(n \log_2 n)$$

Merge Sort - Algorithm

```
void mergeSort(int A[], int first, int last){
  if(first < last){
        mergeSort(A, first, ⌈(first+last)/2⌉ − 1);
        mergeSort(A, ⌈(first+last)/2⌉, last);
        mergeSortedArrays(A, first,⌈(first+last)/2⌉, last)
  }
}
```

Merge Sort - Algorithm

```
void mergeSortedArrays(int A[], int first, int mid, int last){
    int i, j, k;
    int sl = mid - first-1;
    int sr= last - mid;
    int L[sl], R[sr];
    for (i = 0; i < sl; i++) L[i] = A[first + i];
    for (i = 0; i < sr; i++) R[i] = A[mid + i];
    i = j = 0;
    k = first;
    while (i < sl && j < sr){
        if (L[i] <= R[j]){
            A[k] = L[i];
            i++;
        }
        else{
            A[k] = R[j];
            j++;
        }
        k++;
    }
     while (i < sl){ // If R remains
        A[k] = L[i];
        i++;
        k++;
    }
    while (j < n2){ // If L remains
        A[k] = R[j];
        j++;
        k++;
    }
}
```

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Merge Sort – Why External Memory Sort

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- **Problem:** Sorting
- **Class:** Comparison based sorting

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For example, if n is 3, and the possible elements are $\{a, b, c\}$, then its possible permutations are $(a, b, c), (a, c, b), (c, a, b), (b, c, a), (b, a, c)$, and (c, b, a).

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If the given set is $\{a = 5, b = 1, c = 2\}$, then the sorted permutation is (b, c, a).

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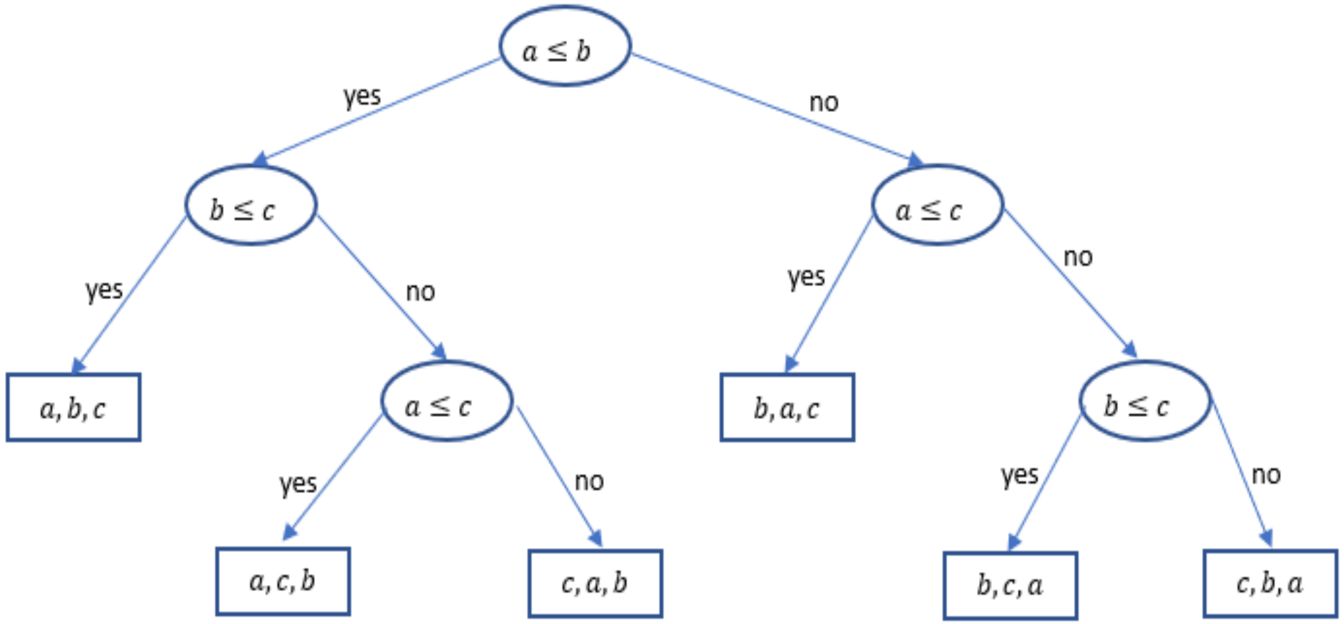

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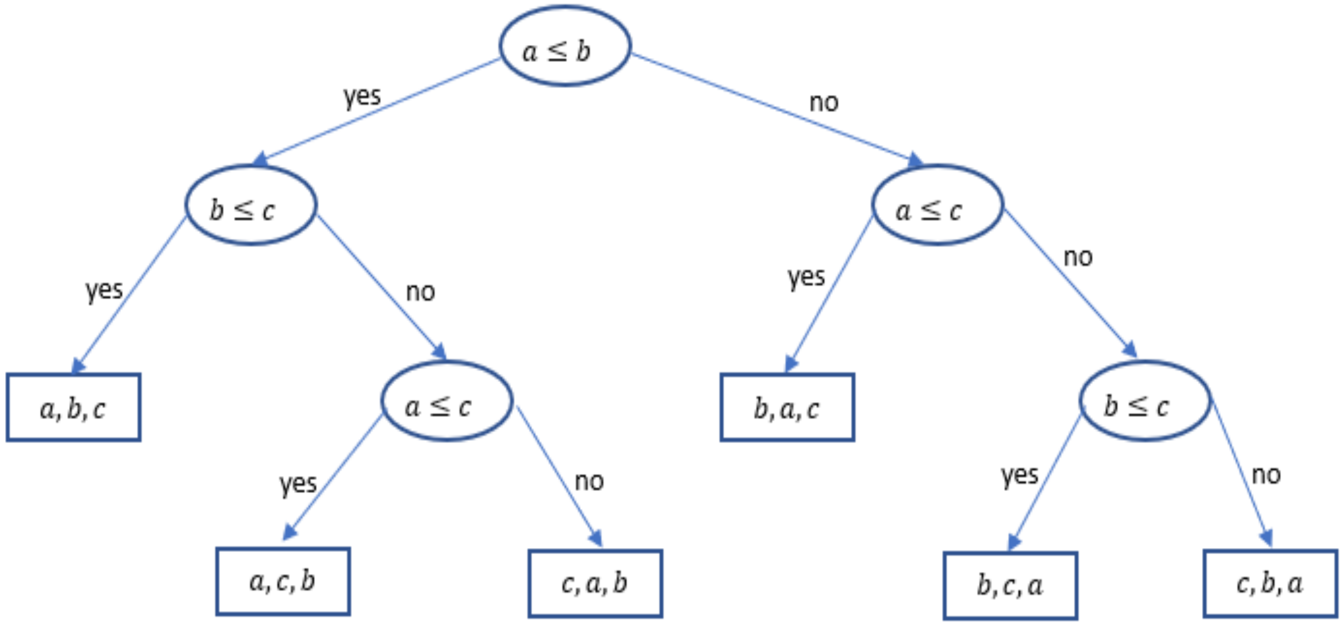

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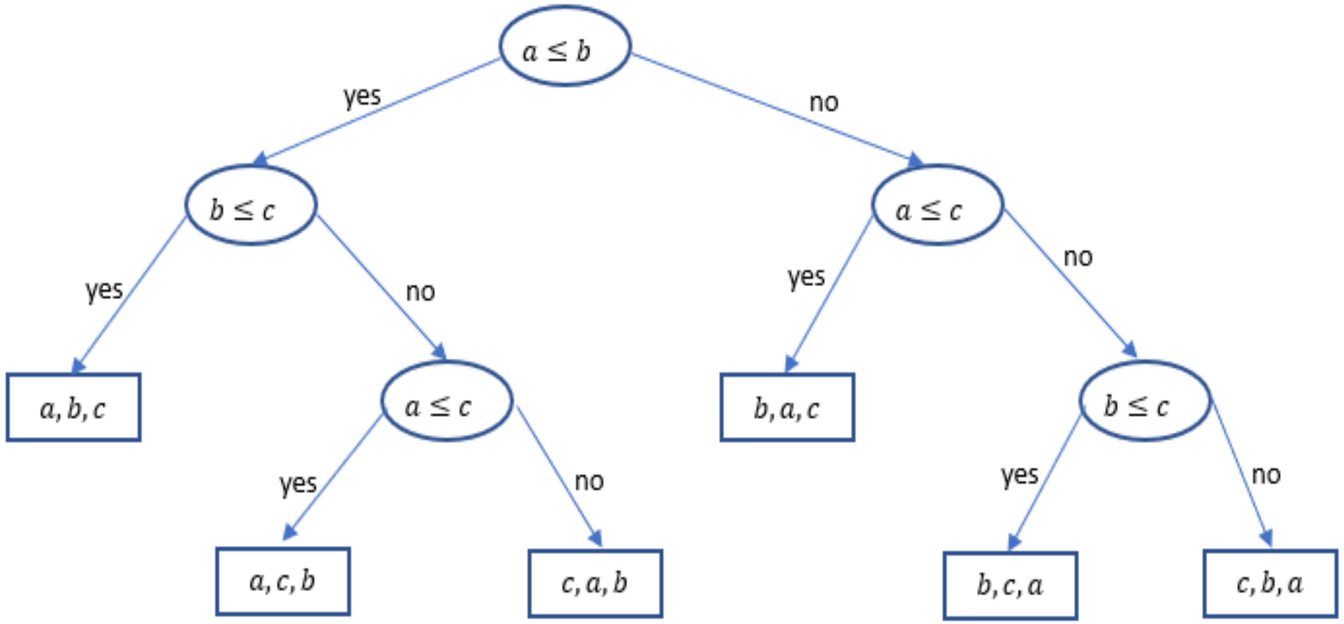

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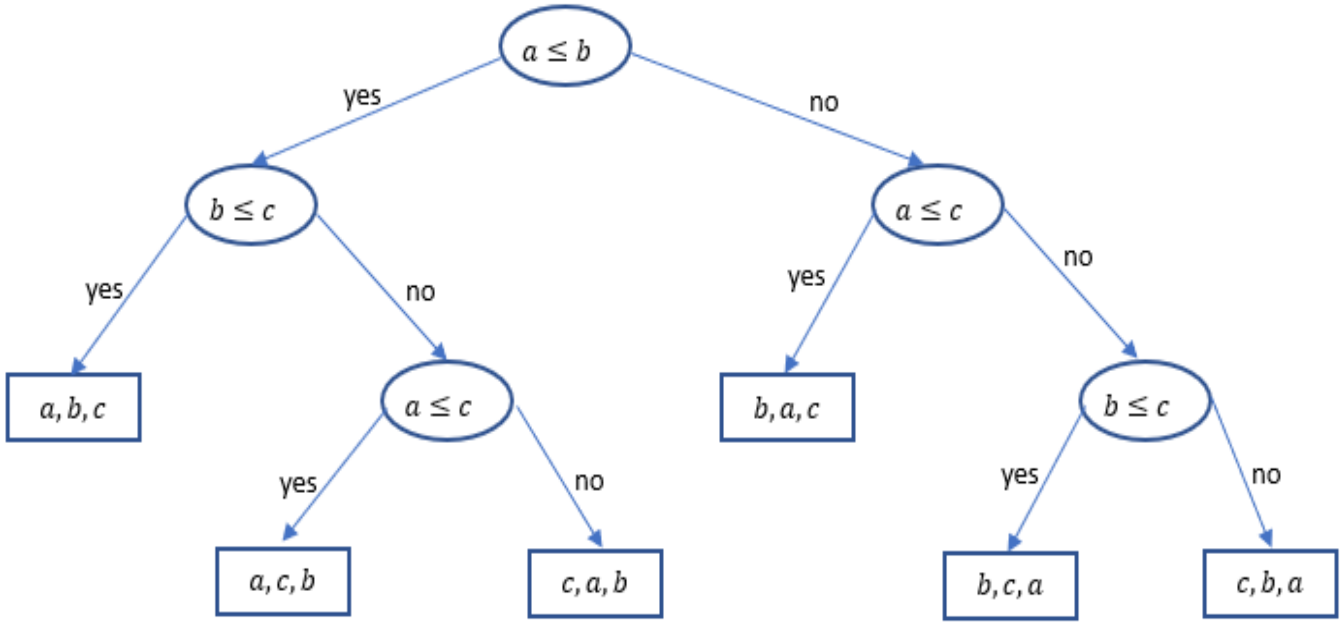

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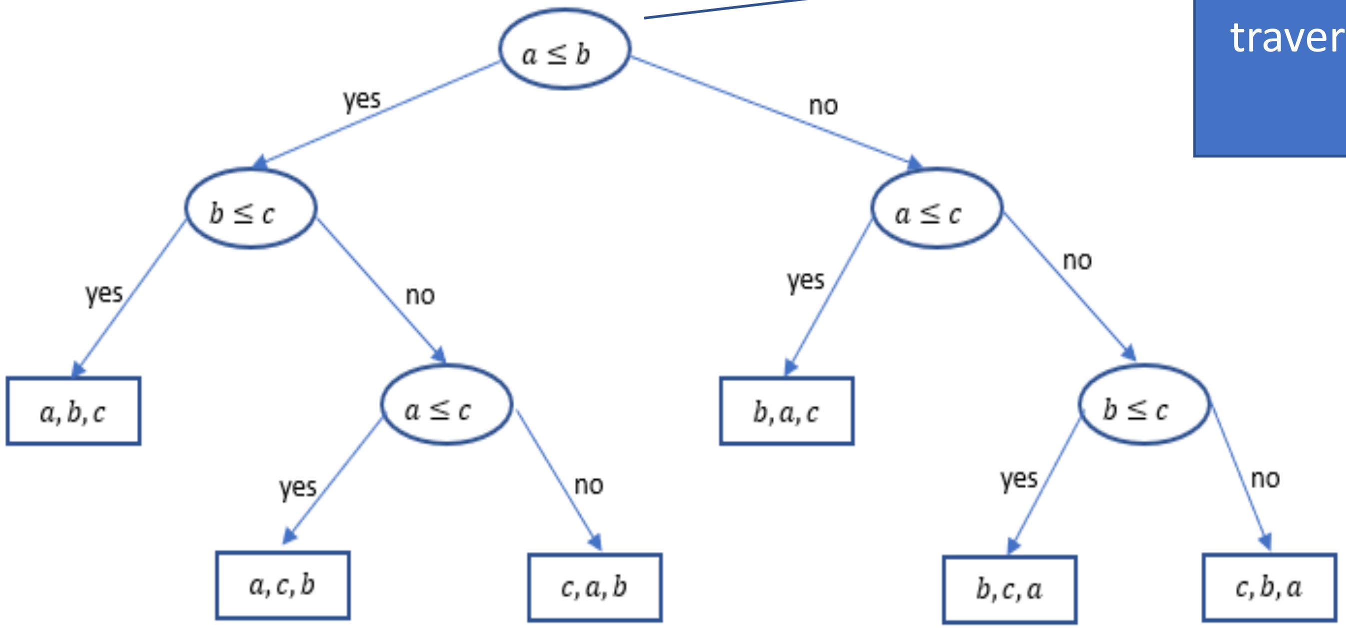
Searching for a permutation is to traverse the tree from the toot to a leaf node



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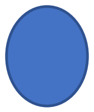

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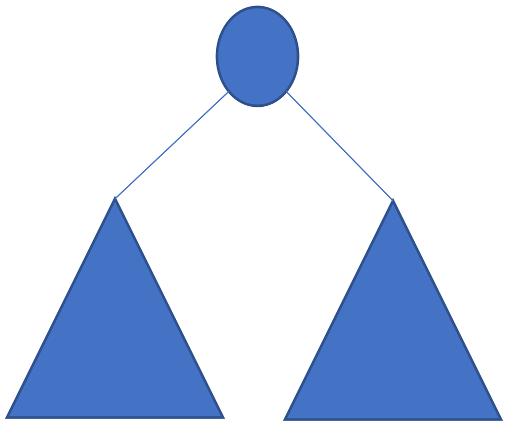

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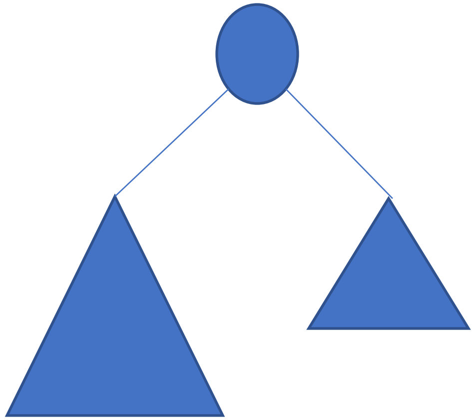

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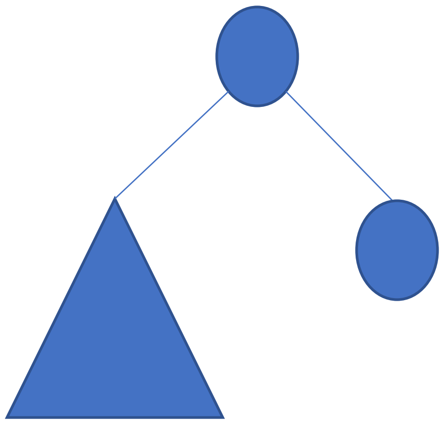

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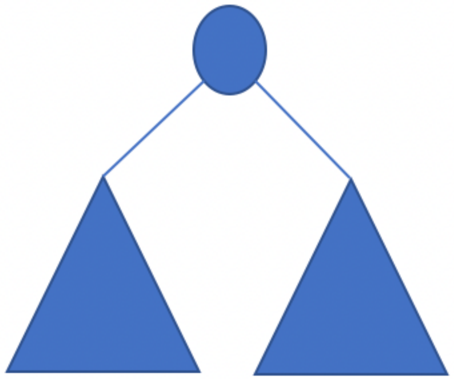

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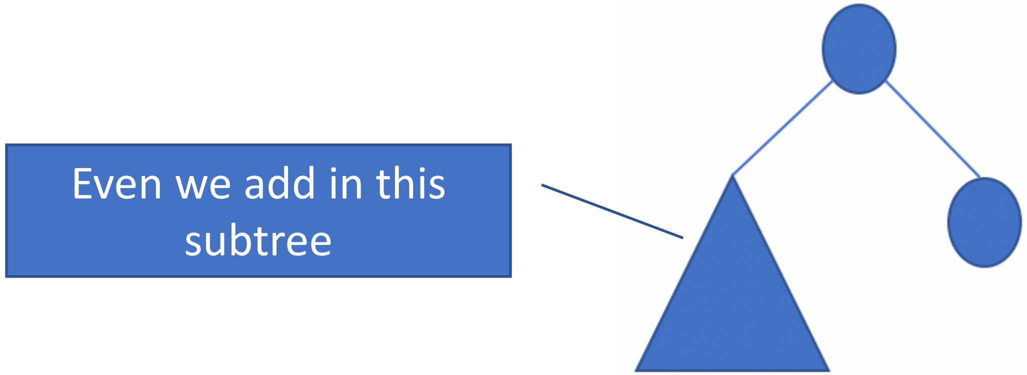

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If we add one more leaf node, in one of the subtrees, the above condition will still hold for the subtrees as $\frac{n}{2} + 1 < n$.

As the height of a binary tree is the height of the tallest child subtree plus one, the height of the binary tree with $n + 1$ leaf nodes is at least $\log_2 \frac{n+1}{2} + 1 = \log_2(n + 1)$.

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To define lower bound, **we need to prove that the *expected height of any decision tree* for sorting n element is at least $log_2 n!$.**

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Then D(T) can be defined recursively as below.

$$D(T) = D(T_i) + D(T_{n!-i}) + n!$$

The $n!$ in the above expression is additional depth of 1 from root to the roots of left and right subtree for all the $n!$ leaf nodes.

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Let us assume that the condition holds for $n - 1$. For n elements, we can define $M(T)$ as

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$$= n! \log_2 \frac{n!}{2} + n! = n! (\log_2 n! - 1) + n! = n! \log_2 n!.$$

So, the expected height of the decision tree which constitutes M(T) is $\frac{M(T)}{n!} \geq \log_2 n!$.

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$$\log_2 n! \geq \log_2 \left(n(n-1)(n-2) \ldots \left\lceil\frac{n}{2}\right\rceil\right) \geq \log_2 \left(\frac{n}{2}\right)^{\frac{n}{2}}$$

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Lower Bound Theory says, any comparison based sorting algorithm will take at least order of $(n \log_2 n)$ time complexity

Non-Comparison based Sorting Algorithms

Counting Sort

Counting Sort

- It is a non-comparison based sorting algorithm.
- It works in asymptotic linear time complexity.
- It works only for *real numbers*.

Counting Sort

Counting sort needs an auxiliary array **Count** which stores the count of each element.

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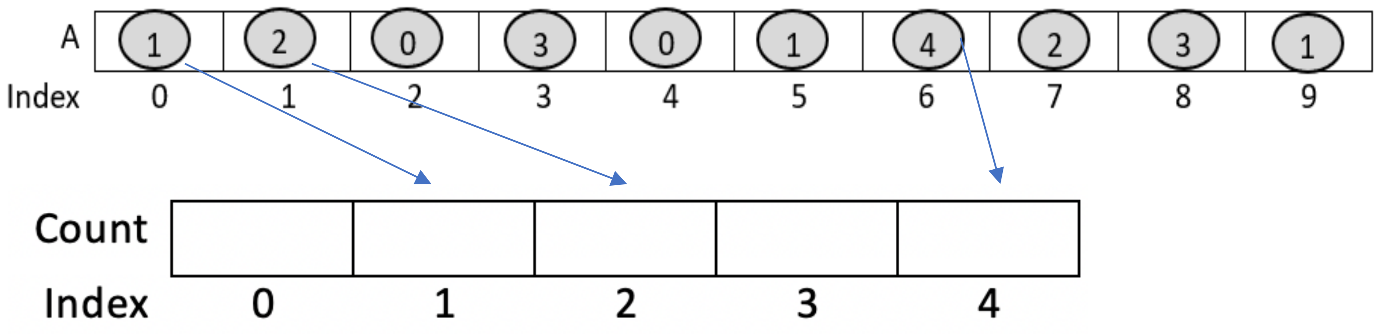

What will count array store?

It stores the count of each element.

The i^{th} **index in Count** will store the **frequency count of i as element in A.**

A	1	2	0	3	0	1	4	2	3	1
Index	0	1	2	3	4	5	6	7	8	9

Count	2	3	2	2	1
Index	0	1	2	3	4

How do you define the Count Array?

Scenario -1

The given array has only positive values.

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A	1	2	0	3	0	1	4	2	3	1
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Count					
Index	0	1	2	3	4

Size of Count array = max(A) + 1

Scenario - 1

The minimum is larger than 0, and elements are not uniformly distributed over indexes

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A	3	10	3	7	10	3	10	5	7	3
Index	0	1	2	3	4	5	6	7	8	9

Count				4		1		2			3
Index	0	1	2	3	4	5	6	7	8	9	10

Scenario - 2

A	3	10	3	7	10	3	10	5	7	3
Index	0	1	2	3	4	5	6	7	8	9

Count	4		1		2			3
Index	0	1	2	3	4	5	6	7

Count[i] = i + min(A)

Scenario – 3: with negative values

A	-1	10	3	7	10	3	10	5	7	3
Index	0	1	2	3	4	5	6	7	8	9

Count	1				3		1		2			3
Index	-1	0	1	2	3	4	5	6	7	8	9	10

LB = min(A)

Count	1				3		1		2			3
Index	0	1	2	3	4	5	6	7	8	9	10	11

Count[i] = i + min(A)

How to use Count array to sort an Array?

A	1	2	0	3	0	1	4	2	3	1
Index	0	1	2	3	4	5	6	7	8	9

Count	2	3	2	2	1
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Counting Sort – An Approach

Count	2	3	2	2	1
Index	0	1	2	3	4

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Counting Sort – An Approach

Count	2	3	2	2	1
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Count	2	3	2	2	1
Index	0	1	2	3	4

A	0	0	1	1	1	2	2			
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Count	2	3	2	2	1
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A	0	0	1	1	1	2	2	3	3	4
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This approach is simple and easy, but it will not be stable.

What is Counting Sort? - Algorithm

A	1	2	0	3	0	1	4	2	3	1
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1. Given an array A, construct
   Count array

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3. For (i=UB; i>=LB; i--)

4.        pos = Count[ A[i] ]

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Count	2	3	2	2	1
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The counts in Count array are sequentially summed, so that Count[i] stores the cumulative sum from Count[0] to Count[i].

Count	2	5	7	9	10
Index	0	1	2	3	4

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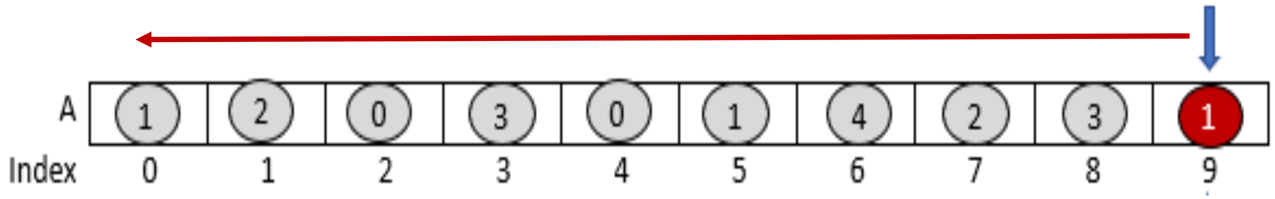


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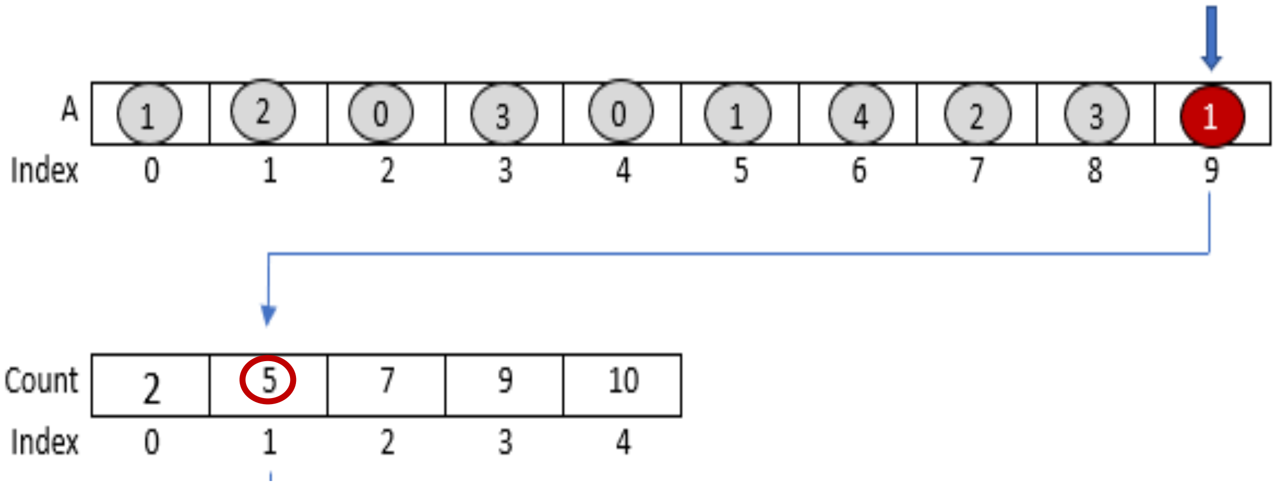

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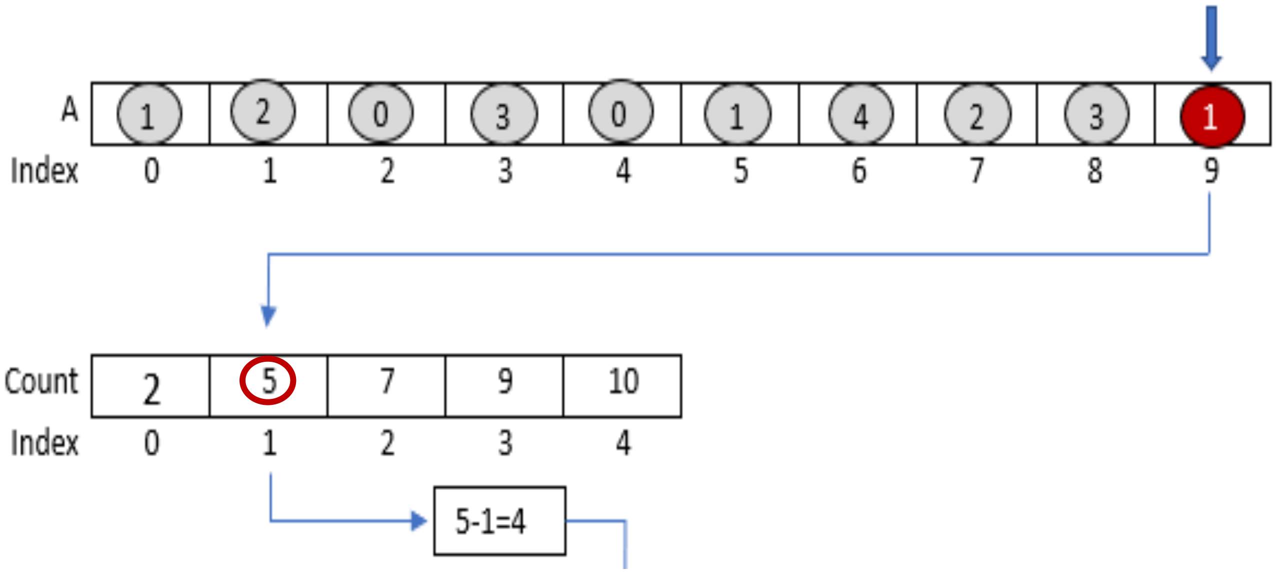

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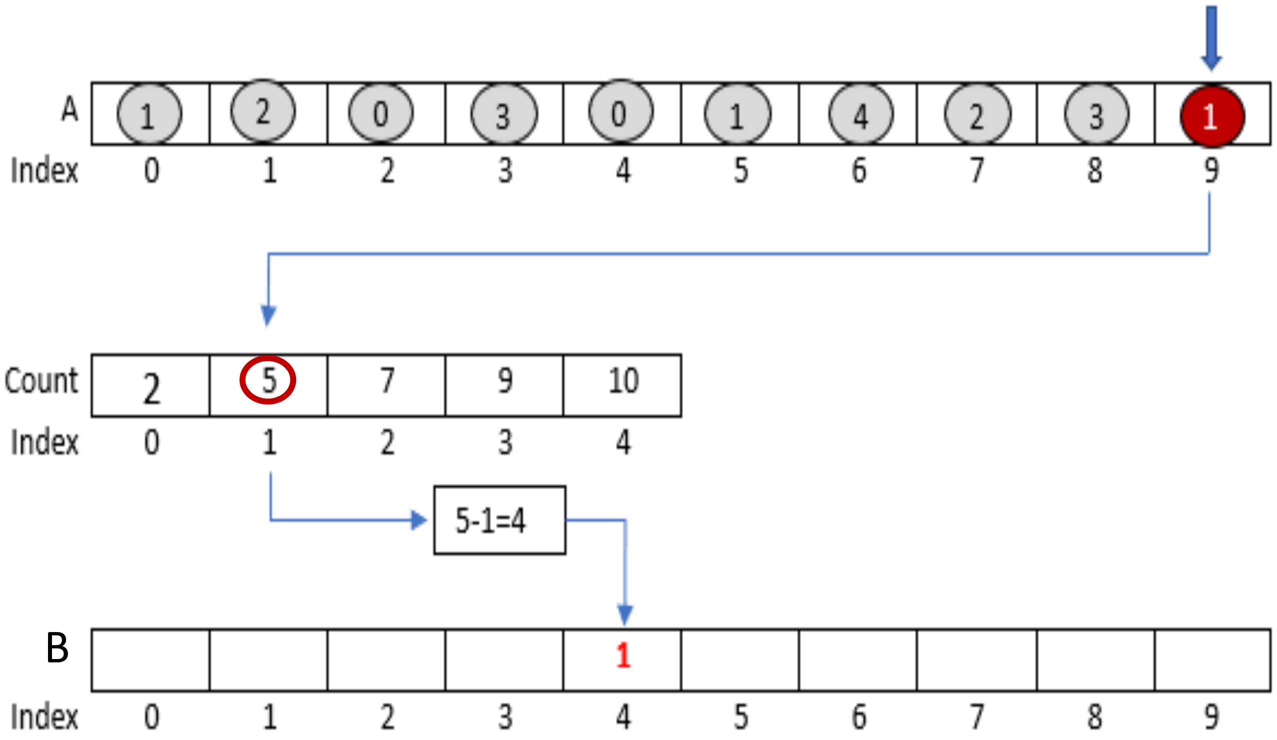

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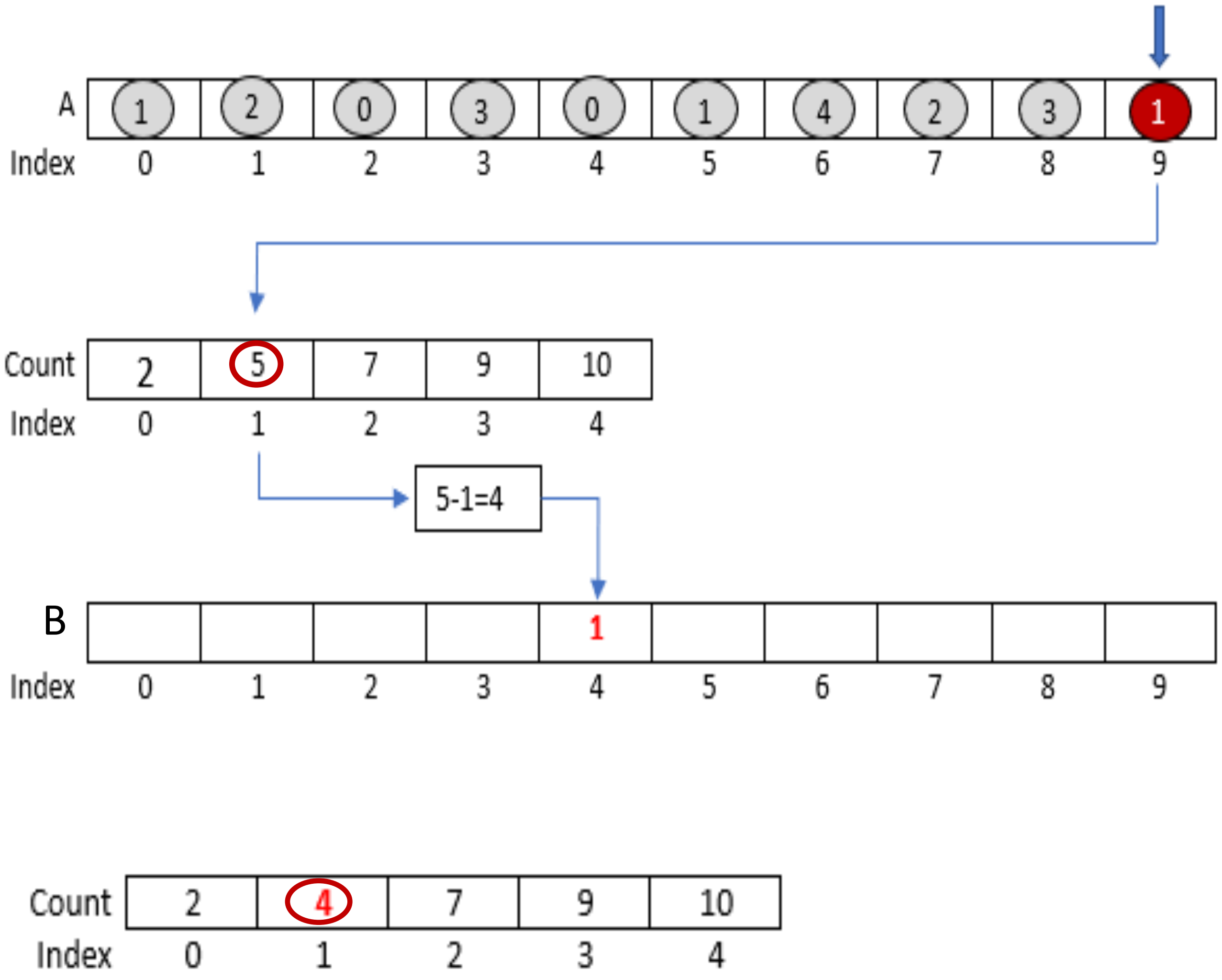


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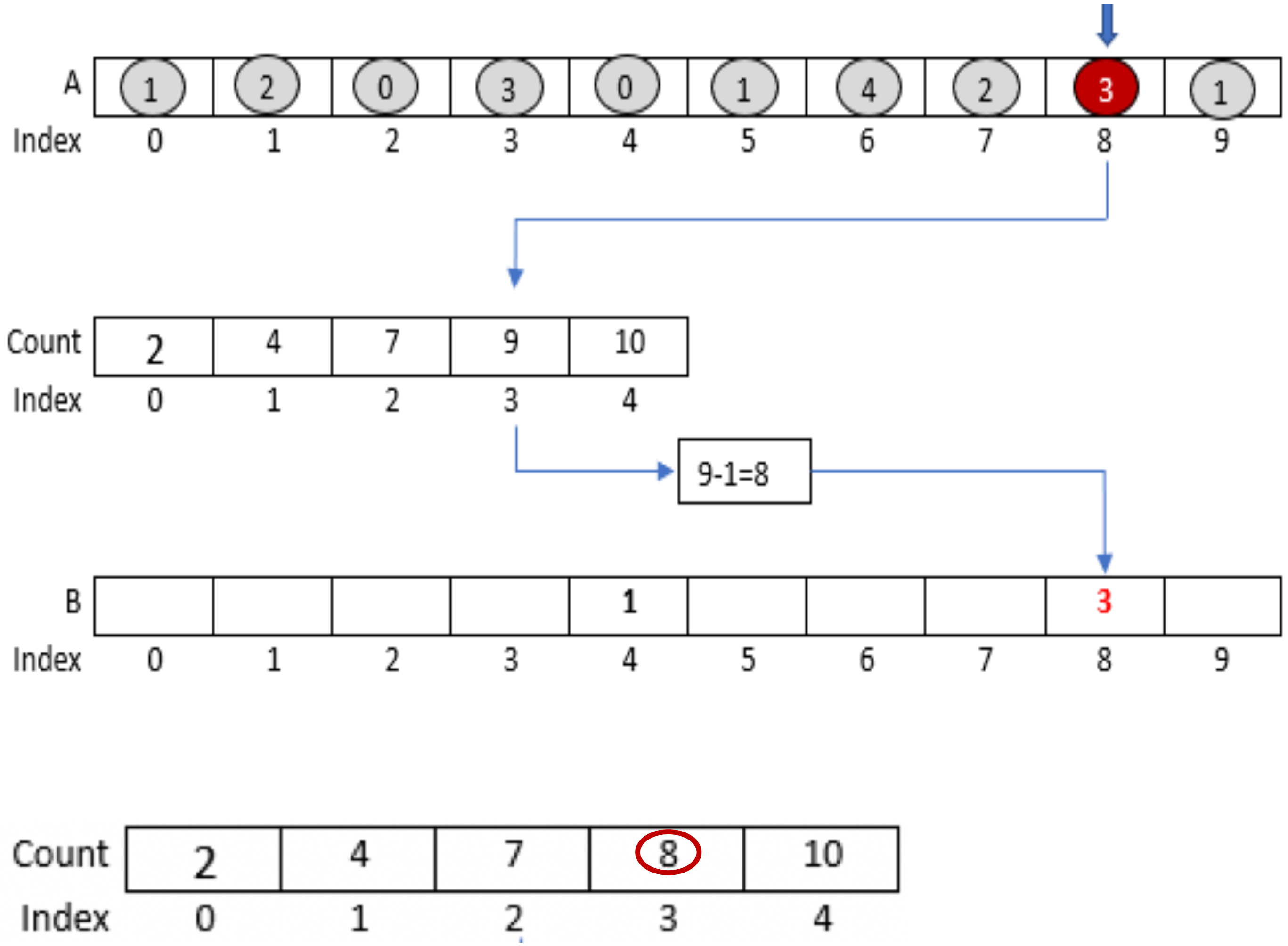

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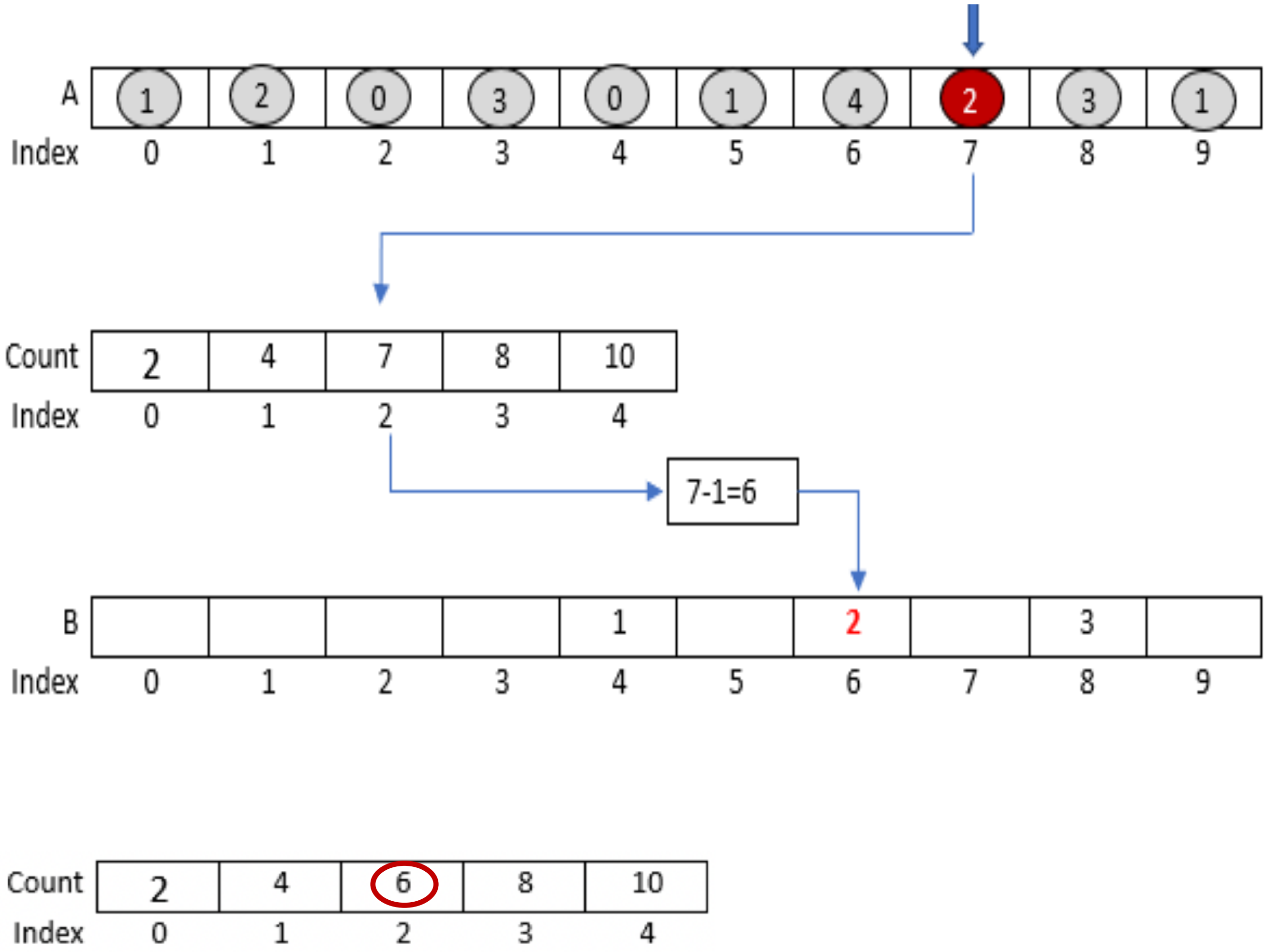

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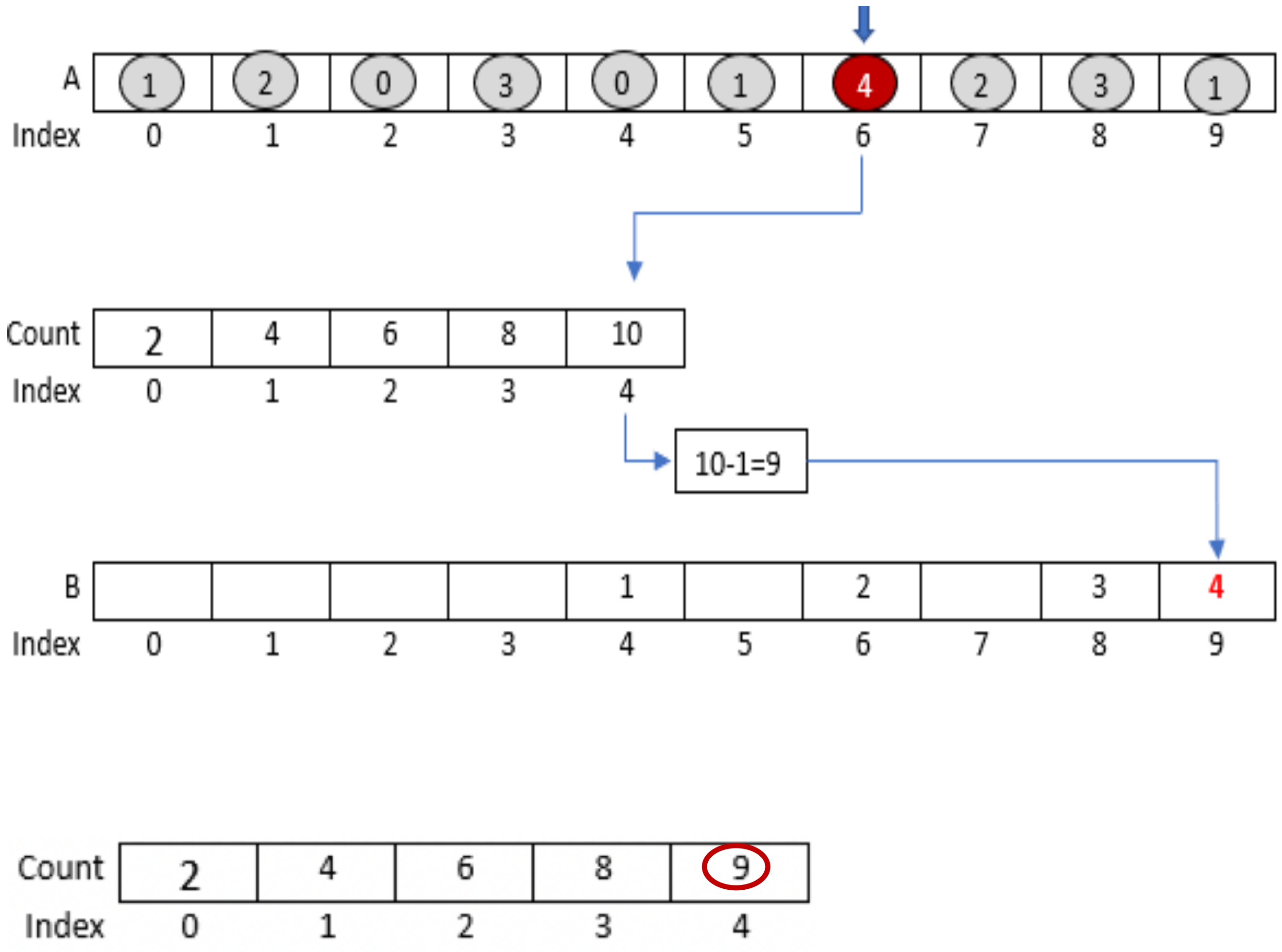

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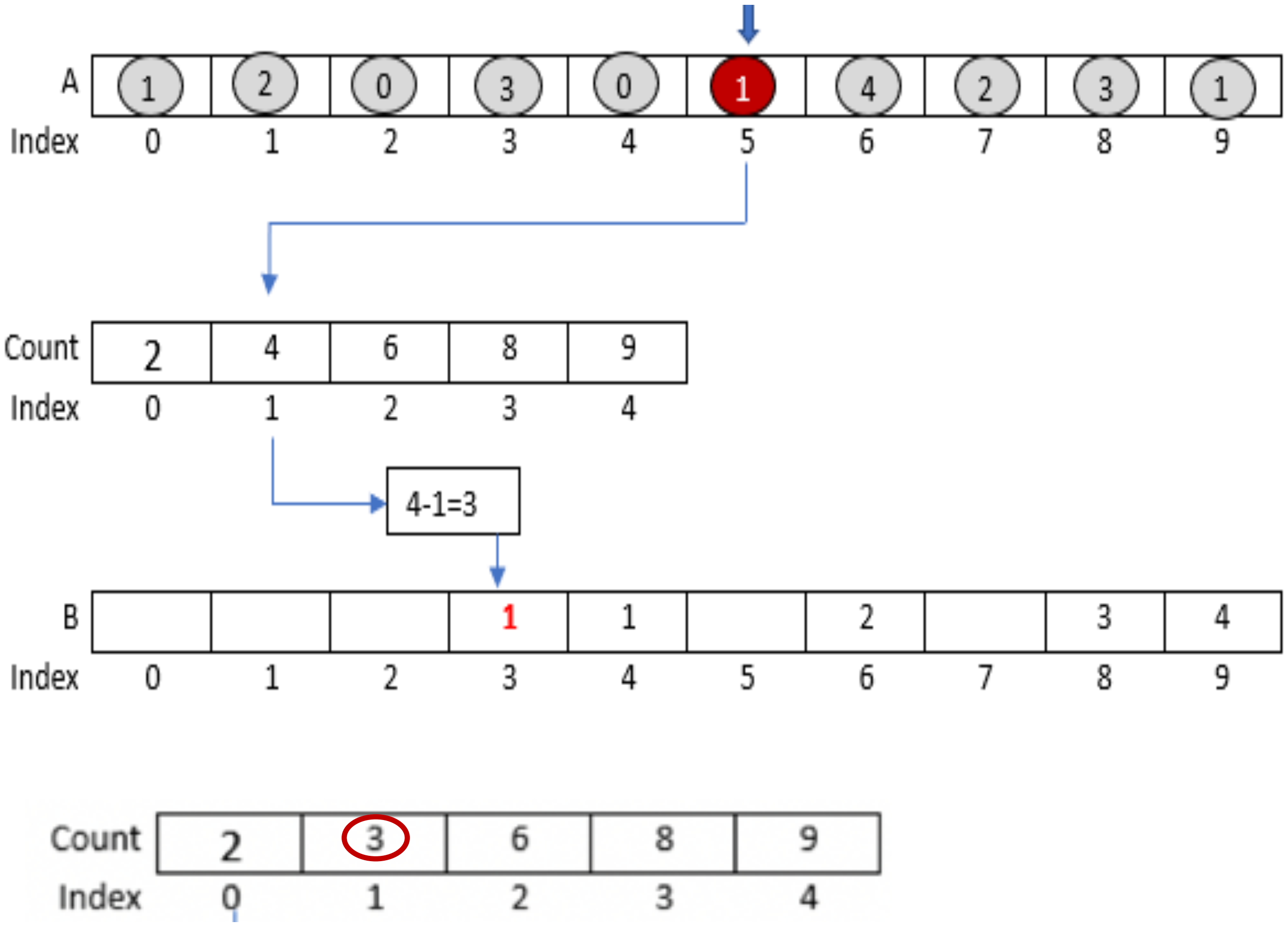

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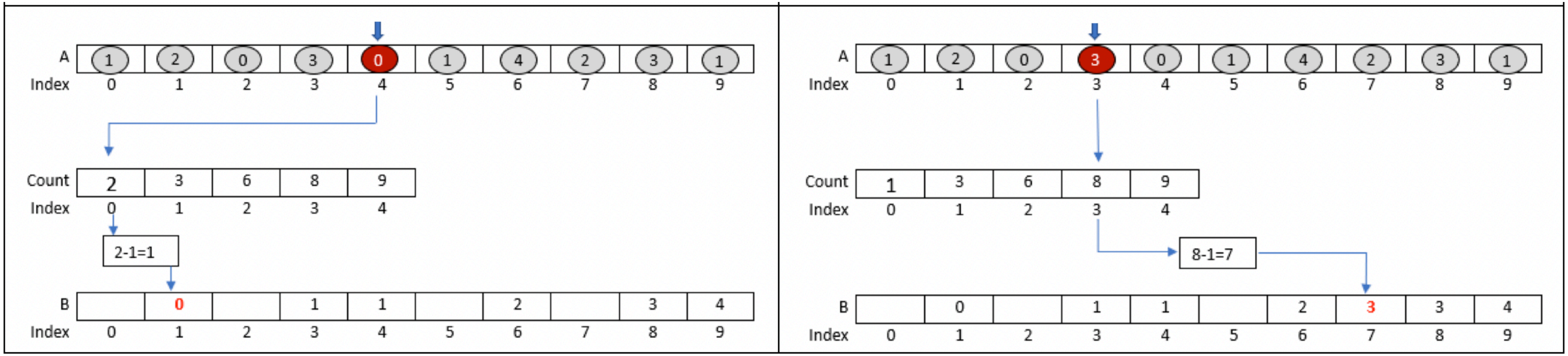

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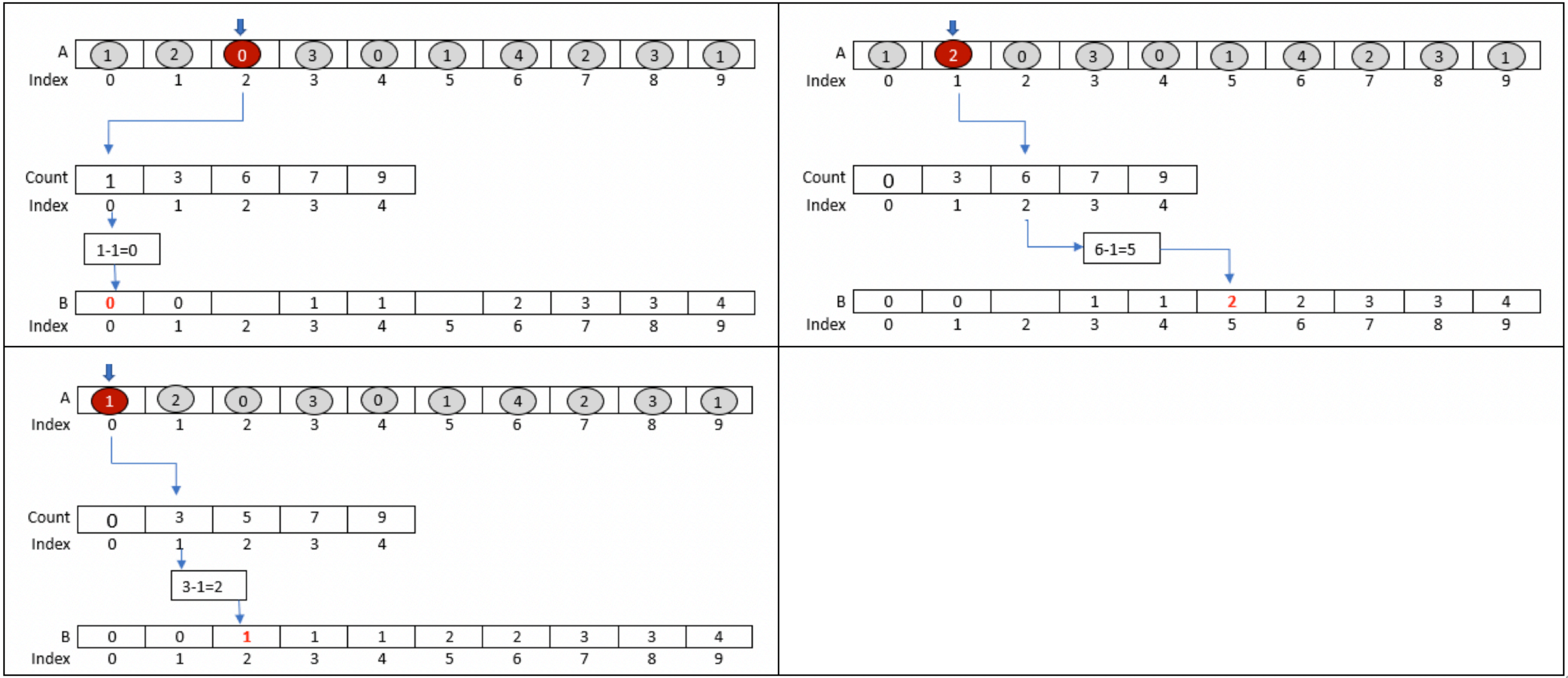

What is Counting Sort?

Once the sorted elements are obtained in B, copy them to A to obtain the final sorted array.

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Time Complexity

Input array is scanned thrice

- First for finding the maximum value,
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- First for finding the maximum value,
- Second for counting frequency, and
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Count array is scanned once for estimating cumulative values.

Time complexity is $\theta(n+k)$, where n is the size of the input array and k is the size of Count array.

Radix Sort

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It is also a non-comparable sorting algorithm, where sorting is done by the index of the data elements.

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$312, 20, 87, 881, 402, 7, 100, 243, 68, 524$

It processes the data elements index by index from the least significant index to the most significant index.

Radix Sort

$312, 20, 87, 881, 402, 7, 100, 243, 68, 524$

Data		
3	1	2
	2	0
	8	7
8	8	1
4	0	2
		7
1	0	0
2	4	3
	6	8
5	2	4

Radix Sort

$312, 20, 87, 881, 402, 7, 100, 243, 68, 524$

Data		
3	1	2
	2	0
	8	7
8	8	1
4	0	2
		7
1	0	0
2	4	3
	6	8
5	2	4

Sort by index 0

Place in order of occurrence

Index 0		
	2	0
1	0	0
8	8	1
3	1	2
4	0	2
2	4	3
5	2	4
	7	7
		7
	6	8

Radix Sort

$312, 20, 87, 881, 402, 7, 100, 243, 68, 524$

Data

3	1	2
	2	0
	8	7
8	8	1
4	0	2
		7
1	0	0
2	4	3
	6	8
5	2	4

Sort by index 0

Place in order of occurrence

Index 0

	2	0
1	0	0
8	8	1
3	1	2
4	0	2
2	4	3
5	2	4
	7	7
		7
	6	8

Sort by index 1

Place in order of occurrence

Index 1

1	0	0
4	0	2
		7
3	1	2
	2	0
5	2	4
2	4	3
	6	8
	7	7
8	8	1

Radix Sort

$312, 20, 87, 881, 402, 7, 100, 243, 68, 524$

Data		
3	1	2
	2	0
	8	7
8	8	1
4	0	2
		7
1	0	0
2	4	3
	6	8
5	2	4

Sort by index 0

Place in order of occurrence

Index 0		
	2	0
1	0	0
8	8	1
3	1	2
4	0	2
2	4	3
5	2	4
	8	7
		7
	6	8

Sort by index 1

Place in order of occurrence

Index 1		
1	0	0
4	0	2
		7
3	1	2
	2	0
5	2	4
2	4	3
	6	8
	8	7
8	8	1

Sort by index 2

Place in order of occurrence

Index 2		
		7
	2	0
	6	8
	8	7
1	0	0
2	4	3
3	1	2
4	0	2
5	2	4
8	8	1

Radix Sort

The time complexity of radix sort is $O(dn)$ where n is the number of elements in the array, and d is the number of digits of the largest element.

It is because, sorting of the elements by an index position can be done in linear time i.e., $O(n)$ using a linear time sorting algorithm, say bucket sort.

We need to repeat the process for d times.

So, the time complexity is $O(dn)$.

Bucket Sort

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It uses a collection of **ordered buckets** which can hold data within **a defined range**.

The order and ranges of the bucket can be defined based on the elements to be sorted

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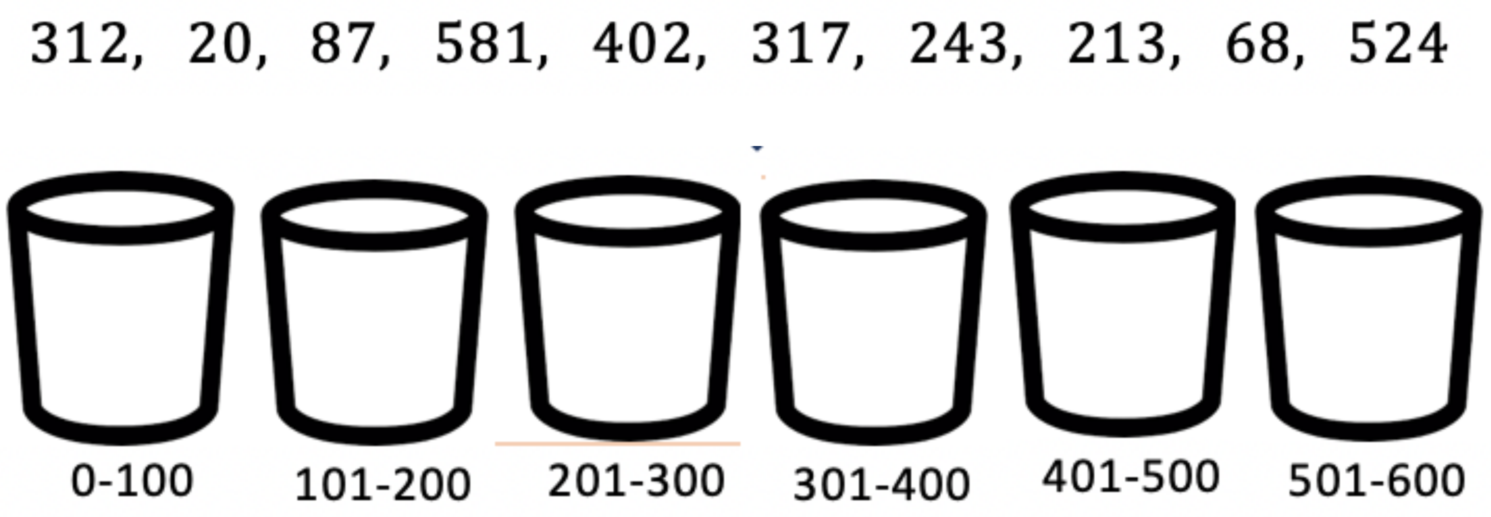
312, 20, 87, 581, 402, 317, 243, 213, 68, 524

Bucket Sort - Algorithm

1. The data elements are distributed into different ordered buckets which can hold data within a defined range.

2. The elements within each bucket are sorted using another sorting algorithm.

3. The sorted list within each of the ordered buckets are simply concatenated to get a bigger sorted list.



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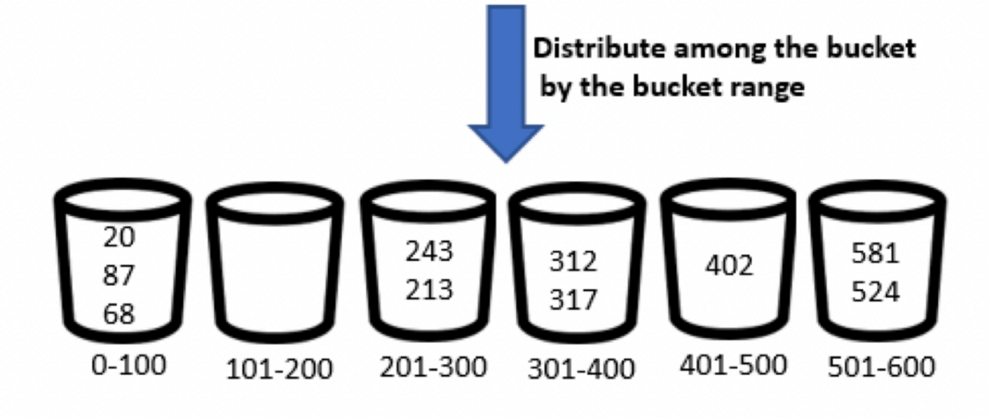

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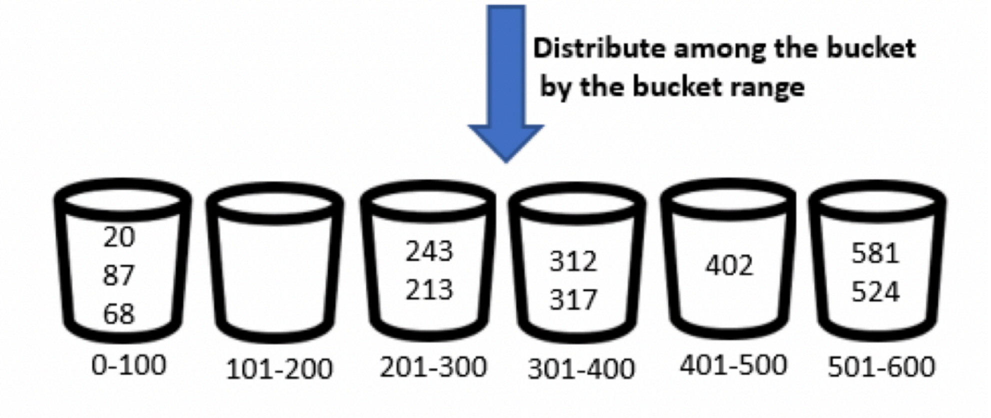

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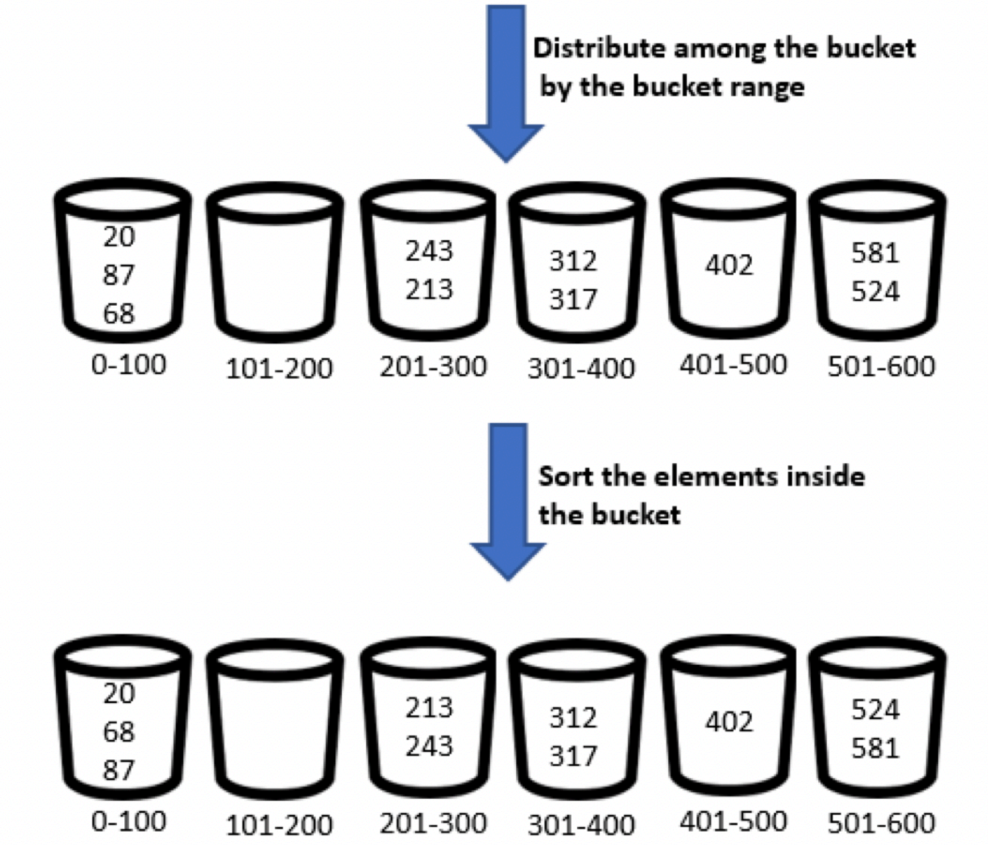

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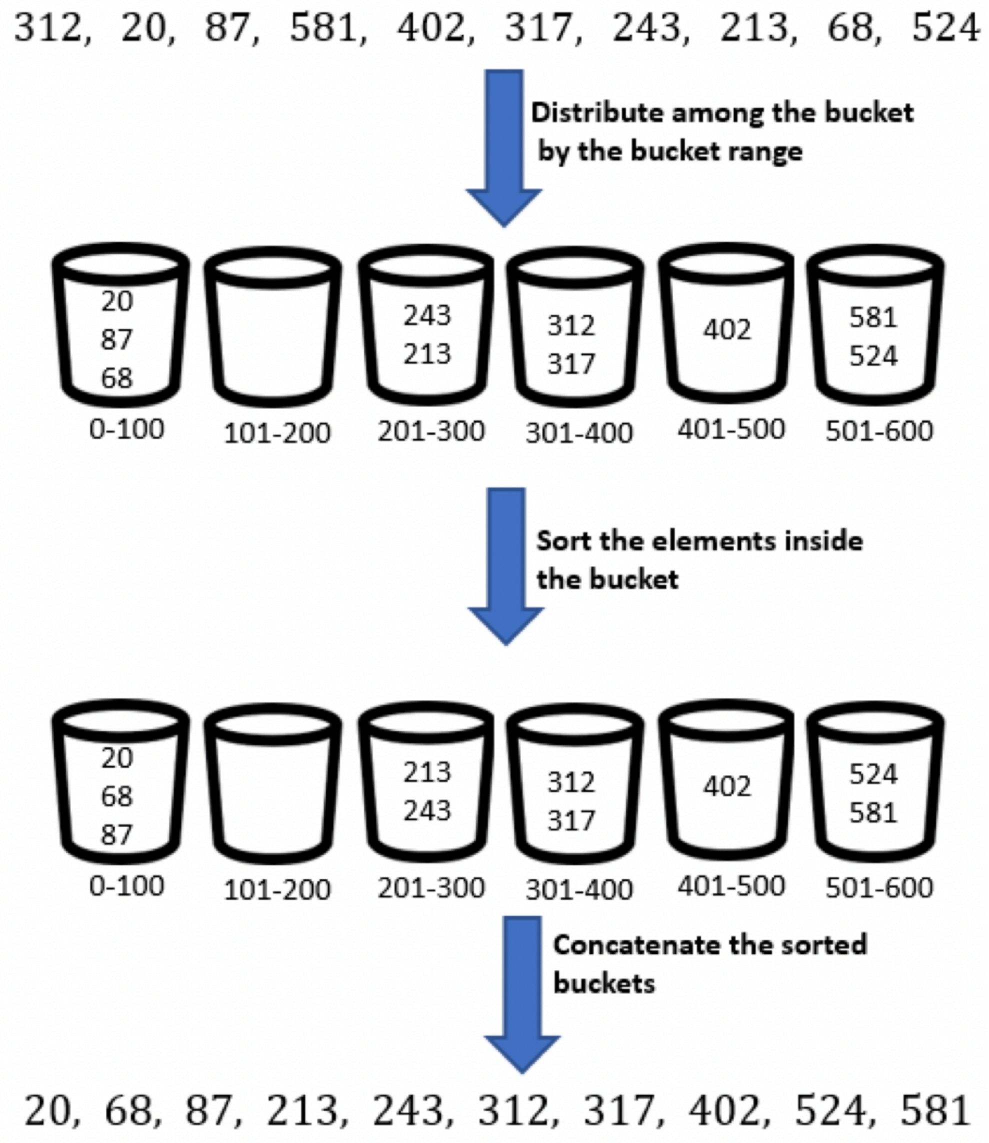

Some of the buckets may be empty or may hold very few data elements.

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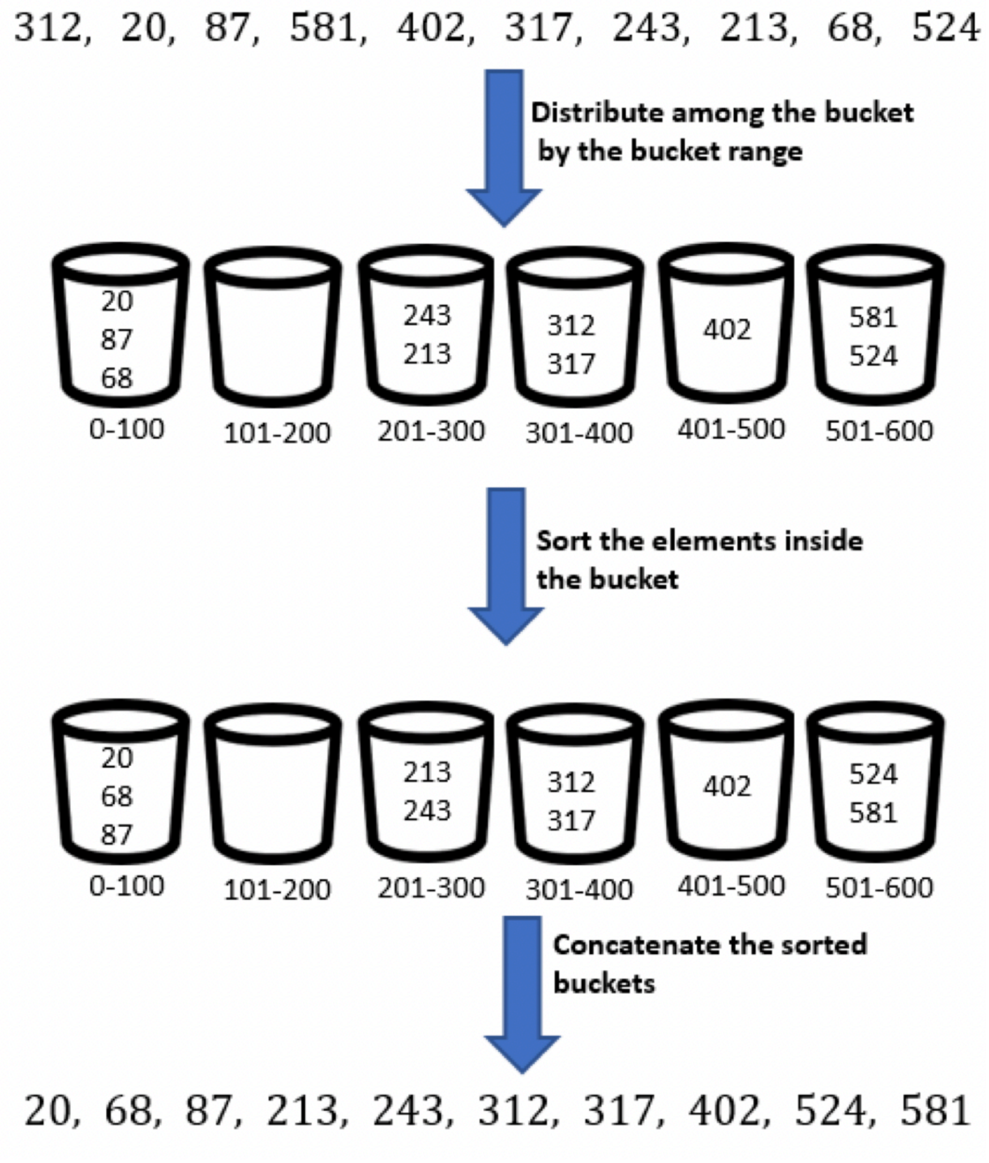
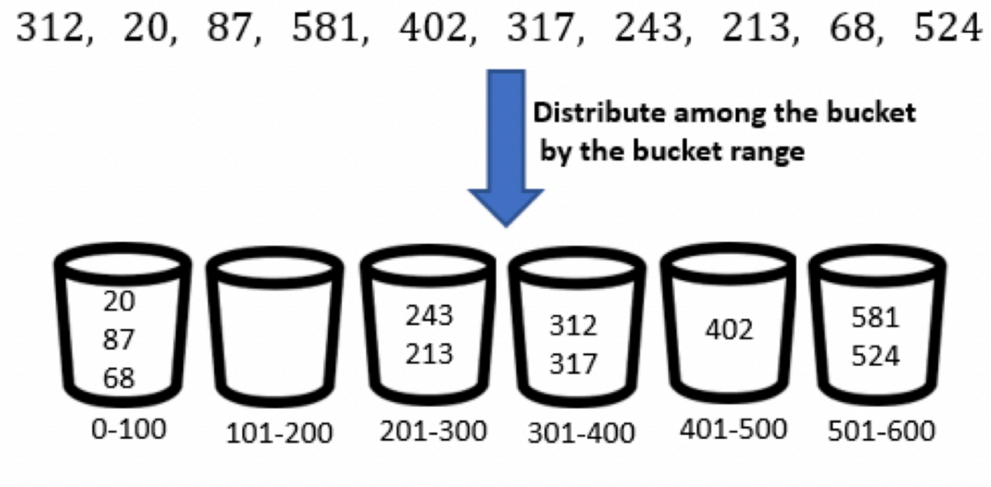


Some of the buckets may be empty or may hold very few data elements.

Time Complexity may depends on several factors such as

- distribution of the elements in the buckets
- Algorithms used for sorting the elements in the buckets
- Number of buckets etc.

Bucket Sort - Algorithm



Bucket Sort - Algorithm

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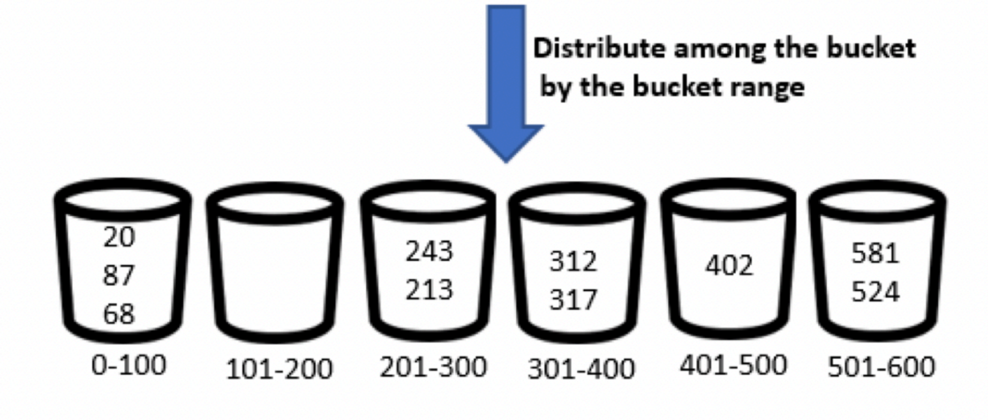
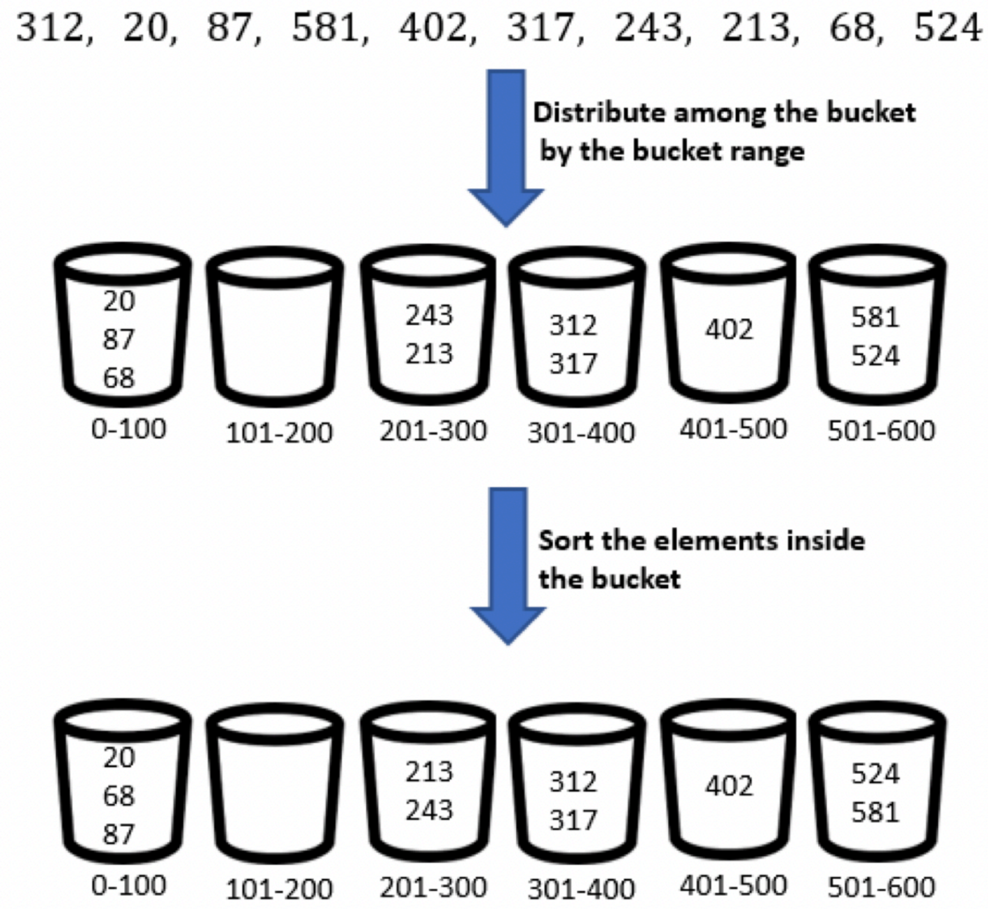


It can be done in $\theta(n)$

Bucket Sort - Algorithm



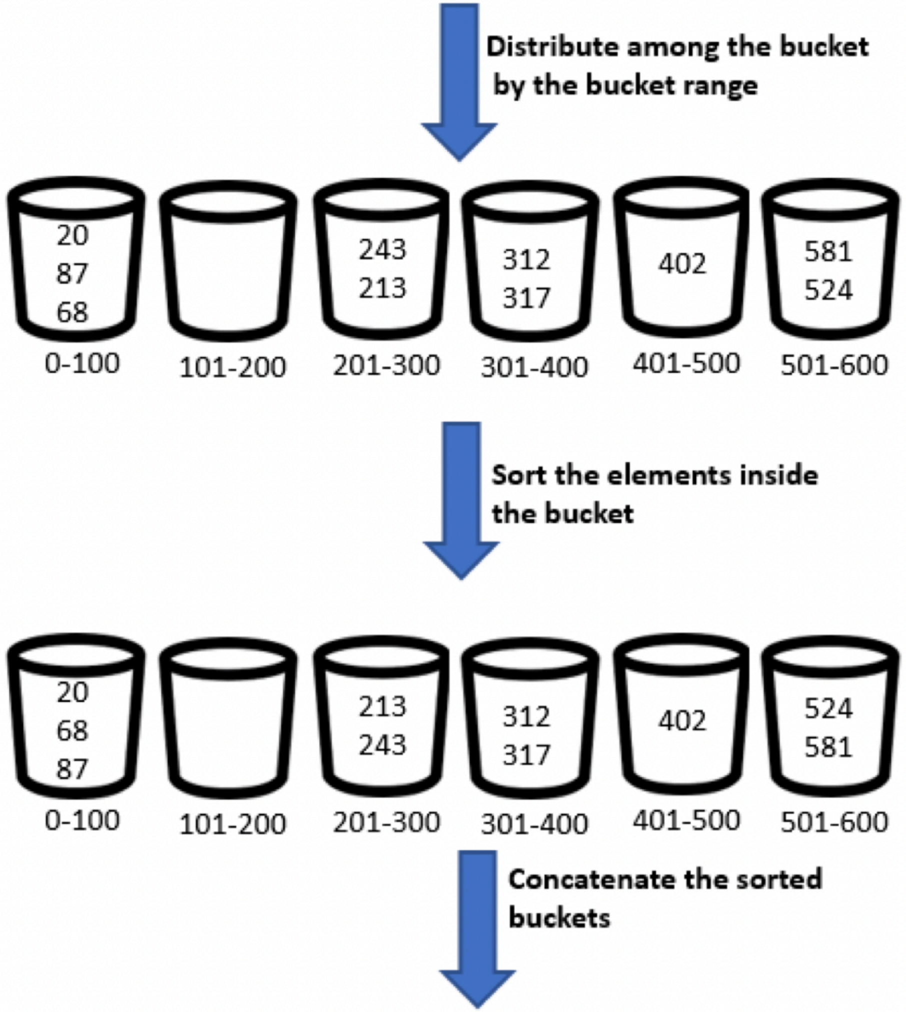
If we assume that the elements are uniformly distributed,

it can be done in $\theta\left(k\frac{n}{k}\right) = \theta(n)$

where k is the number of buckets.

Bucket Sort - Algorithm

312, 20, 87, 581, 402, 317, 243, 213, 68, 524



20, 68, 87, 213, 243, 312, 317, 402, 524, 581

It can be done in $\theta(n)$